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MOSER'S QUESTION ON A SIMULTANEOUS APPROXIMATION OF A SET OF NUMBERS AND A SIMULTANEOUS NORMAL FORMS OF MAPS

広島大学・理学研究科　　吉野　正史 (Masafumi Yoshino)[1]
Graduate School of Sciences
Hiroshima University

1. Introduction

In the paper [5] J. Moser studied the following problem. Let f_ν, $\nu = 1, \dots, d$ be the germs of commuting holomorphic functions $(\mathbb{C}, 0)$ satisfying

$$(1.1) \qquad f_\nu \circ f_\mu = f_\mu \circ f_\nu, \quad \nu, \mu = 1, \dots, d,$$

$$(1.2) \qquad f_\nu(0) = 0, \quad f_\nu'(0) \equiv \lambda_\nu = e^{2\pi i \alpha_\nu}, \ \nu = 1, \dots, d.$$

We want to seek a holomorphic function $u(z)$ such that

$$(1.3) \qquad u(0) = 0, \ u'(0) = 1, (u^{-1} \circ f_\nu \circ u)(z) = \lambda_\nu z, \ \nu = 1, \dots, d.$$

Following Haeflinger [2] and Banghe -Haeflinger [1] the commuting example appears as a holonomy group of codimension one foliation.

In the case of a single map with $\alpha_1 = \theta$ the following theorem is well known.

Theorem 1.(Siegel) *If there exist $C > 0$ and $\tau > 0$ such that*

$$(1.4) \qquad \|\theta q\| := \inf_{p \in \mathbb{Z}} |\theta q - p| \geq C q^{-\tau}, \forall q \geq 2, q \in \mathbb{Z}$$

there exists a unique holomorphic solution $u(z)$ such that

$$(1.5) \qquad u(0) = 0, \quad u'(0) = 1, \quad u(e^{2\pi i \theta} z) = f(u(z)).$$

The difficult part of the proof of this theorem lies in proving the convergence of the formal power series solution u of the so-called homology equation. The condition (1.4) is a sufficient condition in order to show the convergence of the formal power series solution. On the other hand

[1]Supported by Grant-in-Aid for Scientific Research (No. 11640183), Ministry of Education, Science and Culture, Japan and GNAMPA-INDAM, Italy. E-mail: yoshino@math.sci.hiroshima-u.ac.jp

it is a difficult and interesting problem to find a necessary condition for the convergence. We recall a classical result due to Cremer: if

$$\limsup_{k\to\infty} \frac{1}{d^k} \log \frac{1}{|\lambda^k - 1|} = \infty, \quad d \geq 2, integer \tag{1.6}$$

there exists a divergent formal solution u. We note that the left-hand side is expressed by using a Nevalina function. Therefore it is an interesting problem to understand the convergence without a Siegel condition.

We recall two approaches to this problem. The former one is to weaken the Diophantine condition. The typical one is a so-called Bruno condition: there exist $c > 0$ and $\tau > 0$ such that

$$\|\theta q\| \geq \exp\left(-\frac{cq}{(\log(q+1))^{1+\tau}}\right), \quad q \in \mathbb{Z}_+. \tag{1.7}$$

The latter one is to understand from the viewpoint of the symmetry, $\exists h,\ f \circ h = h \circ f$. Namely, if there exist sufficiently many symmetry then we can linearize our map without a Siegel condition or without any Diophantine condition. This approach is closely related with the work of Moser in [5].

We note that a similar Diophantine phenomena happen in the study of the Goursat problem. This was first noted by J. Leray in [3]. More precisely, let us consider the following Goursat problem.

$$\frac{\partial^2}{\partial s \partial t} u = 0, \quad u|_{s+t=0} = u_1(s), \ u|_{\lambda s + t = 0} = u_2(s), \tag{1.8}$$

where $\lambda \neq 0$ be a complex number and $u_j(s)$ are analytic functions near the origin $s = 0, t = 0$. Here s and t are real or complex variables. The Goursat problem is related with a moving boundary problem for a hyperbolic equation.

