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Generation and propagation of interface to a Lotka-Volterra competition diffusion system with large interaction rate

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1 Introduction

This is a joint work with Georgia Karali (University of Toronto), Masato Iida (Iwate university), Masayasu Mimura (Meiji university), Eiji Yanagida (Tohoku university), and Tohru Wakasa (Waseda university) ([7], [9]).

Habitat segregation phenomena in mathematical ecology supply us with various problems which are interesting from the aspect of interfacial dynamics. We mathematically discuss regional partition by competitive two species and their competition for their own habitats. When the competition between two species is bitter, they cannot coexist at the same point. In such cases we can expect that the two species with a suitable initial state segregate their habitats and compete on the interface between both the habitats. Then it is significant to understand the dynamics of the segregation patterns.

In this article we treat a competition-diffusion system for two species in competition of the Lotka-Volterra type:

$$\begin{cases} u_t = d_1\Delta u + (a_1 - b_1 u - c_1 v)u, \\ v_t = d_2\Delta v + (a_2 - b_2 v - c_2 u)v. \end{cases}$$

Here a_k, b_k, c_k and d_k $(k = 1,2)$ are positive constants; $u = u(t,x)$ and $v = v(t,x)$ are the population densities of competitive two species. Our concern is the situation where the interspecific competition is exceedingly bitter: in particular, the situation close to the singular limit as $c_1, c_2 \to \infty$ with c_1/c_2 fixed. Thus we simply rewrite the above system as

$$\begin{cases} u_t = \Delta u + (1-u)u - cMuv, \\ v_t = d\Delta v + (a-v)v - bMuv, \end{cases} \tag{1}$$

where a, b, c, d are fixed positive constants and M is a huge parameter. As seen in the following section, the spatial supports of u and v satisfying (1) become separated from each other by an interface in a short time-period. Then after that the *segregated* (u, v)

behaves like a solution of a two phase free boundary problem for the Fisher equation. We will establish a rigorous mathematical theory both for the formation of interfaces at the initial stage and for the motion of those interfaces in the later stage. More precisely, we will show that, given virtually arbitrary smooth initial data, the solution develops interfaces within the time scale of $O(\epsilon^2)$. We will then prove that the motion of the interfaces converges to the free boundary problem as $\epsilon \to 0$.

There are several related works on singular limits of some reaction-diffusion systems as the effect of interaction tends to infinity: [1], [3], [4], [5] and [11] investigate the *fast reaction limit* of chemical reaction systems (see also the references therein). As for competition-diffusion systems, [2] investigates singular limits of the stationary problems as the interspecific competition rate tends to infinity. The most related work is [6], which we will mention after giving the formal derivation of the singular limit.

2 Formal derivation of the singular limit

In this section we present a formal derivation of the singular limit of (1).

We consider (1) with an initial data $(u(x,0), v(x,0)) = (u_0(x), v_0(x))$. We will put some assumption on the initial data.

Assumption 1 *Let u_0, v_0 be smooth and bounded up to the second derivatives. Consider the situation where both $D_0 = \{\, x |\ bu_0(x) > cv_0(x) \,\}$ and its complement possess interior points. Suppose that*

$$\inf_{\partial D_0} |b\nabla u_0 - c\nabla v_0| > 0.$$

Remark 1 *Assumption 1 assures that ∂D_0 is an $N-1$ dimensional hypersurface with bounded mean curvature.*

When M is sufficiently large, the dynamics of (1) consists of two consecutive stages.

