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Nagata spaces and wN-spaces which are preserved by quasi-perfect maps

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1. きっかけとなった問題と否定的解答

Good, Knight and Stares は[3]で、次の命題を示し、それに関連して『wN-space は quasi-perfect map でその構造が保存されるか？』という問題を提出した。

Proposition 1.1[3, Proposition 18]. *The closed, finite to one image of a wN-space is a wN-space.*

この問題に対して、[14]で Ying and Good は Lutzer が与えた例;

Example 1.2[10, Example 4.3]. *A perfect image of a first countable startifiable space that is not even a q-space.*

が、否定的解答となることを指摘した。それは、Nagata space, wN-space, q-space の定義、そして既に知られている次の事実より明らかである。

Theorem 1.3[1, Theorem 3.1]. *A space is a Nagata space if and only if it is first countable and stratifiable.*

Lutzer が示した Example 1.2 は、次の事実も示している。

Fact 1.4. *Every quasi-perfect image of any Nagata-space is not Nagata.*

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ところで、次の事実はよく知られている。

Fact 1.5. *The quasi-perfect image of a metrizable space is a metrizable space.*

そこで、ここでは wN-space と metrizable space の間に位置し、quasi-perfect map でその構造が保存される空間を定義し、その空間に関する結果を報告する

2. よく知られている空間の定義

ここでは、space は T_1-space を、map は continuous で onto map を意味する。space X の subspace A に対し Cl(A) で A の closure を、N で自然数全体からなる集合を表す。 また、ここで特に定義されていない術語などは[2][6]を参照のこと。

Definition 2.1. For a space (X, τ), a function $g : X \times N \to \tau$ is called a *g-function* if $x \in g(x,n)$ and $g(x,n+1) \subseteq g(x,n)$ for each $(x,n) \in X \times N$.
For a subset A of X and $n \in N$, we put $g(A,n) = \cup \{ g(x,n) \mid x \in A \}$.

この g-function について、いくつかのよく知られている以下の性質を考える;

(N) *If $g(x,n) \cap g(x_n,n) \neq \phi$ for each $n \in N$, then x is a cluster point of the sequence $\langle x_n \rangle$,*

(wN) *If $g(x,n) \cap g(x_n,n) \neq \phi$ for each $n \in N$, then the sequence $\langle x_n \rangle$ has a cluster point,*

(γ) *If $x_n \in g(y_n,n)$ and $y_n \in g(x,n)$ for each $n \in N$, then x is a cluster point of the sequence $\langle x_n \rangle$,*

($w\gamma$) *If $x_n \in g(y_n,n)$ and $y_n \in g(x,n)$ for each $n \in N$, the sequence $\langle x_n \rangle$ has a cluster point,*

(1st) *If $x_n \in g(x,n)$ for each $n \in N$, then x is a cluster point of the sequence $\langle x_n \rangle$,*

(q) *If $x_n \in g(x,n)$ for each $n \in N$, the sequence $\langle x_n \rangle$ has a cluster point,*

(wM) *If $x_n \in g(y_n,n)$, $g(y_n,n) \cap g(z_n,n) \neq \phi$ and $z_n \in g(x,n)$ for each $n \in N$, then the sequence $\langle x_n \rangle$ has a cluster point,*

(α) *For each $x \in X$, $\cap \{g(x,n) \mid n \in N\} = \{x\}$ holds and, if $y \in g(x,n)$, then $g(y,n) \subseteq g(x,n)$.*

