Estimating Property Value with Fuzzy Linguistic Logic

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Abstract

The defect of mathematically defined model of property valuation is lack of complete market information. Fuzzy linguistic logic is applied to reduce the subjectivity of the appraiser in determining the weights of qualitative variables. This paper aims to propose an flexible adjustment method. Section 1 describes the objective and the literature for this study. Section 2 proposes the mathematical model for the fuzzy linguistic property valuation. And section 3 gives conclusion and the further research.

Keywords: Property valuation, Fuzzy linguistic logic, Quantification Theory I

1. Introduction

The qualitative attributes of property such as the desirability of the neighborhood, locational accessibility and attractiveness of the community require many different kinds of adjustment methods for valuation. The appraisers face with insufficient information and limited techniques for the reasonable weights for the variables.

Dilmore (1993) applied the fuzzy logic with the expert system and improved the estimates of the different distance effects for the more accuracy of real estate valuation. Dilmore (1994) discussed the comparable sales method of the adjustment techniques which using the fuzzy logic could define a more elastic way of membership to reduce the lack of information. Bagnoli & Smith (1998) demonstrated the application of fuzzy logic an income-producing property, with a resulting fuzzy set output. The fuzzy sets can be combined to produce reasonable conclusions, and inferences can be made, given a specified fuzzy input function.

2. Mathematical model

2.1 Property valuation model with Quantification Theory I

The linear equation form describes the effect of independent qualitative variables on property value. The qualitative variables are constructed by categories which assumed the

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samples will choose a unique category in the explanations variables. The function form is formulated in Equation (1).

$$Y_v = \hat{Y} + \sum_{i=1}^{r}\sum_{j=1}^{p}\hat{b}_j^{(i)}X_j^{(i)} \tag{1}$$

where

$\hat{Y}$: Predictive value of property value Y_v.

$\hat{b}_j^{(i)} = \hat{B}_j^{(i)} - \sum_{j=1}^{p} B_j^{(i)}$

$X_j^{(i)}$: The jth effecting factor with i preferrence level.

i: Preference level.

$j = 1, \ldots, p$.

$\hat{B}_j^{(i)}$: Partial coefficient of the jth independent factor with ith preference level.

The OLS method is applied to estimate the category score $\hat{B}_j^{(i)}$ of the $B_j^{(i)}$. Equation (1) states the standardizations procedure of $\hat{b}_j^{(i)}$, and the category score represents the level of effects from the independent variable on Y_v. Range score is defined as the significance of the independent variable and is measured by the maximum category score minus the minimum one in terms of the preference level. The range score shows the relative importance of the independent variable. The range score is represented by the $R_g^{(j)}$ in Equation (2).

$$R_g^{(j)} = \underset{1\le j\le p}{Max}\ \hat{b}_j^{(i)} - \underset{1\le j\le p}{Min}\ \hat{b}_j^{(i)} \tag{2}$$

The transformation equation of range score of independent variable is applied to estimate the rage difference in terms of the property value. Equation (3) calculates the range ratio of independent variables relative to the property value. [7]

$$L_j = \frac{R_g^{(j)}}{Y_v} \times 100\% \tag{3}$$

where

L_j: Range ratio of the jth independent factor relative to property value.

$R_g^{(j)}$: Range score of the jth independent factor.

Y_v: The property value.

Equation (4) transforms the range score of the jth independent factor into range difference which states by the preference level ($i = 1,\ldots,5$) and represents the maximum adjustment percentage of the independent factor. All of the qualitative variables in terms of the range differences can be established an adjustment table using the preference level of evaluation in practical application.

$$M_j = \frac{L_j}{k-1} \tag{4}$$

where

M_j: Range difference of the jth independent factor.

L_j: Range difference of the jth independent factor.

k: Preference level rank , this study assumes k=5.

Equation (5) is the empirical study function form.

$$\begin{aligned} Y_v = {} & [B_1^{(1)}X_1^{(1)} + B_2^{(1)}X_2^{(1)} + \ldots + B_j^{(1)}X_j^{(1)}] + [B_1^{(2)}X_1^{(2)} + B_2^{(2)}X_2^{(2)} + \ldots \\ & + B_j^{(2)}X_j^{(2)}] + \ldots + [B_1^{(i)}X_1^{(i)} + B_2^{(i)}X_2^{(i)} + \ldots + B_j^{(i)}X_j^{(i)}] + e_v \end{aligned} \tag{5}$$

where

Y_v: Property value of sample v.