The first stage is a short time-period of the rapid evolution, where u, v, Δu and Δv are negligible compared with Muv so that the ordinary differential equations

$$\begin{cases} \tilde{u}_\tau = -c\tilde{u}\tilde{v}, \\ \\ \tilde{v}_\tau = -b\tilde{u}\tilde{v}, \end{cases} \tag{2}$$

approximates (1) in the time scale $\tau = Mt$. Since $b\tilde{u} - c\tilde{v}$ is independent of τ, $\tilde{u}$ satisfies

$$\tilde{u}_\tau = (\omega - b\tilde{u})\tilde{u}$$

with $\omega = \omega(x) = bu_0(x) - cv_0(x)$ and hence

$$\lim_{\tau\to\infty} \tilde{u} = \max\{0,\ \omega/b\}.$$

Consequently $(u(t,x), v(t,x))$ essentially becomes the continuous function

$$(u_1(x),\, v_1(x)) = \begin{cases} (\omega(x)/b,\, 0) & \text{in } D_0, \\ (0,\, -\omega(x)/c) & \text{in } \mathbb{R}^N \backslash D_0 \end{cases} \tag{3}$$

after a short period of time scale t. The non-degeneracy of $\nabla\omega$ on $\partial D_0 = \{\, x \mid \omega(x) = 0 \,\}$ causes the gap of $(\nabla u_1, \nabla v_1)$ across the surface ∂D_0. Thus sharp transition of $(\nabla u,\, \nabla v)$ appears near ∂D_0. Namely the *corner layer* of $(u(t,\cdot),\, v(t,\cdot))$ is generated along the surface ∂D_0 in a short time-period.

The second stage of the dynamics of (1) describes the propagation of the corner layer. The stretching (u, v) with a suitable scale makes the analysis of the corner layer easier. To rescale the system in the best possible way, we need to estimate the length scale $\epsilon = \epsilon(M)$ of the width of the corner layer. We note that u_1, v_1 are continuous functions with bounded gradients and that the mean curvature of the surface ∂D_0 is bounded. It is natural to assume in the second stage that $u = O(\epsilon)$, $v = O(\epsilon)$, $u_t = O(1)$ and $\Delta u = O(\epsilon^{-1})$ on the corner layer for huge M and that the effects of Δu and Muv in (1) are well-balanced. Then we have $\epsilon = O(M^{-1/3})$. For simplicity we put $M = \epsilon^{-3}$ and rewrite (1) as

$$\begin{cases} u_t = \Delta u + (1-u)u - \dfrac{c}{\epsilon^3} uv, \\ v_t = d\Delta v + (a-v)v - \dfrac{b}{\epsilon^3} uv. \end{cases} \tag{4}$$

Set

$$D^\epsilon(t) = \{\, x \mid bu(t,x;\epsilon) > cv(t,x;\epsilon) \,\}.$$

for the solution $(u(t,x;\epsilon),\, v(t,x;\epsilon))$ of (4) corresponding to the initial datum $(u_0(x),\, v_0(x))$. Taking account of (3) and the argument for the first stage, we can expect that $u(t,x;\epsilon)$ (resp. $v(t,x;\epsilon)$) almost vanishes in $\mathbb{R}^N \backslash D^\epsilon(t)$ (resp. $D^\epsilon(t)$); further the corner layer of $(u(t,\cdot;\epsilon),\, v(t,\cdot;\epsilon))$ remains along the interface $\partial D^\epsilon(t)$. Around each point $y \in \partial D^\epsilon(t)$ we introduce a local orthogonal coordinate system (ξ, σ) such that $\sigma = (\sigma_1, \dots, \sigma_{N-1})$ is a local coordinate along $\partial D^\epsilon(t)$ whereas $\xi = \xi(x,\, \partial D^\epsilon(t))$ is the signed distance from x to $\partial D^\epsilon(t)$ locally defined near y so that $\xi > 0$ in $D^\epsilon(t)$. Around the corner layer we stretch the solution and see it using a moving coordinate system (t, ρ, σ), where $\rho = \xi/\epsilon$ is a rescaled coordinate in the normal direction to $\partial D^\epsilon(t)$. Suppose that $(u(t,x;\epsilon),\, v(t,x;\epsilon))$ is asymptotically written as