A Goursat problem is also related with the Schröder equation as follows. It follows from the equation $\partial_t \partial_s u = 0$ that $u = \exists\phi(t) + \exists\psi(s)$. By the boundary conditions we obtain

$$\phi(-s) + \psi(s) = u_1(s), \quad \phi(-\lambda s) + \psi(s) = u_2(s). \tag{1.9}$$

It follows that

$$\phi(-\lambda s) - \phi(-s) = u_2(s) - u_1(s) \equiv v(-s). \tag{1.10}$$

By setting $s \mapsto -s$ we obtain the Schröder equation

$$\phi(\lambda s) - \phi(s) = v(s). \tag{1.11}$$

It is almost clear that we meet a Diophantine condition if we want to solve (1.11) in a class of analytic functions. Indeed, let

$$\phi(s) = \sum_{n=1}^{\infty} \phi_n s^n, \quad v(s) = \sum_{n=1}^{\infty} v_n s^n$$

be the expansions of ϕ and v, respectively. By inserting the expansions into the equation we obtain

$$\sum_{n=1}^{\infty} \phi_n(\lambda^n - \lambda)s^n = \sum_{n=1}^{\infty} v_n s^n. \tag{1.12}$$

Hence, if $\lambda^n - \lambda \neq 0$ $(n = 1, 2, \dots)$ we can construct a formal solution. As to the convergence of a formal power series solution we need a Diophantine condition.

By a similar argument as in the above we can prove

Theorem 2. (Leray) *If*

$$\rho(\lambda) := \limsup_{k \to \infty} \frac{1}{k} \log \frac{1}{|\lambda^k - 1|} < \infty \tag{1.13}$$

(1.8) has a unique analytic solution for any $u_j(s)$.

We call $\rho(\lambda)$ a Leray -Pisot function. (cf. [4]). The necesssary part is given by

Theorem 3. *(cf. [8]) If* $\rho(\lambda) = \infty$ *then there exist* u_1 *and* u_2 *such that (1.8) has a formal power series solution* u *which does not converge in any neighborhood of the origin.*

Hence it may happen that one can weaken the Cremer's condition for the divergence of a formal power series solution. Leray's result implies us this may be case since Goursat problem is closely related with Schröder's equation, a linearized homology equation.

If we consider the Goursat problem for third order equation we find that the Leray-Pisot function of two variables

$$\rho(\lambda, \mu) := \limsup_{k \to \infty} \frac{1}{k} \log \frac{1}{|\lambda^k - 1| + |\mu^k - 1|} \tag{1.14}$$

plays the same role as $\rho(\lambda)$ in the case of second order equation. In fact, the condition $\rho(\lambda, \mu) > 0$ is necessary and sufficient for the unique local solvability in some neighborhood of the origin for any right-hand side and any boundary conditions, while if $\rho(\lambda, \mu) = 0$ we have a divergence of a formal power series solution.

2. Statement of the results

Simultaneous Diophantine condition. We say that the set of numbers α_j $(j = 1, \dots, d)$ satisfies a simultaneous Diophantine condition if there exist $\exists C > 0$ and $\exists \tau > 0$ such that

$$\max_{\nu=1,\dots,d} \|q\alpha_\nu\| \geq Cq^{-\tau}, q = 1, 2, 3, \dots, \tag{2.1}$$

where

$$\|q\alpha_\nu\| = \min_{p \in \mathbb{Z}} |q\alpha_\nu - p|.$$

This condition is weaker than the so-called simultaneous Siegel condition:

$$\exists C >, \exists \tau >; \|q\alpha_\nu\| \geq Cq^{-\tau}, \nu = 1, \dots, d, q = 1, 2, \dots. \tag{2.2}$$

We say that β is a Liouville number if, for every $\lambda > 0$ there exist infinitely many integers $q \in \mathbb{Z}$ such that

$$0 < \|q\beta\| < q^{-\lambda}. \tag{2.3}$$

Moser's question. Given the germs of commuting holomorphic functions $(\mathbb{C}, 0)$, $f_\nu(z), \nu = 1, \dots, d$ satisfying (1.1) and (1.3). We consider

$$f(z) := f_1(z)^{g_1} \circ \dots \circ f_d(z)^{g_d}, \quad g_1, \dots, g_d \in \mathbb{Z}. \tag{2.4}$$

Suppose that α_j $(j = 1, \dots, d)$ satisfy the simultaneous Diophantine condition. Then Moser asked whether there exist $g_1, \dots, g_d \in \mathbb{Z}$ such that $f(z)$ satisfies a Diophantine condition. If this is the case, the linearization problem in a commuting case is reduced to the case of a single map, hence to Siegel's theorem. The answer to this question is negative. In fact, Moser proved:

Theorem 4. (Moser) *For $d \geq 2$ and a given $\tau > 2/(d-1)$ there exists a set of cardinality of $(\alpha_1, \dots, \alpha_d) \in \mathbb{R}^d$ such that the simultaneous Diophantine condition holds, but such that, for all $g = (g_1, \dots, g_d) \in \mathbb{Z}^d \setminus 0$*

$$r := g_1\alpha_1 + \dots + g_d\alpha_d$$

are Liouville numbers (i.e., non Diophantine).

In [5], Moser raised the question whether this theorem can be extended to case where α_j $(j = 1, \dots, d)$ are n-dimensional vectors, $\alpha_j = (\alpha_{j,1}, \dots, \alpha_{j,n})$. More precisely we consider a commuting system of maps

$$f_\nu : (\mathbb{C}^n, 0) \longrightarrow (\mathbb{C}^n, 0), f_\nu(z) = A_\nu z + O(z^2), \nu = 1, \dots, d. \tag{2.5}$$

Let λ_j^ν, $(j=1,\dots,n)$ be the eigenvalues of A_ν with multiplicity, $(\nu = 1,\dots,d)$. We write

$$\lambda_j^\nu = \exp(2\pi i \theta_j^\nu), \quad 0 \le \theta_j^\nu \le 1, \tag{2.6}$$

and set $\theta^\nu = (\theta_1^\nu,\dots,\theta_n^\nu)$. We define

$$\langle \alpha, \theta^\nu \rangle := \sum_{j=1}^{n} \alpha_j \theta_j^\nu, \quad \alpha = (\alpha_1,\dots,\alpha_n) \in \mathbb{Z}^n. \tag{2.7}$$

We say that $\{\theta^\nu\}_{\nu=1}^d$ satisfies a simultaneous Diophantine condition if there exist $C>0$ and $\tau>0$ such that

$$\min_{k=1,\dots,n} \sum_{\nu=1}^{d} \|\langle \alpha, \theta^\nu\rangle - \theta_k^\nu\| \ge C|\alpha|^{-\tau}, \quad \forall |\alpha| \ge 2, \alpha \in \mathbb{Z}_+^n, \tag{2.8}$$

where $\|t\| = \inf_{p\in\mathbb{Z}} |t-p|$.

Let $p_\nu \in \mathbb{Z}$, $(\nu = 1,\dots,d)$ and set

$$\delta_j = \sum_{\nu=1}^{d} \theta_j^\nu p_\nu, \quad \delta = (\delta_1,\dots,\delta_n). \tag{2.9}$$

We say that δ is a Liouville vector, if for every $\lambda > 0$ the inequality

$$0 < \min_{k=1,\dots,n} \|\langle \alpha, \delta\rangle - \delta_k\| < |\alpha|^{-\lambda} \tag{2.10}$$

holds for infinitely many $\alpha \in \mathbb{Z}_+^n$. Note that δ gives the eigenvalues of a map $f = f_1^{p_1} \circ \cdots \circ f_d^{p_d}$. Then we have

Theorem 5. *Suppose that $d > n \ge 2$. Then there exists a set of linearly independent vectors $\theta_j = (\theta_j^1,\dots,\theta_j^d)$ $(j=1,\dots,n)$ with the density of continuum satisfying a simultaneous Diophantine condition for which, for any $p = (p_1,\dots,p_d) \in \mathbb{Z}^d \setminus 0$ the $\delta = (\delta_1,\dots,\delta_n)$, $\delta_j = \sum_{\nu=1}^d \theta_j^\nu p_\nu$ is a Liouville vector.*

We note that $f_\nu(z)$, $\nu = 1,\dots,d$ satisfies a simultaneous Diophantine condition while, for any $p = (p_1,\dots,p_d) \in \mathbb{Z}^d$ $f := f_1^{p_1} \circ \cdots \circ f_d^{p_d}$ does not satisfy a Diophantine condition.

