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Why Mathematics in Ancient China?*

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1. Introduction

Since the beginning of the last century hundreds of scholars have devoted themselves to the discipline of the history of mathematics in China. Two approaches to the history of mathematics in China have been made, namely, discovering *what* mathematics was done and recovering *how* mathematics was done, respectively. Both approaches, however, focus on the same problem, that is mathematics in history.[1]

It is often said that, compared with Greek mathematics, Chinese mathematics was characterized by a practical tradition. Many scholars hold that this tradition is the fatal weakness of Chinese mathematical science, one that prevented it from developing into modern science. Some historians of mathematics have argued that certain fundamental factors of the Greek theoretical tradition, such as *proof* and *principle*, can also be detected in the *Nine Chapters of Arithmetic* (《九章算术》, 1 century BC) and Liu Hui's *Annotations* (《刘徽注》, 263 AD). However, it seems to us that many people are still not convinced.

For a better understanding of the value of Chinese mathematics from a historical perspective, we need to figure out the issue of *why mathematics* was done in ancient civilizations. Following the topic of *why mathematics*, researches would be shifted, to some extent, from the mathematics in history to the history of mathematics. Under these circumstances, mathematics in ancient China and other old civilizations will be placed in the whole history of mathematics. The diversity of mathematics in different civilizations would make us a more distinct picture of the history of mathematics.

In this article, we will try to explore the reason why a practical tradition of mathematics was chosen in ancient China.

2. The Aim of mathematical science in ancient China

2.1 "Royal science" and professional scientists

Royal science is the science that was controlled and encouraged by emperors. Of the exact sciences, mathematical astronomy was the only subject that attracted a great attention of rulers in ancient and medieval China.

Mathematical astronomy was an art related to the calendar-making system in ancient periods. In every dynasty, the Royal Observatory 司天监 was an indispensable part of the state. Three

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kinds of expert, mathematicians 算学家, astronomers 天文家 and astrologers 太乙家, were employed as professional scientists by the emperor. Those who were called mathematicians took charge of establishing the algorithms of the calendar-making system; those who were classified as astronomers dealt with the astronomical instruments and observation. Astrologers made divination of the supposed influences of the stars and planets on human affairs and terrestrial events. None of them made researches in pure mathematics.

All of these professional scientists were royal officers. Biographies of the leading scholars who worked in the Royal Observatory were recorded in the official history of each dynasty. Among them are Zu Chongzhi (祖冲之，429-500) who compiled the *Daming calendar-making system* 《大明历》 in 463 AD, Yi-xing (一行，683-727) who compiled the *Dayan calendar-making system* 《大衍历》 in 724 AD and Guo Shoujing (郭守敬，1231-1316) who compiled the *Shoushi calendar-making system* 《授时历》 in 1280 AD.

On the contrary, even the most outstanding mathematicians were ordinary people, such as Liu Hui (刘徽，fl. 3rd century, see *DSB*, v.8, pp.418-425), Li Ye (*Li Chih*, 李冶，1192-1279, see *DSB*, v.8, pp.313-320), Qin Jiushao (*Ch'in Chiu-shao*, 秦九韶，1202-1261, see *DSB*, v.3, pp.249-256,), Yang Hui (杨辉，fl. 13th century, see *DSB*, v.14, pp.538-546) and Zhu Shijie (*Chu Shih-chieh*, 朱世杰，fl.13th century, see *DSB*, v.3, pp.265-271).[2] Few facts are known about them from the historical records except their mathematical works. [3-4]

2.2 Mathematics for mathematical astronomy

It is said that there were two peaks of traditional Chinese mathematics. The *Nine Chapters of Arithmetic* (c. 1st Century BC) and its *Annotations* (263 AD) by Liu Hui featured the first one. Qin J., Li Y., Yang H. and Zhu S. belonging to the same generation formed another splendid peak in the 13th century.

Basic topics of mathematics related to civil life were main theme in time of the *Nine Chapters* and Liu Hui. But in the 12th and 13th centuries, mathematics was highly developed for the purpose of application of mathematics to astronomy. For instances:

Indeterminate problem. Found in Qin Jiushao's book, the Chinese Remainder Theorem is an algorithm for dealing with a set of linear congruence. It is related with the problem to seek for the superior epoch —— a special moment when the earth, the moon and the five planets gather near the point of winter solstice, and usually with other conditions. A set of linear congruence to find such an epoch was established.