$$(u,v) = \begin{cases} (u^*, v^*) + O(\epsilon) & \text{away from the layer (outer expansion),} \\ \epsilon(U_1, V_1) + \epsilon^2(U_2, V_2) + O(\epsilon^3) & \text{around the layer (inner expansion),} \end{cases}$$

where (u^*, v^*) is a bounded continuous function of the fixed coordinate (t,x) and (U_1, V_1) and (U_2, V_2) are smooth functions of the moving coordinate (t, ρ, σ) with a bounded gradient; all of them are independent of ϵ. By a formal argument based on the *matched* *asymptotic expansion* method, we can formally conclude that (u^*, v^*) satisfy

$$\begin{cases} u_t^* = \Delta u^* + (1-u^*)u^*, \quad v^* \equiv 0 \quad \text{in } D(t), \\ v_t^* = d\Delta v^* + (a - v^*)v^*, \quad u^* \equiv 0 \quad \text{in } \mathbb{R}^N \backslash D(t), \\ b\dfrac{\partial u^*}{\partial \nu^i} = cd\dfrac{\partial v^*}{\partial \nu^o} \quad \text{on } \partial D(t), \end{cases} \tag{5}$$

and (U_1, V_1) satisfy

$$\begin{cases} U_{1\rho\rho} = cU_1V_1, \quad -\infty < \rho < +\infty, \\ dV_{1\rho\rho} = bU_1V_1, \quad -\infty < \rho < +\infty, \\ (U_1(t,\rho,\sigma),\, V_1(t,\rho,\sigma)) = \left(0,\, -\rho\dfrac{\partial v^*}{\partial \nu^o}(t,y)\right) \quad \text{as } \rho \to -\infty, \\ (U_1(t,\rho,\sigma),\, V_1(t,\rho,\sigma)) = \left(\rho\dfrac{\partial u^*}{\partial \nu^i}(t,y),\, 0\right) \quad \text{as } \rho \to +\infty, \end{cases} \tag{6}$$

and (U_2, V_2) satisfies (10) which is given later.

Here $D(t)$ is the formal limit of $D^\epsilon(t)$ as $\epsilon \to +0$, $\nu^i(\nu^o)$ inner (outer) normal to $\partial D(t)$, and y a point on $\partial D(t)$ corresponding to the coordinate $(0, \sigma)$. In (6) the boundary conditions at $\rho = \pm\infty$ reflect the request that (u^*, v^*) and $\epsilon(U_1, V_1)$ should be matched. The boundary condition on $\partial D(t)$ in (5) is requested for (u^*, v^*) in order that the elliptic boundary value problem (6) possesses a solution. Consequently, in the second stage the supports of $u(t, \cdot; \epsilon)$ and $v(t, \cdot; \epsilon)$ are almost separated by the corner layer which remains in a narrow range of $O(\epsilon)$ along the propagating interface $\partial D(t)$. The dynamics of the segregation pattern is essentially determined by the free boundary problem (5). We see from the elliptic equations in (6) that the population on the interface supplied by the diffusion from both the habitats instantly disappears by the strong competition between two species.

3 Main result

The formal derivation of the free boundary problem (5) from (4) as $\epsilon \to +0$ is justified by [6] on a bounded domain in $\mathbb{R}^N$ under the no-flux boundary condition in the framework of weak topology of H^1. It also gives a result on the uniqueness and existence of a Hölder-continuous weak solution to (5). However we need to justify the derivation of (5) at least in the framework of C^0-topology in order to investigate the dynamics of the segregating interface. To accomplish this end we impose the existence of a classical solution to (5) as follows.