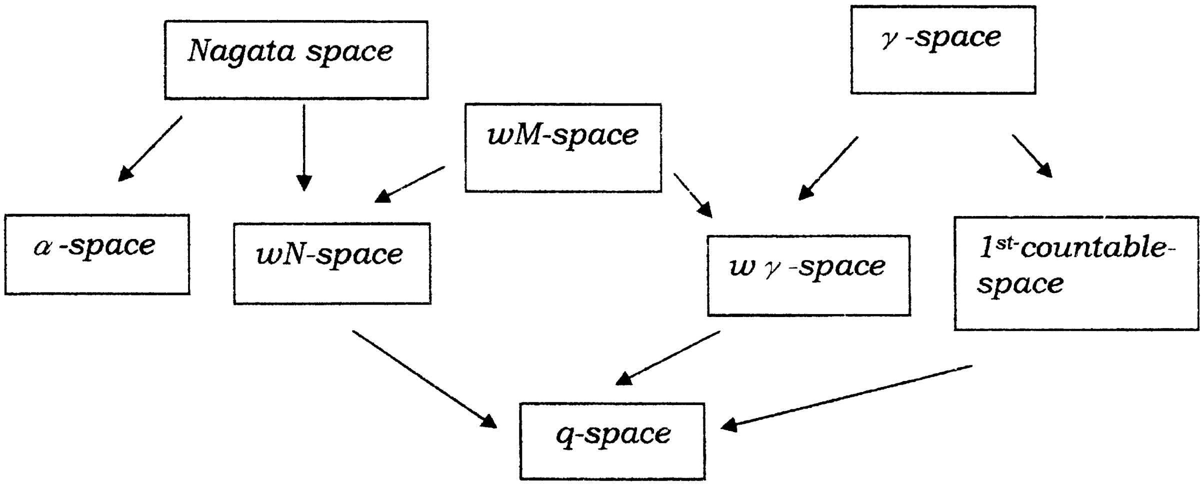
ここでは、g-function を用いて空間の定義を与える。

Definition 2.2. For a space (X, τ) with a *g*-function $g : X \times N \to \tau$,

(1) X is a *Nagata space* if g satisfies the condition (N),
(2) X is a *wN-space* if g satisfies the condition (wN),
(3) X is a γ*-space* if g satisfies the condition (γ),
(4) X is a $w\gamma$*-space* if g satisfies the condition ($w\gamma$),
(5) X is a *1st-counable space* if g satisfies the condition (1st),
(6) X is a *q-space* if g satisfies the condition (q),
(7) X is a *wM-space* if g satisfies the condition (wM),
(8) X is an α*-space* if g satisfies the condition (α).

Nagata space については[1][4][6]、wN-space、γ- space、$w\gamma$-space、1st-countable space、q-space についてはそれぞれ[6]、wM-space については[6][7]、そして α-space については[5]にオリジナルの定義または *g*-function による特徴づけがある。

これらの空間の関係は次の通りである。



3. Kotake の定理と新しい空間の定義

Nagata space と wN-space の関係について、Kotake は次の定理を示した。

Theorem 3.1[9, Theorem1.3] *A space is a Nagata space if and only if it is a regular,* α *and wN-space.*

ここで、α-space の概念に着目して、wN-*space* の構造を含む空間を定義する。

Definition 3.2. For a space (X, τ),

(1) X is called *α_0-wN-space*, if there exists a *g*-function g: $X \times N \rightarrow \tau$ satisfying the conditions (α_0) and $(w\mathrm{N})$, where
(α_0) if $y \in g(x,n)$, then $g(y,n) \subseteq g(x,n)$,

(2) X is called *α-wN-space*, if there exists a *g*-function g: $X \times N \rightarrow \tau$ satisfying the conditions (α) and $(w\mathrm{N})$,

(3) X is called *sα-wN-space*, if there exists a *g*-function g: $X \times N \rightarrow \tau$ satisfying the conditions $(s\alpha)$ and $(w\mathrm{N})$, where
$(s\alpha)$ For for each $x \in X$, $\cap \{Cl(g(x,n)) \mid n \in N\} = \{x\}$ holds and,
if $y \in g(x,n)$, then $g(y,n) \subseteq g(x,n)$.

定義より、これらの空間の関係は次の通りである。

sα-wN-space	→	*α-wN-space*	→	*α_0-wN-space*

上の定義から α-space に関連して、次の空間が定義される。ここで、strongly α-space は Yoshioka が[15]で定義した。

Definition 3.3. For a space (X, τ),

(1) X is called *α_0-space*, if there exists a *g*-function g: $X \times N \rightarrow \tau$ satisfying the condition (α_0),

(2) X is called *strongly α-space*, if there exists a *g*-function g: $X \times N \rightarrow \tau$ satisfying the condition $(s\alpha)$.

Remark 3.4. Every strongly α-space is T_2 (hence, sα-*w*N-space is T_2).

ここで、注意しなければならない事柄は、《α-*w*N-space》と《α, *w*N-space》は同じ空間を意味しないことである。前者は条件(α)と(*w*N)を同時に満たす *g*-function が存在する空間を、後者は条件(α)を満たす *g*-function g と条件(*w*N)を満たす *g*-function h が存在する空間を意味する。

4. sα-wN-space, α-wN-space そして α_0-wN-space

Definition 3.3 で定義した空間が、よく知られている空間とどのような関係にあるかを調べると、つぎの結果が得られる。

Proposition 4.1. *For a space X, the following statements hold:*

(1) *If X is a countably compact space, then X is an* α_0*-wN-space.*
(2) *If X is an* α_0*-wN-space, then X is a wM-space..*

Proof. (1): For each $(x,n)\in X\times N$, define a *g*-function $g(x,n)=X$. Then this *g*-function g satisfies the conditions (α_0) and $(w\mathrm{N})$.
(2):Let g be *g*-function satisfying condition $(\alpha_0)$and$(w\mathrm{N})$. To show that X is wN, it is sufficient that g-function g satisfies condition $(w\gamma)$, because of [12; Theorem 5.2] . Let $x_n\in g(y_n,n)$ and $y_n\in g(x,n)$ for each $n\in N$, then $g(y_n,n)\subseteq g(x,n)$ by the (α_0)-ness of *g*-function g. So $x_n\in g(x,n)\cap g(x_n,n)$, and g satisfies the condition $(w\mathrm{N})$. Thus, $\langle x_n\rangle$has a cluster point.