Numerical solution of algebraic equations, known as Ruffini-Horner method. In order to transform the length of arc on a circle to its corresponding chord, a polynomial equation of the 4th order was established by Guo Shoujing in the *Shoushi calendar-making system* (1280 AD). It was the highest order algebraic equation that appeared in traditional Chinese mathematical astronomy.

Interpolation and series summation. The formula for higher order series summation is found in Zhu Shijie's work. Series summation is related with finite differences. Interpolation functions from Liu Zhuo's 刘焯 quadratic (600 AD) to Guo Shoujing's cubic (1280 AD) were constructed by finite differences.

2.3 Mathematical astronomy under imperial authority

Compilation and promulgation of calendar symbolized the imperial authority in ancient China. The calendar of a dynasty had to be replaced by the new dynasty. Ordinary people were strictly prohibited to construct a calendar-making system. They were not even allowed to acquire the knowledge of mathematical astronomy.

The technical details of calendar-making were kept secret to public, with the result that nobody knew how a traditional Chinese calendar-making system was compiled after the tradition was discontinued by the impact of Jesuit science in the Ming and Qing dynasties (16-19 century).

Calendar-makers were asked to maintain a high precision in prediction. It was judged by the calculated positions of the celestial bodies and the predictions of astronomical phenomena, such as eclipse. It often happened that a calendar-making system was replaced by another one simply because it failed to predict a solar eclipse. In the past 2000 years, more than 100 systems appeared.

Competition among calendar-makers did not concern cosmic or geometric model but numerical method. The former might tell one how a celestial body moves, the topic that people did not really care about. People were interested in the latter simply because it could tell them where a celestial body was. Thus few people paid much attention to cosmic model building except some of those who worked at the Royal Observatory.

In ancient China, most mathematicians were trained as calendar-makers. Mathematics, except those which astrologers were interested in, was never a part of philosophy. It was developed for mathematical astronomy besides for ordinary application.

The aim of calendar-makers and mathematical astronomy was accurate prediction but not cosmic model building. In this way, a practical tradition to mathematics was formed in ancient China.

3. Practical tradition: evolution of numerical method

In his *Qianxiang calendar-making system* (乾象历，206AD), linear interpolation was employed by Liu Hong 刘洪 to calculate the equation of center of the moon after the irregular lunar motion was discovered in the 2nd century. His method made the change of the moon's velocity from a level line to a step like pattern.

By the end of the 6th century, after Zhang Zixin 张子信 found that the apparent motion of the sun was irregular, linear interpolation was applied to the calculation of the equation of center of the sun in several calendar-making systems. The piecewise linear interpolation made the mean solar motion presented by a level line be replaced by 24 step like lines in a tropical year that was divided into 24 parts (*qi*).

It was Liu Zhuo (in 600 AD) who changed the pattern of the sun's apparent velocity from step like lines to a slant line. Thus the linear method was replaced by the piecewise quadratic interpolation. Liu Zhuo's method was used so extensive that it appeared in everywhere it could be used afterward.

One may image from his intuition that the higher the order of an interpolation function is, the higher the precision is. Unfortunately, it is far from the truth. The fact is that the higher the order of an interpolation function is, the bigger the amplitude of vibration around the points of interpolation is. It is called the *Runge Phenomenon* in mathematics texts. In practice, the lower order piecewise interpolation is much more welcome than a higher order one, say the fourth or higher.

Although the piecewise parabolic interpolation is such an important numerical algorithm which is still widely used in modern science, the problem is how to determine the partition properly since we do not know what its original function is. On the one hand, too many small pieces made by partition would be inconvenient for application. On the other hand, if the length of the piece made by partition is too wide, the interpolation function could not be sufficiently precise to its original one.