Let $D(t)$ be a one-parameter family of open subsets of $\mathbb{R}^N$, and denote $\partial D(t)$ by $\Gamma(t)$ for simplicity. and let $u^*(t,x)$ and $v^*(t,x)$ be nonnegative continuous functions defined on $[0,T] \times \mathbb{R}^N$ with some $T > 0$. We assume the following hold for $t \in [0,T]$:

Assumption 2 *The boundary of $D(t)$, which is denoted by $\Gamma(t)$, is in C^2 for each t and in C^1 with respect to t;*

Assumption 3 *(u^*, v^*) satisfies (5) in the classical sense;*

Assumption 4 *$|u^*|, |\nabla u^*|, |\Delta u^*|$ are bounded in $D(t)$ uniformly with respect to t, and $|v^*|, |\nabla v^*|, |\Delta v^*|$ are bounded in $\mathbb{R}^N \backslash D(t)$ uniformly with respect to t;*

Assumption 5 $\inf_{y\in\partial D(0)} \lim_{\substack{x\to y \\ x\in D(0)}} |\nabla u^*(x)| > 0, \quad \inf_{y\in\partial D(0)} \lim_{\substack{x\to y \\ x\in \mathbb{R}^N\backslash\overline{D(0)}}} |\nabla v^*(x)| > 0.$

If the free boundary condition in (5) is replaced by

$$\mu\frac{d}{dt}\Gamma(t) = b\frac{\partial u^*}{\partial \nu^i} - cd\frac{\partial v^*}{\partial \nu^o} \quad \text{on } \Gamma(t),$$

where μ is a positive constant and $\frac{d}{dt}\Gamma(t)$ denotes the propagation speed of $\Gamma(t)$ in the outer normal direction, then the regularity of $\Gamma(t)$ will be assured by the parabolicity as treated in [8] and [10]. However, in our case which corresponds to the case $\mu = 0$, it is not easy to deduce the regularity of $\Gamma(t)$ in (5), because the parabolicity is partially broken on $\Gamma(t)$. Nevertheless, a recent result in [11] suggests that the partial regularity of $\Gamma(t)$ in the classical sense can hold also for (5). Thus we believe the above assumptions natural.

Now we will give our main theorem.

Theorem 1 *Under Assumptions 1-5, there exist a positive constant $C > 0$ such that for sufficiently small $\epsilon > 0$, the following hold:*

$$|u(t,x;\epsilon) - u^*(t,x)| < C\epsilon|\log\epsilon|,$$

$$|v(t,x;\epsilon) - v^*(t,x)| < C\epsilon|\log\epsilon| \quad \textit{for } (t,x)\in[\epsilon^2, T]\times\mathbb{R}^N,$$

where $(u(t,x;\epsilon),\ v(t,x;\epsilon))$ is a nonnegative solution of (4).

Theorem 1 shows that , for virtually arbitrary smooth initial data, the solution develops interfaces in time $t = \epsilon^2$ and the motion of the interface is approximated by the free boundary problem (5) for $t \in [\epsilon^2, T]$.

Our main tool for deriving the above results is the method of upper and lower solutions. We will use two different pairs of upper and lower solutions, namely $(u^\pm, v^\pm)$ and $(U^\pm, V^\pm)$. The first one $(u^\pm, v^\pm)$ is used to analyze the generation of the interface that takes place in a very fast time scale. The second one $(U^\pm, V^\pm)$ is used to study the motion of the interface in a relatively slow time scale. The transition from the initial stage to the second stage occurs within a time scale of ϵ^2. Since the behaviors of solutions are so different between the two stages, it is important to construct suitable upper and lower solutions for each stage and to know the right timing to switch from $(u^\pm, v^\pm)$ to $(U^\pm, V^\pm)$.

In the following Section 4, we deal with the generation of the interface, and in Section 5, the motion of the interface. Section 4 is depend on [9], and Section 5 is on [7].

4 Generation of interface

In this section we study the generation of interface that takes place in the initial stage. We will construct an upper and lower solution for this stage.