$X_j^{(i)}$: Preference level scale describing the jth independen variable.

$$X_j^{(i)} = \begin{cases} i = 1\text{: independent variable } j \text{ states worst.} \\ i = 2\text{: independent variable } j \text{ states inferior.} \\ i = 3\text{: independent variable } j \text{ states medium.} \\ i = 4\text{: independent variable } j \text{ states good.} \\ i = 5\text{: independent variable } j \text{ states best.} \end{cases}$$

j=1,...,p

$B_j^{(i)}$: Partial coefficient of jth independent factor and ith preference level.

e_v: Error term of sample v.

The preference level of evaluation is divided into 5 ranks which states worst, inferior, medium, good and best. This is a qualitative measurement and can not be directly implemented the traditional OLS method owing to the discrete preference level.

2.2 The weightings of comparable variables decision under fuzziness

The comparable variables with implicit qualitative characteristics require various adjustments in property valuation. The fuzzy linguistic logic and professional interview questionnaires are applied for the group decision under fuzziness.

2.2.1 Construction of preference relations in property valuation model - the ill known consequences

Alternative property valuation results are revealed a crisp consequence. However in some situations it is not possible to reach a consensus among experts in the consequence. And a single value is not enough to reflect the diversity of different judgments from several comparable variables.

The crisp preference relation R corresponds to a mapping R: $A \times A \to \{0,1\}$. We define m potential alternatives of a set $A = \{a_1, ..., a_m\}$ results from the aggregation of $(R_1, ..., R_n)$. The preference analysis is conducted on the Cartesian product $A \times A$. A fuzzy constraint is characterized by $G = \{(x, g(x)), x \in X\}$. The membership function $g : X \to [0,1]$.

The range of possible consequence of the estimate for each alternative can be investigated the possibilities of discrimination between these alternatives even they are not completely defined. A possibility distributions in a natural attitude can be represented the ill-consequences. The fuzzy consequence of the alternative a, on a given dimension is the fuzzy set of the evaluation scale X defined by:

$$W_a = \{(x, \pi W_a(x)), x \in X\} \quad (6)$$

where πW_a represents the possibility degree $\pi W_a(x)$ of the punctual event $W_a = x$, such that

$$\sup_{x \in X} \{\pi W_a(x)\} = 1 \quad (7)$$

The attractiveness of a fuzzy consequence relatively to a fuzzy objective function may be evaluated as compatibility between these two fuzzy sets. [6] The compatibility level between a fuzzy consequence W_a and a fuzzy objective G defined on the same scale X can be approximate by the quantities:

$$\prod(G, W_a) = \sup_{x \in X} \min(g(x), \pi W_a(x)) \quad (8)$$

$$N(G, W_a) = \inf_{x \in X} \max(g(x), 1 - \pi W_a(x)) \quad (9)$$

where $\prod(G, W_a)$ and $N(G, W_a)$ are respectively the possibility and necessity of the fuzzy set G relatively to W_a. Equation (8) measure the possibility of the G event relatively to the consequence W_a, and Equation (9) measures the certitude of the G event relatively to consequence W_a. The two equations are possible to state "the alternative a fits into the decision maker's objective" and "the alternative a does not fit into the decision maker's objective". By definition $\mathrm{N}(G, W_a)=1-\prod(G', W_a)$ where $G'=\{(x, 1\text{-g}(x), x \in X\}$ is the complement of the G.

In practical use the attributes contributed the importance to the property value is modeling with Quantification Theory I. Using a qualified response by the individual professional states the effecting scale of the property value is expressed as a linguistic possibility value as 'Very unimportant', 'Unimportant', 'Medium', 'Important' and 'Very important'. We aggregate the possibility and necessity of the fuzzy event W in a single value. Thus the consequence of objective compatibility level by the score is defined as:

$$G^{\alpha}(a)=(1-\alpha)\prod(G, W_a)+\alpha\mathrm{N}(G, W_a) \tag{10}$$

α is a technical parameter and allows performing a convex combination of theses two equations. α =0 represents optimistic evaluation and α =1 reflects more pessimistic evaluation. Parameter α is a degree of prudence when modulating the confidence we have in our evaluation. Criterion G^{α} allows a total preorder to be defined on A. We further replace G^{α} by the interval $[g^{-}(a), g^{+}(a)], a \in A$ bounded by the following compatibility level:

$$\begin{cases} g^{-}(a)=\mathrm{N}(G, W_a) \\ g^{+}(a)=\prod(G, W_a) \end{cases} \tag{11}$$

We assume the interval type is of equal length, and then the triangular membership function can be characterized by the possibility distribution. Table 1 defines the linguistic scale and its consequence of comparability.