In order to solve such a problem, a new algorithm was invented by Bian Gang 边冈 in his *Chongxuan calendar-making system* (崇玄历，892 AD). His method constitutes a major innovative concept, even by the standards of modern computational mathematics. We name it *piecewise iterated quadratic interpolation.*[5]

Since Liu Zhuo's quadratic interpolation was constructed with the aid of a geometric model under an astronomical background, the problem is that no similar geometric model exists for constructing a higher order interpolation following in Liu's steps. That hindered Chinese mathematicians from going further for hundreds of years.

In 1280 AD, a purely algebraic method was made use to transform the third-order problem to second-order by Wang Xun 王恂 and Guo Shoujing in their *Shoushi li*《授时历》. This idea with which a cubic interpolation was constructed was just that which people employ to construct Newton's interpolation. [6]

Ceaseless efforts to improve the numerical method were made in order to guarantee that the algorithm could satisfy the precision of astronomical observation. Polynomial interpolation was by no means the only numerical method which was used by calendar-makers in ancient China. Compound functions and finite difference tables, for instance, were also found in the Chinese calendar-making system.[7]

Table 1. A chronological table for the evolution of interpolation in China

Time	Scientists	Events	Source
2^{nd} century	LI Fan 李梵 & SU Tong 苏统	Irregular lunar motion.	*The Han History* 续汉书律历志
+206	LIU Hong 刘洪	Linear interpolation	*The Qianxiang li* 乾象历
c. +560	ZHANG Zixin 张子信	Irregular solar and planetary motions	*The Sui History* 隋书天文志
+600	LIU Zhou 刘焯	Parabolic interpolation for points at equal intervals	*Thr Huangji li* 皇极历
+724	Yi-xing 一行	Parabolic interpolation for points at unequal intervals	*The Dayan li* 大衍历
+892	BIAN Gang 边冈	Piecewise iterated parabolic interpolation	*The Chongxuan li* 崇玄历
+1280	GUO Shoujing 郭授敬 & WANG Xun 王恂	Cubic interpolation	*The Shoushi li* 授时历

It was neither necessary nor possible that a geometric model could replace the numerical method which occupied the principal position in the Chinese calendar-making system. The reason was that it was only the numerical method that satisfied the ruler's aim: high accuracy in prediction and computation. As the subject closely related to numerical method, algebra, instead of geometry, became the most developed field of mathematics in ancient China.

4. Evolution of planetary theory

The planetary theory has occupied an important position in the history of science. It is said that there would be no modern science without the planetary theory. The Copernican revolution was the revolution of the planetary theory from Ptolemy's geocentric model to heliocentric. Great attentions have been paid to this topic again and again.

It is obvious that historians of science would be interested in a comparative approach between Greek and Chinese planetary theory. Such an approach, unfortunately, has not been undertaken due to the fact that the planetary theory in ancient China was not figured out properly.

In this section, the theoretical model and the evolution of planetary theory in ancient Chinese mathematical astronomy will be discussed briefly. We will take this model as a case study of another scientific tradition, namely, *practical tradition.*

The aim of planetary theory in traditional Chinese mathematical astronomy was to calculate the apparent position of planets at any given time, namely, to determine the geocentric longitude of planets. Neither declination nor latitude of planets was cared much in ancient China.

In about the fifth century BC, Chinese mathematical astronomer already designed an algorithm for calculating the planetary movement. Every effort had been made to improve the planetary theory until the 13th century AD.

A historical material shows that the precision requirement of planets should be up to 1 *du* (= 360°/365.25) in the 12th century (see Table 2). Since the largest error of Mars in Copernicus's theory could be up to 5°, the precision requirement of 1° for medieval Chinese astronomers seems to be too high to be a reality. But it is true, and the fact is that the precision required by the rulers could not be negotiated. What Chinese astronomers challenged was to keep their planetary theory improved so that it could catch up with the requirement of the emperor.

Table 2. Observations of Jupiter, Mars and Saturn in the third lunar month in 1168AD.