Consider two functions $\phi(\tau;\xi,\eta)$ and $\psi(\tau;\xi,\eta)$ defined by

$$\begin{cases} \dot{\phi} = -c\phi\psi, & \phi(0) = \xi > 0, \\ \dot{\psi} = -b\phi\psi, & \psi(0) = \eta > 0. \end{cases}$$

We can observe that $A = A(\phi(\tau), \psi(\tau)) = b\phi - s\psi$ is preserved for any $\tau > 0$, so we have

$$\phi(\tau;\xi,\eta) = \frac{\xi A e^{A\tau}}{A + b\xi(e^{A\tau} - 1)}, \quad \psi(\tau;\xi,\eta) = \frac{\eta A e^{-A\tau}}{A + c\eta(1 - e^{-A\tau})},$$

and

$$\lim_{\tau\to+\infty} \phi(\tau;\xi,\eta) = \max\left\{\frac{A(\xi,\eta)}{b}, 0\right\}, \quad \lim_{\tau\to+\infty} \psi(\tau;\xi,\eta) = \max\left\{0, -\frac{A(\xi,\eta)}{c}\right\}.$$

As we have mentioned in the introduction, we can expect that the solution $(u(x,t), v(x,t))$ would be approximated by

$$(\phi(\frac{t}{\epsilon^3}; u_0(x), v_0(x)),\ \psi(\frac{t}{\epsilon^3}; u_0(x), v_0(x))) \tag{7}$$

by a formal argument. The upper and lower solutions in this stage is given by modifying the approximated solution (7):

$$\begin{aligned} u^+(x,t) &= \phi(\frac{t}{\epsilon^3}, u_0(x) + c_1\epsilon\exp(\frac{t}{\epsilon^2}),\ v_0(x) - c_2\epsilon\exp(\frac{t}{\epsilon^2})), \\ v^+(x,t) &= \psi(\frac{t}{\epsilon^3}, u_0(x) + c_1\epsilon\exp(\frac{t}{\epsilon^2}), v_0(x) - c_2\epsilon\exp(\frac{t}{\epsilon^2})), \\ u^-(x,t) &= \phi(\frac{t}{\epsilon^3}, u_0(x) - c_1\epsilon\exp(\frac{t}{\epsilon^2}),\ v_0(x) + c_2\epsilon\exp(\frac{t}{\epsilon^2})), \\ v^-(x,t) &= \psi(\frac{t}{\epsilon^3}, u_0(x) - c_1\epsilon\exp(\frac{t}{\epsilon^2}),\ v_0(x) + c_2\epsilon\exp(\frac{t}{\epsilon^2})), \end{aligned} \tag{8}$$

where $c_1, c_2 > 0$ are constants to be determined.

Theorem 2 (*Nakashima-Wakasa [9]*) *Suppose that Assumption 1 holds. Then there exists $c_1, c_2 > 0$ such that for sufficiently small $\epsilon > 0$, (u^+, v^+), (u^-, v^-) are pair of upper* *and lower solutions of* (4) *for* $0 \le t \le \epsilon^2$. *Moreover the following estimates hold:*

$$|u^{\pm}(x,\epsilon^2) - \max\left\{\frac{\omega(x)}{b}, 0\right\}| < C_1\epsilon, \quad x \in \mathbb{R}^N$$

$$|v^{\pm}(x,\epsilon^2) - \max\left\{0, -\frac{\omega(x)}{c}\right\}| < C_1\epsilon \quad x \in \mathbb{R}^N$$

$$|u^{\pm}(x,\epsilon^2)| < C_2\epsilon^5, \qquad in\ \{x \in \mathbb{R}^N \backslash D_0\ ;\ dist(x, \partial D_0) > C_3\epsilon\},$$

$$|v^{\pm}(x,\epsilon^2)| < C_2\epsilon^5, \qquad in\ \{x \in D_0\ ;\ dist(x, \partial D_0) > C_3\epsilon\}$$

where $C_1, C_2, C_3 > 0$ *are positive constant independent of* $\epsilon > 0$, *and*

$$\omega(x) = A(u_0(\cdot), v_0(\cdot)) = bu_0(x) - cv_0(x).$$

Theorem 2 shows that, for virtually arbitrary initial data, the solution forms interfaces in time $t = \epsilon^2$. More precisely, at time $t = \epsilon^2$, $(u^{\pm}, v^{\pm})$ stays between another pair of upper and a lower solution which are given in the next section, Motion of interface. This makes it possible to combine two different pairs of upper and lower solutions.