Table 1

Linguistic scale and its consequence of comparability

Linguistic scale $W_a(x)$	Consequence of comparability
Very unimportant	(0, 0, 0.25)
Unimportant	(0, 0.25, 0.5)
Medium	(0.25, 0.5, 0.75)
Important	(0.5, 0.75, 1)
Very important	(0.75, 1, 1)

We suppose the evaluation of the preference level for comparable variables is equally distributed and characterized by the following possibility distribution.

$$\pi W_a(x)=\begin{cases}1-4x, & \text{if} \quad 0\le x\le 0.25\\ 0, & \text{if} \quad x>0.25\end{cases}$$

$$\pi W_a(x)=\begin{cases}4x, & \text{if} \quad 0\le x\le 0.25\\ 2-4x, & \text{if} \ 0.25\le x\le 05\\ 0, & \text{otherwise}\end{cases}$$

$$\pi W_a(x)=\begin{cases}4x-1, & \text{if}\ 0.25\le x\le 0.5\\ 3-4x, & \text{if} \ 0.5\le x\le 0.75\\ 0, & \text{otherwise}\end{cases} \tag{12}$$

$$\pi W_a(x)=\begin{cases}4x-2, & \text{if} \ 0.5\le x\le 0.75\\ 4-4x, & \text{if}\ 0.75\le x\le 1\\ 0, & \text{otherwise}\end{cases}$$

$$\pi W_a(x)=\begin{cases}4x-3, & \text{if} \ 0.75\le x\le 1\\ 0, & \text{if} \quad x>1\end{cases}$$

2.2.2 The estimate of the fuzzy linguistic weights

The individual professional states preferences in the questionnaire with the importance level in linguistic possibility value. Equation (13) is used to figure out the fuzzy weights of the effecting factors.

$$W_{as}=\frac{1}{N}\left[n_{s1}\left(0,0,\frac{1}{k-1}\right)+n_{s2}\left(\frac{0}{k-1},\frac{1}{k-1},\frac{2}{k-1}\right)+....+n_{si}\left(\frac{i-2}{k-1},1,1\right)\right] \tag{13}$$

W_{as} : The fuzzy weight of the sth factor.

N : Total samples.

n_{si} : The sth factor with the i linguistic scale.

k : The preference level rank of k.

2.2.3 The order of the weights

The effecting factors weights and membership degree are established and resolved by the order. The fuzzy multiple attributes sorting theory is used to transform the membership function as a crisp number by means of the maximum membership set and minimum membership set. [5]

(1) Membership function of the factors

The maximum membership function is defined as $W_{\max}(x)$ and minimum membership function is defined as $W_{\min}(x)$. The $W_{\max}(x)$ will intersect with the fuzzy weight W_{as} in the right margin and the $W_{\min}(x)$ will also intersect with the fuzzy weight W_{as} in the left margin. W_{as} = (a, b, c) is assumed and represented by (a, 0), (b, 1) and (c, 0). The (a, 0) and (b,1) can figure out the membership $y=\frac{x-a}{b-a}$, (b,1) and (c,0) can figure out the membership $y=\frac{c-x}{c-b}$. Fig.1. demonstrates the result.

$$W_{\max}(x)=\begin{cases} x, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$
$$W_{\min}(x)=\begin{cases} 1-x, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

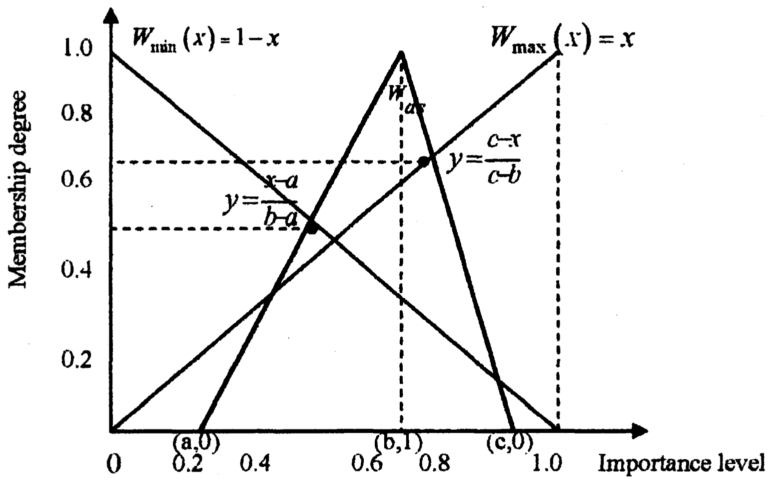

Fig. 1. Illustration of the membership weights sorting process of factors

(2) Right and Left score estimate

The maximum membership in equation (15) resolves the right score. W_{as} is calculated from $y=\frac{x-a}{b-a}$ and $y=\frac{c-x}{c-b}$ represented intersect with $W_{\max}(x)=x$. In other words it represent the alternative s fits into the decision maker's objective is "very true". The solution is $\left(\frac{a}{1+a-b},\frac{a}{1+a-b}\right)$ and $\left(\frac{c}{1+c-b},\frac{c}{1+c-b}\right)$ in the dimension. We decide the higher score of the membership as the W_{as} right score $\mu_R(S)$.