Date	Planet	Observation In lunar mansion		*Jiyuan li* 纪元历 1106		*Tongyuan li* 统元历 1135		*Qiandao li* 乾道历 1167	
11	Jupiter	14.00 *du*	In *shi*	13.25	R			14.25-	F
24		16.75 *du*	In *shi*	16.25	R	0.25 in *bi*	R	16.5+	F
27		0.25 *du*	In *bi*	0.0	R	0.75+	R	0.25+	F
20	Mars	6.0+ *du*	In *wei*	6.75	P	7.25	R	6.00	P
24		9.50 *du*	In *wei*	9.00	P	10.00	R	9.00	F
27		11.00 *du*	in *wei*	11.00	P	12.0+	R	11.50	R
11	Saturn	6.0- *du*	in *xu*	7.0+	R			6.25-	F
20		6.50 *du*	in *xu*	7.50-	R	8.75+	R	6.50	P
24		6.5- *du*	in *xu*	7.50	R	9.00	R	6.5+	F
27		6.50 *du*	in *xu*	7.50	R	9.00	R	6.5+	F

Note: R, F, and P represent rough , fine, and perfect, respectively. 1 *du* = 360°/365.25.
Source: *Chapter of Acoustics and Calendar-making*, in the *Song History*, vol. 81.

Briefly speaking, the planetary theory after 600 AD consisted of two parts:

S1. Mean geocentric longitude of the planets.

S2. Difference between the mean and true geocentric longitude of the planets.

The algorithms for S1 were almost the same in the long history, while that for S2 changed gradually following the advancement in astronomical observation.

In order to construct the algorithm for S1, a synodic period of planet was divided into several parts (see Fig.1). The key points of the division included: Conjunction, the sun and the planet at the same longitude; Appearance in the morning, the ending point of invisibility; Station in the morning, the starting point of retrogradation; Opposition, the earth between the sun and the planet; Station in the evening, the starting point of prograde motion; Disappearance in the evening, the starting point of invisibility.

Since the period between appearance and station in the morning (or station and disappearance in the evening) was too long, it was often divided into several smaller intervals.

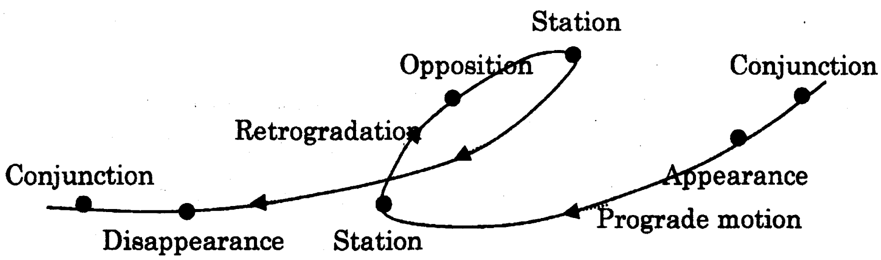

Fig. 1 Apparent motion of the superior planet

An astronomical table of S1 is found in each calendar-making system. Observed data of mean motion on the division in a synodic period are given in this table. Making use of the data in the table of synodic period of planets, interpolation function would be constructed in each interval. The mean geocentric longitude of a planet at any given time was then calculated with these functions.

Table 3. Synodic period of Mars in the *Jiyuan calendar-making system* (纪元历, 1106AD)

Interval	Time (in day)	x	Distance (in *du*)	β	v_a (*du*/day)
Conjunction	67	0	48	0	0.7192
Fast 1	63	67	44.60	48	0.7136
Fast 2	58	130	40.09	92.60	0.7024
Fast 3	52	188	34.06	132.69	0.6800
Fast 4	45	240	26.32	166.75	0.6300
Slow 1	37	285	16.68	193.07	0.5400
Slow 2	28	322	5.75	209.75	0.3726
Station	11	350		215.5	
Retrogradation	28.96	361	-8.16	215.5	-0.4130
Opposition		389.96		207.34	

Note: time x: days after the mean conjunction; distance β : displacement in *du* from the position of the mean conjunction; velocity v_a: apparent speed of planet at the beginning of the interval.
Source: *Chapter of Acoustics and Calendar-making*, in the *Song History*, vol. 80.

Since the Mars was always a troublemaker for ancient and medieval astronomers, let us take it in the *Jiyuan calendar-making system* (纪元历, 1106AD) as an example to show how such a table was constructed. The period between appearance and station of the Mars was broken into 6 pieces (see Table 3). A theoretical analysis shows that the largest error of the mean longitude (β) of Mars in Table 3 is 12', and its average absolute error in a synodic period is only 4'.