5 Motion of interface

In this section we construct another pair of upper and lower solutions for the second stage, motion of interface. This upper and lower solutions $(U^{\pm}, V^{\pm})$ has interface near $\Gamma(t)$, the solution of the free boundary problem (5).

We first construct upper and lower solutions $(U^{\pm}_{in}, V^{\pm}_{in})$ in a tubular neighborhood of $\Gamma(t)$ by modifying the first two terms of the inner expansion. After that we construct an upper and a lower solution $(U^{\pm}_{out}, V^{\pm}_{out})$ outside the tubular neighborhood using the first term of outer expansion. Then we match $(U^{\pm}_{in}, V^{\pm}_{in})$ and $(U^{\pm}_{out}, V^{\pm}_{out})$, then obtain $(U^{\pm}, V^{\pm})$. Once $(U^{\pm}, V^{\pm})$ are obtained, they will later be combined with another set of upper and lower solutions $(u^{\pm}, v^{\pm})$ that take care of the generation of interface at the initial stage.

5.1 An upper and a lower solution near the interface

Let $d(x,t)$ be the signed distance function with respect to the interface $\Gamma(t)$, namely,

$$d(x,t) = \begin{cases} -\mathrm{dist}(x,\Gamma(t)), & x \in D(t), \\ \mathrm{dist}(x,\Gamma(t)), & x \in \mathbb{R}^N \backslash D(t). \end{cases} \tag{9}$$

Here $\mathrm{dist}(x,\Gamma(t))$ is the distance from x to the hypersurface $\Gamma(t)$ in $\mathbb{R}^N$. Since $\Gamma(t)$ is a smooth hypersurface that depends smoothly on t, $d(x,t)$ is a smooth function of (x,t) near $\Gamma(t)$. In what follows we fix a constant $d^* > 0$ such that $d(x,t)$ is smooth in the N-

dimensional tubular neighborhood $\{(x,t) \in \mathbb{R}^N \times [0,T];\ \mathrm{dist}(x,\Gamma(t)) \le d^*\}$. Note that $|\nabla d| = 1$ in this neighborhood. We seek for upper and lower solutions in the following form:

$$\begin{aligned}
U_{in}^{+}(x,t) &= \epsilon U_1\left(\frac{d(x,t)}{\epsilon} - \eta(t), \sigma\right) + \epsilon^2 U_2\left(\frac{d(x,t)}{\epsilon} - \eta(t), \sigma, t\right) + \epsilon^3 q(t),\\
V_{in}^{+}(x,t) &= \epsilon V_1\left(\frac{d(x,t)}{\epsilon} - \eta(t), \sigma\right) + \epsilon^2 V_2\left(\frac{d(x,t)}{\epsilon} - \eta(t), \sigma, t\right) - \epsilon^3 \hat{q}(t),\\
U_{in}^{-}(x,t) &= \epsilon U_1\left(\frac{d(x,t)}{\epsilon} + \eta(t), \sigma\right) + \epsilon^2 U_2\left(\frac{d(x,t)}{\epsilon} + \eta(t), \sigma, t\right) - \epsilon^3 q(t),\\
V_{\epsilon}^{-}(x,t) &= \epsilon V_1\left(\frac{d(x,t)}{\epsilon} + \eta(t), \sigma\right) + \epsilon^2 V_2\left(\frac{d(x,t)}{\epsilon} + \eta(t), \sigma, t\right) + \epsilon^3 \hat{q}(t).
\end{aligned}$$