上の命題(1)に関連して、coutably compact spaces と α_0-wN-spaces の間に位置する空間について、後で関連する問題として述べる。
また、(2)の逆は成り立つかどうか？　不明である。

Question 4.2. *Does there exist a wM-space which is not* α_0*-wN ?*

なお、wM-space と α_0-wN*-space* の関係については、次が示される。

Proposition 4.3. *Every subparacompact wM-space is an* α_0*-wN-space.*

Proof. Let X be a subparacompact wM-space. Since X is wN, X is matacompact by [6;Corollary 3.5]. Since X is wM, there is a sequence $\langle \gamma_n\rangle$of open covers of X such that $x_n\in st^2(x,\gamma_n)$ for each $n\in N$, then $\langle x_n\rangle$ has a cluster point (this is the original definition by Ishii [7]). For each $n\in N$, let δ_n be a point-finite open refinement of γ_n. Let $g(x,n)=\cap\{U\in\delta_n \mid x\in U\}$ for each $(x,n)\in X\times N$. Then it is easily seen that *g*-function g satisfies the condition (α). Now let $g(x,n)\cap g(x_n,n)\neq\phi$ for each $n\in N$, then $x_n\in st^2(x,\delta_n)\subseteq st^2(x,\gamma_n)$ for each $n\in N$. Thus $\langle x_n\rangle$has a cluster point, so X is α_0-wN.

sα-wN-space と α-wN-space については次が成り立つことがわかる。

Theorem 4.4. *For a space X, the following conditions are equivalent:*

(1) *X is a metrizable space.*
(2) *X is a s*α*-wN-space.*

(1) X is a regular α-wN-space.

Proof. (1)⇒(2):For each $n\in N$, let $\beta_n=\{B(x;1/n)\mid x\in X\}$, where $B(x;1/n)$ is the $1/n$-neighbourhood of x and let ζ_n be a locally finite closed refinement of β_n. For each $(x,n)\in X\times N$, define a g-function $g(x,n)=X\setminus\cup\{F\in\zeta_n\mid x\notin F\}$. To verify this g-function satisfies condition (N), let $g(x,n)\cap g(x_n,n)\neq\phi$ for each $n\in N$. There exist $y_n\in g(x,n)\cap g(x_n,n)$, $F\in\zeta_n$ and $B\in\beta_n$ such that $x_n, x\in F\subseteq B$. Then $g(x,n)\subseteq st(x,\beta_n)$ and $x_n\in st(x,\beta_n)$. It follows that the sequence $\langle x_n\rangle$ clusters at x. So, g satisifies the condition (wN). And it is obvious that this g-function satisfies the condition ($s\alpha$).
(2) ⇒(3):Let g be a g-function with conditions($s\alpha$)and(wN). To show the regulality of X, let any $x\in X$ and any open set U with $x\in U$. Suppose that for each $n\in N$, there exist $x_n\in Cl(g(x,n))\setminus U$. Then $g(x,n)\cap g(x_n,n)\neq\phi$ for each $n\in N$, so there is a cluster point $p\in X\setminus U$ of the sequence $\langle x_n\rangle$. Now we have for each $n\in N$, $p\in cl(\{xk\mid k\geq n\}\subseteq Cl(g(x,n))$, so $p\in\cap\{Cl(g(x,n))\mid n\in N\}=\{x\}$. Hence p=x, this is a contradiction.
(3) ⇒(1):Since X is regular α and wN-space, X is Nagata by Theorem 3.1. And X is wM by Proposition 4.1(2), it follows that X is $w\gamma$. By [6; Theorem 4.7] , X is metrizable.