Before the phenomenon of the irregular revolution of planets was discovered by Zhang Zixin in about 560 AD, only the mean geocentric longitude was cared about in the planetary theory. In this case, revolutions of the earth and the planet were supposed to be in mean motion. We name the planetary theory in this period *double-mean model.*

After Liu Zhuo's *Huangji calendar-making system* (皇极历, 600 AD) at latest, an algorithm was designed to calculate the difference between the mean and true geocentric longitude in every traditional Chinese calendar-making system. However, its astronomical meaning could not be figured out until Yi-xing's *Dayan calendar-making system* (大衍历, 724 AD).

In Yi-xing's planetary theory, the algorithm was designed only for the equation of center of planets. Yi-xing's model can be explained as follows: the sun was surround by the earth in its mean motion and by planets in their true motion. We name his theory *mean-true model.*

In his *Chongxuan calendar-making system* (崇玄历, 892 AD), Bian Gang was conscious that Yi-xing's *mean-true model* was incorrect. Several calendar-makers made their efforts to improve the planetary theory one after another since then. In 1106 AD, the algorithm for the deviation in planetary theory was finalized by Yao Shunfu 姚舜辅 in the *Jiyuan calendar-making system* 纪元历. His algorithm corresponds to the fact that both revolutions of the earth and planets were supposed to be in their true motion. We name the last planetary theory in ancient China *double-true model.*

5. Scientific tradition

5.1 Theoretical tradition and practical tradition

There are two traditions in science, namely, *theoretical* and *practical.*

In the theoretical tradition, science features to explain the natural phenomena. Theory is judged by its function of the explanation. Observation is employed to verify the correctness of the theoretical hypothesis. Old model is always replaced if the new one is more reasonable in the explanation of natural phenomena.

In the practical tradition, science features to solve the concrete problems. Theory is judged by its accuracy of computation. Scientific progress follows the advancement of observation. A theoretical model is always improved to meet the precision requirement step by step.

These two traditions differ mainly in their starting and ending points.

In the theoretical tradition, model is built up from the hypothesis to account for natural phenomena. For a better understanding of natural phenomena, the model is revised on the base of a new hypothesis from time to time. The closer the model to the truth is, the more accurate the theory is.

In the practical tradition, on the other hand, model is built up from the observation to solve concrete problems. For a more accurate prediction, uninterrupted efforts are made to explore unknown factors, and the related numerical analyses were improved. The more accurate the theory is, the closer the model to the truth is.

Through different courses they approach the same goal (see Fig.2).

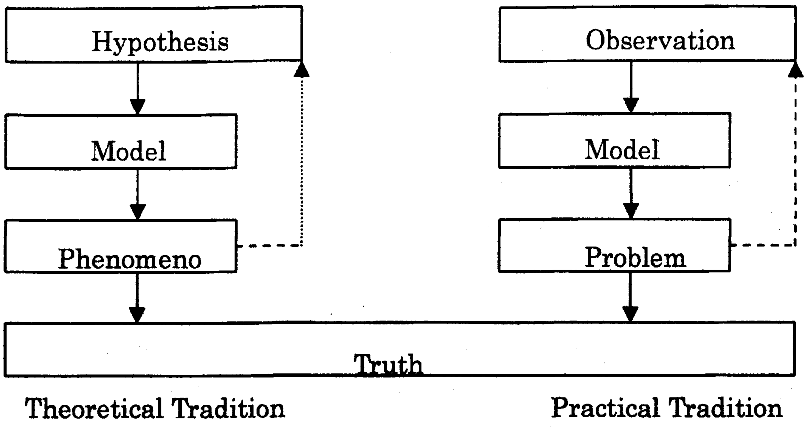

Fig.2 Scientific tradition

5.2 Practical tradition in China

Chinese mathematical astronomy provides a typical example for the practical tradition.

Science in ancient China was supposed to solve concrete problems, such as planetary positions. The function of explaining the natural phenomena never dominated the scientific tradition. What Chinese scientists really cared about was how to solve the problem they faced as accurately as possible. It was true that numerical analyses won their favor over the cosmic or geometric model building. The evolutions of numerical method and the planetary theory as we described above show the reason why the practical tradition was formed in ancient China.