Here

$$\eta(t) = \left(\log \frac{1}{\epsilon}\right)\gamma \exp(Mt)$$

$$q(t) = \sigma \exp(Mt), \quad \hat{q}(t) = \hat{\sigma} \exp(Mt),$$

where $\gamma, \sigma, \hat{\sigma}$ and M are positive constants to be determined appropriately, and (U_1, V_1) satisfies (6) and (U_2, V_2) satisfies

$$\begin{cases}
-U_{2\xi\xi} + c(U_1V_2 + U_2V_1) = -U_{1\xi}(d_t - \Delta d) & -\infty < \rho < +\infty,\\
-dV_{2\xi\xi} + b(U_1V_2 + U_2V_1) = -V_{1\xi}(d_t - d\Delta d) & -\infty < \rho < +\infty,\\
(U_2(t,\rho,\sigma),\ V_2(t,\rho,\sigma)) = (0,\ 0) & \text{as } \rho \to -\infty,\\
(U_2(t,\rho,\sigma),\ V_2(t,\rho,\sigma)) = (0,\ 0) & \text{as } \rho \to +\infty.
\end{cases} \tag{10}$$

(10) is obtained by the formal argument based on the *matched asymptotic expansion.* The following lemma assures the existence of the first and second term of upper and lower solutions, whose proofs are omitted.

Lemma 1 *(i) There exists a unique positive solution of* (6).
(ii) There exists a solution of (10).

Since the first two terms of $(U_{in}^{\pm}, V_{in}^{\pm})$ are determined, we choose appropriate q and $\hat{q}$ so that $(U_{in}^{\pm}, V_{in}^{\pm})$ are an upper and lower solutions.

5.2 Upper and lower solutions away from the interface

In this subsection we will construct upper and lower solutions away from the interface modifying the first term of outer expansion.

Let g be a smooth function satisfying

$$g(s) = 0 \text{ if } s < 0, \quad g(s) = 1 \text{ if } s > 1$$

$$g'(0) = g'(1) = 0, \quad g'(s) \geq 0 \text{ for } 0 \leq s \leq 1$$

and set

$$\lambda_1(s) = g(\frac{s}{\epsilon} + \tilde{R}|\log\epsilon|), \quad \lambda_2(s) = g(-\frac{s}{\epsilon} - \tilde{R}|\log\epsilon|).$$

Moreover let δ satisfy $0 < \delta \ll d^*$ and define

$$\theta(s) = \begin{cases} -\beta\epsilon|\log\epsilon|(s+\delta)^2 + \beta\delta\tilde{R}\epsilon^2|\log\epsilon|^2 + \dfrac{\beta\delta^2}{\tilde{R}}\epsilon|\log\epsilon|, \\ \qquad -\delta - \tilde{R}\epsilon|\log\epsilon| \leq s \leq -\tilde{R}\epsilon|\log\epsilon|, \\ \beta\delta\tilde{R}\epsilon^2|\log\epsilon|^2 + \dfrac{\beta\delta^2}{\tilde{R}}\epsilon|\log\epsilon|, \qquad s \leq -\delta - \tilde{R}\epsilon|\log\epsilon|. \end{cases}$$

Now we will define upper and lower solutions in the following form:

$$U^+_{out}(x,t) = \begin{cases} u^*(x,t) + \epsilon|\log\epsilon|\alpha\exp(Lt) - \theta(d(x,t)), & d(x,t) \leq -\tilde{R}\epsilon|\log\epsilon| \\ (1 - \lambda_1(d(x,t)))U^+_\epsilon + \lambda_1(d(x,t))\epsilon^4, & d(x,t) > \tilde{R}\epsilon|\log\epsilon| \end{cases}$$

$$V^+_{out}(x,t) = \begin{cases} 0, & d(x,t) \leq -\tilde{R}\epsilon|\log\epsilon|, \\ v^*(x,t) - \epsilon|\log\epsilon|\alpha\exp(Lt) + \theta(-d(x,t)), & d(x,t) > \tilde{R}\epsilon|\log\epsilon| \end{cases}$$