3. Sketch of the proof

We will give the sketch of the proof of Theorem 5. We need lemmas in [5]. (For the detail, see [5]). Let $E^n \subset \mathbb{R}^d$ be a real subspace in $\mathbb{R}^d$. With the standard Euclidean norm $|\cdot|$ in $\mathbb{R}^n$ we define

$$\operatorname{dist}(x, E^n) = \min_{y\in E^n} |x-y|, \qquad x \in \mathbb{R}^n.$$

Definition. *We define $\mu := \mu(E^n)$ as the supremum of the numbers λ for which*

$$\text{(3.1)} \qquad dist(j, E^n) < |j|^{-\lambda}, \qquad j \in \mathbb{Z}^d$$

possesses infinitely many solutions. Here $\mu = \infty$ is admitted.

Clearly, the definition is independent of the norm. Note that, if $\mathbb{Z}^d \cap E^n = \{0\}$ and $\tau > \mu$ then there exists a positive constant c such that

$$\text{(3.2)} \qquad \operatorname{dist}(j, E^n) \geq c|j|^{-\tau}, \quad \text{for all } \ j \in \mathbb{Z}^d \setminus \{0\}.$$

A subspace E^n satisfying $\mathbb{Z}^d \cap E^n = \{0\}$ and (3.2) is called a Diophantine subspace with respect to $\mathbb{Z}^d$. The following theorem is given in Moser [Theorem 2.1, 5]. (See also [6]).

Theorem. *For almost all E^n in the Grassmann manifold $G_n(\mathbb{R}^d)$ one has $\mu(E^n) = \frac{n}{d-n}$.*

Proof of Theorem 5. Let us assume that there exists a subspace E^n in $\mathbb{R}^d$ generated by the linearly independent vectors $\theta_j = (\theta_j^1, \dots, \theta_j^d)$, $(j = 1, \dots, n)$ such that $\mu(E^n) = \frac{n}{d-n}$. Let τ be such that $\tau > \frac{n}{d-n}$. Then we have (3.2). We consider the left-hand side of (2.8)

$$\text{(3.3)} \qquad \min_{1 \leq k \leq n} \sum_{\nu=1}^{d} \|\langle \alpha, \theta^\nu \rangle - \theta_k^\nu\| = \min_{1 \leq k \leq n} \sum_{\nu=1}^{d} \inf_{p_\nu \in \mathbb{Z}} |\langle \alpha, \theta^\nu \rangle - \theta_k^\nu - p_\nu|.$$

We set

$$y = y_k = (\langle \alpha, \theta^\nu \rangle - \theta_k^\nu)_{\nu\downarrow 1,\dots,d} \in E^n, \ k = 1, \dots, n.$$

Let $j = (p_\nu)_{\nu\downarrow 1,\dots,d} \in \mathbb{Z}^d$ be a multiinteger for which the infimum in the right-hand side of (3.3) is taken. Then the right-hand side of (3.3) is bounded from the below by $c_1 \min_{1 \leq k \leq n} |j - y_k|$ for some positive constant c_1 independent of j and k. By the inequality $|j - y_k| \geq \operatorname{dist}(j, E^n)$ for $k = 1, \dots, n$ and (3.2) we can estimate the right-hand side of (3.3) from the below in the following way

$$\text{(3.4)} \qquad \geq c_1 \min_{1 \leq k \leq n} |j - y_k| \geq c_1 \operatorname{dist}(j, E^n) \geq c_2 |j|^{-\tau},$$

for some positive constant c_2 independent of j. Because the infimum in (3.2) is taken for j such that $|j - y_k| \leq M|y_k|$ for some constant M independent of k, we obtain, by the condition $|\alpha| \geq 2$

$$|j| \leq (1 + M)|y_k| \leq c'(1 + |\alpha|) \leq c''|\alpha|$$

for some positive constants c' and c''. It follows that the right-hand side of (3.3) is bounded from the below by $c|\alpha|^{-\tau}$ for some positive constant c independent of α. This proves (2.8).

We want to show that there exists E^n satisfying $\mu(E^n) = \frac{n}{d-n}$ and the Liouville property (2.10) for any $p = (p_1, \dots, p_d) \in \mathbb{Z}^d \setminus 0$. For the detail we refer to [10].