5.3 Theoretical tradition in Greece

Studies by historians of science in the last century showed that the Copernican Revolution made no essential progress in terms of accuracy of planetary positions. The reason is that Copernicus and Ptolemy made use of the same epicycle-eccentric method. And,

> *"It has been argued that as formalizations, Copernicus and Ptolemaic theories were strictly equivalent (D. J. de S. Price), geometrically equivalent (A. R. Hall), even 'absolutely identical' (J. L. E. Dreyer). ...... With Freud, man lost his Godlike mind; with Darwin his exalted place among the creatures on earth; with Copernicus man had lost his privileged position in the universe."*[8]

Why was Copernicus's heliocentric system bound to triumph over Ptolemy's geocentric system?

It is obvious that Copernicus carried on the Greek astronomical tradition. As Neugebauer once said that "One cannot read a single chapter in Copernicus or Kepler without a thorough knowledge of the *Almagest*."[9] In Ptolemy's *Almagest,*

> *"This notion of uniform, circular motion is fundamental. In fact, the history of Greek theoretical astronomy is, to a very large extent, the history of a long series of efforts at explaining away the observed irregularities in the heavens by resolving even the most complex celestial motion into a set of uniform, circular components."*[10]

Copernicus's goal was to give a complete and physically correct description of the planetary theory.

"The heliocentric theory—philosophical quibbles aside—gives the order and distances of the planets unambiguously and under the reasonable assumption that the equation of the anomaly shows the ratio of radii of the planet's and earth's orbits. In so doing, it makes the planetary system into a single whole in which no parts can be arbitrarily rearranged. By contrary, in the geocentric theory the radii of the eccentrics and epicycles are known only relatively, one planet at a time, and only by additional assumptions, such as the contiguity of successive spheres, can the order and distances of the planets be determined."[11]

As a consequence of Copernicus's heliocentric model, the phases of an inferior planet like that of the moon was never observed until in the fall of 1610 when Galileo turned his telescope to Venus. Galelio's observation not only provided evidence for the plausibility of Copernicus's system, but also began to threaten the cosmological system of Ptolemy.

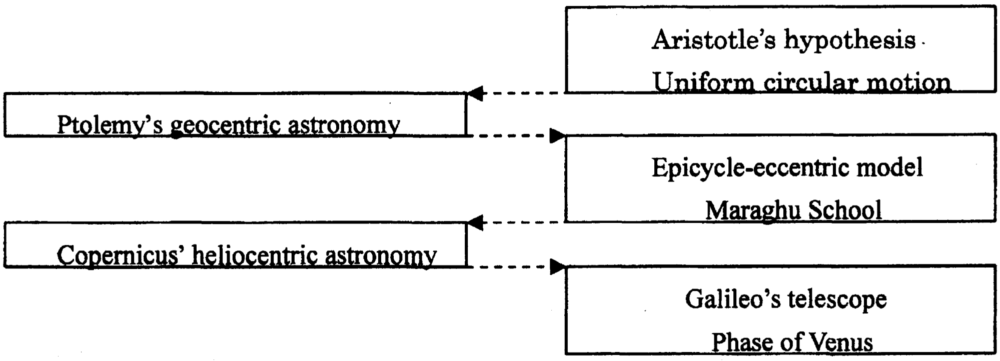

Fig.3 Evolution of planetary theory in Greek tradition

In the aspects of the accountability of the phenomena, certain features of planetary models that are unexplained in the geocentric theory are understood as the necessary consequences of the transformation from a geocentric to a heliocentric arrangement. For instances, why the radii of the epicycles of the superior planets remain parallel to the direction from the earth to the mean sun.

These facts show that the function of explanation in Copernicus's model is better than that in Ptolemy's model. This is the reason why Copernicus heliocentric hypothesis was finally widely accepted and drove away Ptolemy's geocentric hypothesis.

The Copernican revolution on planetary theory provides a typical example of the theoretical tradition.

6. Scientific progress: linear and dualistic

A scientific tradition represents the principal aspects of scientific methodology, spirit, and style. When we speak of the theoretical tradition of Greek science, we do not mean that there was no applied science at all. What we mean is that compared with this theoretical tradition, the practical tradition was of trivial importance in the development of Greek science.