$$\mu_R(S)=\inf_{x\in X}\left[W_{\max}(x)\wedge W_s(x)\right] \tag{15}$$

The minimum membership in Equation (16) resolves the left score. W_{as} is calculated from the intersection of the $W_{\min}(x) = 1 - x$. In other words it represents the alternative s fits into the decision maker's objective is "very false". The solution is $\left(\frac{b}{1+b-a}, \frac{1-a}{1+b-a}\right)$ and $\left(\frac{b}{1+b-c}, \frac{1-c}{1+b-c}\right)$ in the dimension. We decide the higher score of the membership as the W_{as} left score $\mu_L(S)$.

$$\mu_L(S) = \sup_{x \in X}\left[W_{\min}(x) \wedge W_s(x)\right] \tag{16}$$

(3) Medium score definition of the membership

When the right and left score were derived by Equation (15) and (16), Equation (17) was applied to represent the medium score for the several factors fuzzy weights.

$$\mu_T(S) = \frac{\left[\mu_R(S) + 1 - \mu_L(S)\right]}{2} \tag{17}$$

The property valuation model with Fuzzy Quantification Theory I is developed in Equation (18) combined the left score, medium score and right score for the membership of the weights in fuzzy linguistic logic. And the derived results with the fuzzy linguistic logic offer a more flexible adjustment for the qualitative factors.

$$\hat{Y}_v = \sum_{i=1}^{r} \sum_{j=1}^{p} W_s^{(i)} X_j^{(i)} \tag{18}$$

$\hat{Y}_v$: Fuzzy linguistic estimate property value $v(=1, 2, \ldots, p)$.

$W_s^{(i)}$: The ith fuzzy linguistic weights of effecting factors X_j.

$X_j^{(i)}$: Effecting factors $i(=1, 2, \ldots, r)$.

The property valuation takes place in a complex environment where conflicting systems of logic, uncertain and imprecise knowledge and possibly vague preferences have to be considered. The preference modeling used in this study can provide the adjustment table based on multi-valued logic and fuzzy set theory for building the preference level modeling. The property valuation model with Quantification Theory I can also be integrated with the fuzzy linguistic form and give a more flexible adjustment for the appraiser to give not so precise information of the property.

Equation (18) uses the left score, medium score and right score for the membership of the weights. Equation (3) is calculated for the range differences of each factor. The adjustment range can be applied to the practical use in valuation. This results offer a flexible adjustment in linguistic form and a crisp value for the final decision of the property value.

3. Conclusions

The property value is a composite measurement of several different variables. The effecting factors are discussed in many literatures and show different results. The study focuses on the vagueness of the qualitative factors in linguistic form. And the fuzzy linguistic logic can be translated in a reasonable crisp value range for the practical use in the property valuation. The qualitative variables measures are applied in fuzzy linguistic logic. The adjustment by the fuzzy theory can alleviate the uncertain conditions made by human knowledge and lack of information.

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References

[1] Bagnoli C., H. C. Smith, 1998, The theory of fuzz logic and its application to real estate valuation, Journal of Real Estate Research, Vol. 16, No. 2, pp. 169-199.

[2] Chen, S. J., C. L. Hwang, 1992, Fuzzy multiple attribute decision making methods and applications, Springer-Verlag, Germany.

[3] Dilmore, G, 1993, Fuzzy set theory: an introduction to its application for real estate analysts, Paper Presented at the Annual Conference of the American Real Estate Society in Key West, Florida.

[4] Dilmore, G, 1994, Fuzzy logic sequel: clarifying the three approaches to value with fuzzification, Paper Presented at the Annual Conference of the American Real Estate Society in Santa Barbara, Florida.

[5] Liang, G S., M. J. Wang, 1991, A fuzzy multi-criteria decision-making method for facility site selection, International Journal of Production Research, Vol. 29, No. 11, pp. 2312-2330.

[6] ŁOWIŃSKI R., 1998, Fuzzy sets in decision analysis, Operation Research and Statistics, The Handbooks of Fuzzy Sets Series, Kluwer Academic Publishers.

[7] Seikoku F. D. Chuki D. Sei and B. Shinchi., 1999, The residential fixed asset evaluation applied with fuzzy theory, Operations Research, pp.308-316. (in Japanese)