4. Commuting system of vector fields

In the case of a commuting vector fields the situation is completely different from the case of maps. For the sake of simplicity, let us consider a system of holomorphic commuting system of vector fields $\mathcal{X}_\nu$ $(\nu = 1, \dots, d)$, $[\mathcal{X}_\nu, \mathcal{X}_\mu] = 0$ $(\nu, \mu = 1, \dots, n)$ which are singular at the origin. With a standard coordinate in $\mathbb{C}^n$ we write $\mathcal{X}_\mu = \sum_{j=1}^n X_j^\mu(x)\partial_{x_j}$ $(\mu = 1, \dots, d)$. Define $X^\mu := (X_1^\mu, \dots, X_n^\mu)$ and $\Lambda^\mu = \nabla_x X^\mu(0)$. Note that $x\Lambda^\mu$ is the linear part of X^μ. We assume that $\mathcal{X}$ is singular at the origin. Hence we can write

(4.1)
$$X^\mu(x) := X^\mu = (X_1^\mu(x), \dots, X_n^\mu(x)) = x\Lambda^\mu + R^\mu(x), \qquad 1 \le \mu \le d,$$

where $R^\mu(x)$ is analytic in x in some neighborhood of the origin such that

$$R^\mu(0) = \partial_x R^\mu(0) = 0, \qquad 1 \le \mu \le d. \tag{4.2}$$

Let λ_j^μ $(j = 1, \dots, n, \mu = 1, \dots, d)$ be the eigenvalues with multiplicities of Λ^μ. We set $\lambda^\mu = (\lambda_1^\mu, \dots, \lambda_n^\mu)$, $(\mu = 1, \dots, d)$. For a multiinteger $\alpha = (\alpha_1, \dots, \alpha_n) \in \mathbb{Z}_+^n$ we set $\langle \lambda^\nu, \alpha \rangle = \sum_{j=1}^n \lambda_j^\nu \alpha_j$ and define

$$\omega(\alpha) = \min_{1 \le j \le n} \sum_{\nu=1}^d |\langle \alpha, \lambda^\nu \rangle - \lambda_j^\nu|. \tag{4.3}$$

Definition. *We say that $\mathcal{X} := \{\mathcal{X}_\nu; \nu = 1, \dots, d\}$ is non simultaneously resonant if $\omega(\alpha) \neq 0$ for all $\alpha \in \mathbb{Z}_+^n$, $|\alpha| \ge 2$. The set of $\alpha \in \mathbb{Z}_+^n$, $|\alpha| \ge 2$ such that $\omega(\alpha) = 0$ is called a simultaneous resonance of $\mathcal{X}$.*

Definition. *Let ω_k $(k = 2, 3, \dots))$ be given by*

$$\omega_k = \inf\left\{\omega(\alpha); \omega(\alpha) \neq 0,\ \alpha \in \mathbb{Z}_+^n, 2 \le |\alpha| < 2^k\right\}. \tag{4.4}$$

We say that the system $\mathcal{X}$ satisfies a simultaneous Siegel condition, a simultaneous Bruno type condition and a simultaneous Bruno condition respectively if,

$$\omega_k \ge C(1 + 2^k)^{-\tau},$$

$$\omega_k \ge \exp(-C2^k/(k+1)^{1+\tau}),$$

for some constants $C > 0$ and $\tau > 0$ independent of k, and

$$-\sum_{k=2}^{\infty} ln\, \omega_k/2^k < \infty.$$

In the case $d = 1$ we say that the vector field $\mathcal{X} = \mathcal{X}_1$ satisfies a Siegel condition, a Bruno type condition and a Bruno condition, respectively if the corresponding simultaneous condition is verified. Then we have

Theorem 6. *The system $\mathcal{X}_\nu$ ($\nu = 1, \dots, d$) satisfies one of a simultaneous Siegel condition, a simultaneous Bruno condition and a simultaneous Bruno type condition if and only if there exist numbers $c_\nu (\nu = 1, \dots, d)$ such that the following conditions are satisfied:*
(i) the vector field $\mathcal{X}_0 := \sum_{\nu=1}^{d} c_\nu \mathcal{X}_\nu$ satifies a Siegel condition, a Bruno condition and a Bruno type condition, respectively.
(ii) the resonance of $\mathcal{X}_0$ coincides with the simultaneous resonance of the system $\mathcal{X}_\nu$ ($\nu = 1, \dots, d$).

We note that the case of vector fields shows a sharp contrast to that of maps. Because we can choose a Diophantine vector field from the Lie algebra generated by a system of vector fields if the given system satisfies a simultaneous Diophantine condition.