Roughly speaking, the scientific progress in ancient civilization featured a linear course, either in practical or in theoretical tradition. However, the situation in modern time was never so simple.

It is believed that modern science, to a large extent, benefits from the heritage of Greek science. Nevertheless, it is hard to say that the theoretical tradition dominates the development of modern science in all aspects. In fact, the practical tradition also plays an important role.

It is obvious that the numerical analyses are more frequently used than theoretical hypotheses in modern science. For instance, as the consequences of tidal friction, the velocity of earth's rotation is slowing down. Since the earth's rotation is affected by too many factors that have never been figured out, it is rather the numerical method than theoretical model that is in practice.

The task for modern scientists is not only to account for natural phenomena but also to solve concrete problems. The researches arising from the search for solutions to scientific problems have led in two directions: those which are concerned with finding general theorems concerning the problems, and those which are searching for good approximations for solutions.

Both explaining natural phenomena and solving concrete problems are the goals that modern scientists fight for. Observations have occupied a substantial position in the development of modern science. The evolution of the planetary theory from Kepler to Einstein could be an example to show how the theoretical tradition and the practical tradition have merged into a single whole after renaissance:

Johannes Kepler (1571-1630) was a Copernican from his twenties on, and was destined to bring about acceptance of the heliocentric concept. As the German assistant and successor to Tycho Brahe (1546-1601), he developed his empirical laws from Brahe's observations on Mars. However, Kepler then generalized saying that his laws applied to all the planets, including the earth.

It was Isaac Newton (1642--1727) who tried to answer some basic questions, such as what keeps the planets in their elliptical orbits. He found that a fundamental force called *gravity* operating between all objects made them move the way they do. Newton, then, derived his law of universal gravitation.

Kepler's three laws can be obtained as an application of the law of universal gravitation in an approximated case, a 2-body problem where the bodies are point-like, the sun is still and the planets don't interact. Newton himself had solved geometrically the 2-body problem for two spheres moving under their mutual gravitational attraction.

In the 18th century, driven by the needs in navigation for knowledge about the motion of the moon, scientists were working on the higher-dimensional problem. Meanwhile, the predictions based on Newton's theory of planetary motion could not match astronomical observations well. These problems quickly stimulated the early research into the 3-body problem. It led Euler, Largrange, and Laplace to establish the theory of perturbation.

In 1859, Le Verrier noted that the orbit of Mercury did not behave as required by Newton's equations. By 1882, the advance of the precession of the perihelion of Mercury was more accurately known, i.e. 43" per century. The precession of the orbits of all planets except for Mercury's can, in fact, be understood using Newton's theory. Only Mercury seemed to be an exception. The precession of the perihelion of Mercury, therefore, was a problem in the study of the Solar System.

From 1911, Einstein realized the importance of astronomical observations for his theories and he worked with Freundlich to make measurements of Mercury's orbit which were required to confirm the general theory of relativity. According Einstein's theory, Mercury, as the closest planet to the sun, orbits a region in the solar system where space-time is disturbed by the sun's mass. Mercury's elliptical path around the sun shifts slightly with each orbit such that its closest point to the sun (or *perihelion*) shifts forward with each pass.

Newton's theory predicted an advancement only half as large as the one actually observed. Einstein's predictions exactly matched the observation.

It is obvious that most of the turning points in the development of planetary theory in modern science are brought forth by observation. It advances in a gradual but not revolutionary manner.