$$U^-_{out}(x,t) = \begin{cases} u^*(x,t) - \epsilon|\log\epsilon|\alpha\exp(Lt) + \theta(d(x,t)), & d(x,t) \leq -\tilde{R}\epsilon|\log\epsilon| \\ 0, & d(x,t) > \tilde{R}\epsilon|\log\epsilon| \end{cases}$$

$$V^-_{out}(x,t) = \begin{cases} (1 - \lambda_2(d(x,t)))W^-_\epsilon + \lambda_2(d(x,t))\epsilon^4, & d(x,t) \leq -\tilde{R}\epsilon|\log\epsilon| \\ v^*(x,t) + \epsilon|\log\epsilon|\alpha\exp(Lt) - \theta(-d(x,t)), & d(x,t) > \tilde{R}\epsilon|\log\epsilon|. \end{cases}$$

Here $\alpha, \beta, \tilde{R}$ are positive constants to be specified appropriately.

$(U^\pm_{out}, V^\pm_{out})$ are chosen so as to satisfy the following condition.

- $(U^\pm_{out}, V^\pm_{out})$ is an upper and a lower solution for $|d(x,t)| > \tilde{R}\epsilon|\log\epsilon|$.
- The entire upper and lower solution given by (11) below is not smooth for $|d(x,t)| = \tilde{R}\epsilon|\log\epsilon|$. (We need to care about the derivative of $(U^\pm_{in}, V^\pm_{in})$ and $(U^\pm_{out}, V^\pm_{out})$ at $|d(x,t)| = \tilde{R}\epsilon|\log\epsilon|$.) $(U^\pm_{out}, V^\pm_{out})$ are determined so that $(U^\pm, V^\pm)$ given below become an upper and a lower solutions.
- $(U^\pm_{out}, V^\pm_{out})$ has the following estimate.

$$(U^\pm_{out}, V^\pm_{out}) = (u^*, v^*) + O(\epsilon|log\epsilon|).$$

5.3 Entire solution for the motion of interface

The entire solution is given by

$$(U^{\pm}, V^{\pm}) = \begin{cases} (U^{\pm}_{in}, V^{\pm}_{in}) & |d(x,t)| \leq \tilde{R}\epsilon|\log \epsilon|, \\ (U^{\pm}_{out}, V^{\pm}_{out}) & |d(x,t)| > \tilde{R}\epsilon|\log \epsilon|. \end{cases} \tag{11}$$

Now we give the following theorem:

Theorem 3 (*Iida-Karali-Mimura-Nakashima-Yanagida* [7]) *There exists $C > 0$ such that for sufficiently small $\epsilon > 0$, and any $t \in [\epsilon^2, T)$, $(U^+(x,t), V^+(x,t))$ and $(U^-(x,t), V^-(x,t))$ are pair of an upper and a lower solutions for* (4) *and satisfy the following estimate;*

$$|U^{\pm}(t,x;\epsilon) - u^*(t,x)| < C\epsilon|\log \epsilon|,$$

$$|V^{\pm}(t,x;\epsilon) - v^*(t,x)| < C\epsilon|\log \epsilon| \quad \textit{for } (t,x) \in [\epsilon^2, T] \times \mathbb{R}^N.$$

6 Proof of Theorem 1

Combining the estimate in Theorem 2 and expressions of $(U^{\pm}, V^{\pm})$, we have

$$U^-(x,\epsilon^2) \leq u^-(x,\epsilon^2) \leq u^+(x,\epsilon^2) \leq U^+(x,\epsilon^2),$$

$$V^-(x,\epsilon^2) \geq v^-(x,\epsilon^2) \geq v^+(x,\epsilon^2) \geq V^+(x,\epsilon^2).$$

This and Theorems 2 and 3 implies that for arbitrarily chosen initial data satisfying Assumption 1, the solution of (4) stays between (u^-, v^-) and (u^+, v^+) for $t \in (0, \epsilon^2]$, and stays between (U^-, V^-) and (U^+, V^+) for $t \in [\epsilon^2, T]$. Using the estimate in Theorem 3, the proof is completed.

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