5. Sketch of the proof

We will give a sketch of the proof of Theorem 6. We will show the necessity of (i) and (ii). We note that the commutativity of $\mathcal{X}_\nu$ implies that the linear parts of $\mathcal{X}_\nu$ are pairwise commuting. Without loss of generality we may assume that the linear part A_1 of $\mathcal{X}_1$ is put in a Jordan normal form.

Let $c_1, \dots, c_d$ be complex numbers. By the commutativity, the eigenvalues of the linear part of $\mathcal{X}_0 := \sum_{\nu=1}^{d} c_\nu \mathcal{X}_\nu$ are given by $\sum_{\nu=1}^{d} c_\nu \lambda_j^\nu$ ($j = 1, \dots, n$). For $c = (c_1, \dots, c_d) \in \mathbb{C}_+^d$ and $\alpha \in \mathbb{Z}_+^n$ we define

$$\Omega(\alpha, c) = \min_{1 \le j \le n} \left| \sum_{\nu=1}^{d} c_\nu (\langle \alpha, \lambda^\nu \rangle - \lambda_j^\nu) \right|. \tag{5.1}$$

Let $\omega(\alpha)$ and ω_k be given by (4.3) and the definition in the above, respectively. Then we define

$$\begin{aligned} A_k &= \{c = (c_1, \dots, c_d) \in \mathbb{C}_+^d ; \exists \alpha \in \mathbb{Z}_+^n, 2 \le |\alpha| < 2^k \\ &\qquad \text{such that } \omega(\alpha) \neq 0,\ \Omega(\alpha, c) < 2^{-nk-k} \omega_k \}. \end{aligned} \tag{5.2}$$

We can easily show that the Lebesgue measure of the set $A := \overline{\lim}_{k\to\infty} A_k$ is equal to zero. Therefore, if $c \notin A$ there exists $k_0 \geq 1$ such that

$$\Omega(\alpha, c) > \omega_k 2^{-nk-k}, \quad \forall k \geq k_0.$$

This proves that $\mathcal{X}_0$ satisfies a Siegel, a Bruno type and a Bruno condition, respectively.

In order to show (ii) we note that if α is not in a simultaneous resonance set of $\mathcal{X}_\nu$ $(\nu = 1, \ldots, d)$, the set of $c \in \mathbb{C}^n$ such that $\sum_{\nu=1}^{d} c_\nu(\langle\alpha, \lambda^\nu\rangle - \lambda_j^\nu) = 0$ is a hyperplane for each j. The Lebesgue measure of the sum of these hyperplanes is zero. By adding A to the sum of these hyperplanes we can choose $c \notin A$ such that the resonance of $\mathcal{X}_0$ is equal to the simultaneous resonance of $\mathcal{X}_\nu$ $(\nu = 1, \ldots, d)$.

We will prove the sufficiency. We define $\tilde{\omega}(\alpha)$ by

$$\tilde{\omega}(\alpha) = \min_j |\langle\alpha, \sum_\nu c_\nu \lambda^\nu\rangle - \sum_\nu c_\nu \lambda_j^\nu|.$$

We also define $\tilde{\omega}_k$ by (4.4) with $\omega(\alpha)$ replaced by $\tilde{\omega}(\alpha)$. We can easily show that $\tilde{\omega}(\alpha) \leq M\omega(\alpha)$ for some $M > 0$ independent of α. It follows from the assumption (ii) that $\tilde{\omega}_k \leq M\omega_k$. This implies that if $\mathcal{X}_0$ satisfies a Siegel condition (or Bruno type condition) the system $\mathcal{X}$ also satisfies a simultaneous Siegel and Bruno type condition, respectively. Now, let us assume that $\mathcal{X}_0$ satisfies a Bruno condition. Because $\ln \tilde{\omega}_k < \ln M + \ln \omega_k$, it follows that $-\sum_k \ln \tilde{\omega}_k/2^k > -\sum_k(\ln M + \ln \omega_k)/2^k$. Hence $\mathcal{X}$ satisfies a simultaneous Bruno condition. This ends the proof.

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Present adress: Graduate School of Sciences, Hiroshima University, Higashi-Hiroshima, 739-8526, Japan. e-mail: yoshino@math.sci.hiroshima-u.ac.jp
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