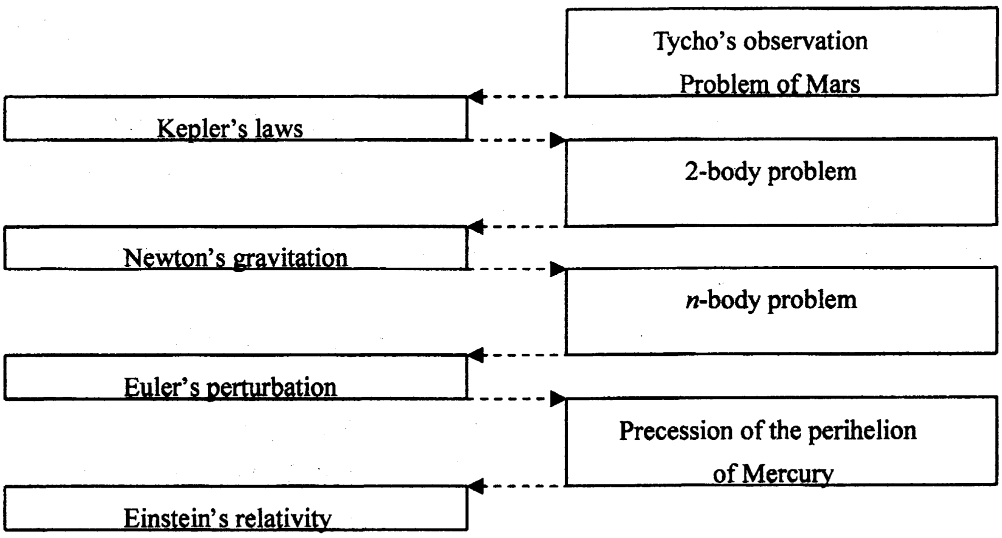

Fig.4. Planetary theory: From Kepler to Einstein

Science in ancient civilizations was often characterized by a distinctive tradition, either the theoretical tradition like Greek, or the practical tradition like Chinese. It developed in a linear course.

The diversity of modern science features the blending of the two traditions. It develops in a dualistic model. The origin of the dualistic scientific tradition might be traced back to the Islamic science before renaissance.

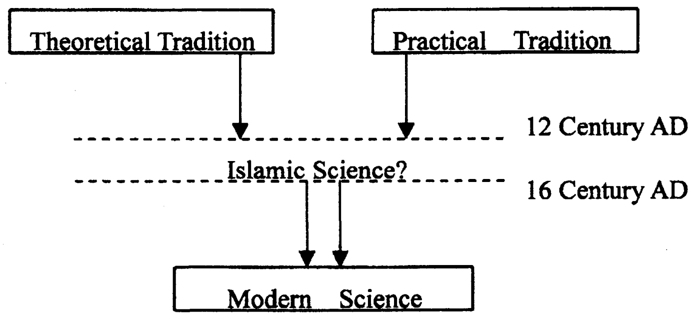

Fig.5. Model of scientific progress

References

[1] Anjing Qu (2002) The Third Approach to the History of Mathematics in China, in: Li Tatsien (ed.) *Proceedings of the International Congress of Mathematicians, 2002*, Vol. III, Higher Education Press, Beijing, pp. 947-958

[2] C. C. Gillispie (ed.) (1976) *Dictionary of Scientific Biography*, Charles Scribner, New York

[3] Ulrich Libbrecht. *Chinese Mathematics in the Thirteenth Century*. Cambridge: MIT Press, 1973

[4] Jean-Claude Martzloff. *A History of Chinese Mathematics*. Berlin, Springer-Verlag, 1997

[5] Anjing Qu (1996) Bian Gang: A mathematician of the 9th century, *Historia Scientiarum* 2, pp. 17-30

[6] Anjing Qu (1996) Zhongguo gudai lifa zhong de sanci neichafa (Cubic Interpolation in Ancient Chinese Calendars), *Ziran kexueshi yanjiu* (*Studies in the History of Natural Sciences*) 2, pp. 131-143

[7] Anjing Qu (1998) Numerical Methods in Medieval Chinese Mathematical Astronomy, *Journal of Northwest University* (Natural Science Edition) 2, pp. 99-104

[8] Paul Edwards (ed.) (1967) *The Encyclopedia of Philosophy*, v.2, The Macmillan Company, New York, p. 122

[9] Otto Neugebauer (1969) *The Exact Sciences in Antiquity*, Dover, New York, pp. 3-4

[10] Olaf Pedersen (1974) *A Survey of the Almagest*, Odense University Press, Denmark, p. 34

[11] Noel M. Swerdlow & Otto Neugebauer (1984) *Mathematical Astronomy in Copernicus's De Revolutionibus*. Springer-Verlag, Berlin, p. 59