$s\alpha$-wN-space(=metrizable space), α-wN-space そして α_0-wN-space の概念は互いに異なるものであることが、次の例によって示される。

Example 4.5. There exists an *α-wN-space* which is not *$s\alpha$-wN.*

Proof. Let X be the space N with the cofinite topology $\{U\subseteq N\mid |N\setminus U| \text{ is finite}\}\cup\phi$. It is well known that X is compact T_1 but not T_2. Since X is not T_2, X is not $s\alpha$-wN. To show that X is a α-wN-space, let $g(x,n)=\{x\}\cup\{k\geq n\}$ for each $(x,n)\in X\times N$. Then g is a g-function satisfying the condition(α) and by the compactness of X, g satisfies the condition (wN) (see [15;Example 4.10] .

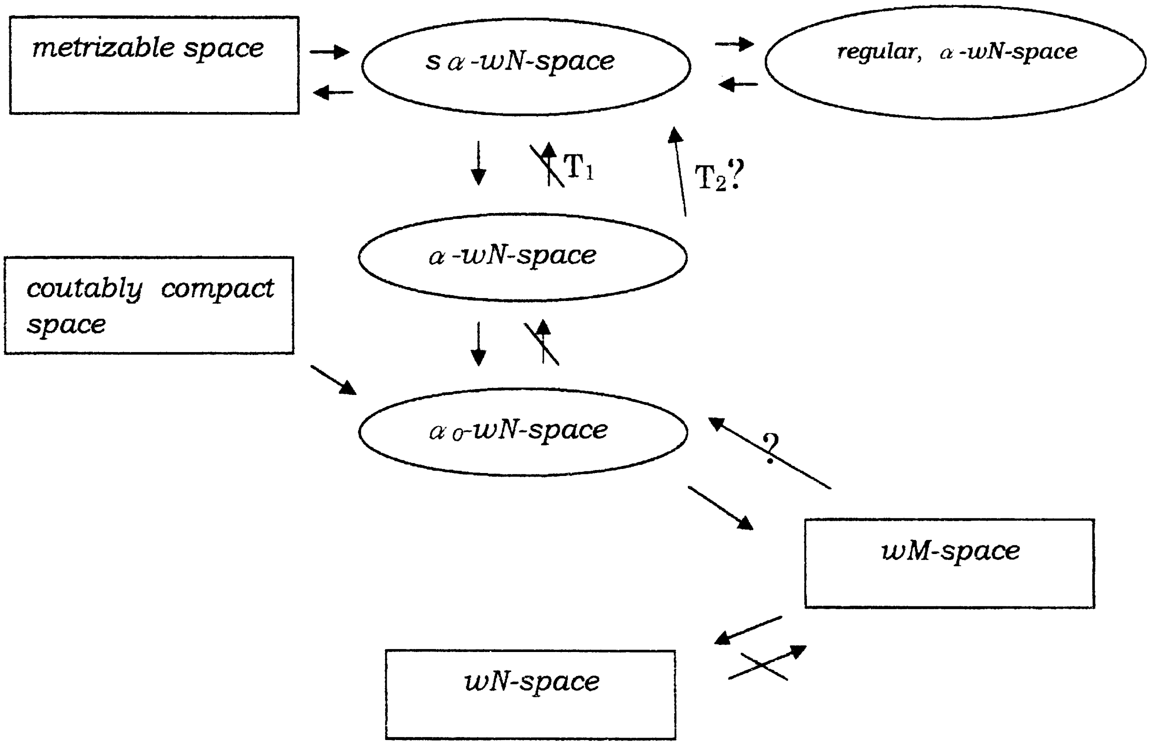
Example 4.5 の空間は T_2-space でない。Theorem 4.4 の結果より次の質問が考えられる。

Question 4.6. *Is every T_2 α-wN-space, metrizable ?*

Example 4.7. There exists an *α_0-wN-space* which is not *α-wN.*

Proof. Let X be the space of all coutable ordinal numbers with order topology. Since X is countably compact, X is α_0–wN by Proposition 4.1(1). Suppose that X is an α-wN-space. Then X is metrizable by Theorem4.4. This is a contradiction.

このセクションの内容は次のようになる。



なお、wN-space であって、wM でない空間の例は Remark 5.4 を参照のこと。

5. α-*wN*-space および α_0-*wN*-space の quasi-perfect image

Theorem 4.4 より sα–wN-space は metrizable であるから sα–wN-space の quasi-perfect image は sα–wN-space(=metrizable space) である。

α_0-wN-space および α-wN-space の quasi-perfect image については次の定理が成り立つ。

Theorem 5.1. *The following statements hold:*

(1) *The quasi-perfect image of an α_0–wN-space is α_0–wN.*

(2) *The quasi-perfect image of an α–wN-space is α–wN.*

Proof. (1): Let $f : X \to Y$ be a quasi-perfect map, and X an α_0–wN-space. The space X has a g-function g satisfying the conditions (α_0) and (wN). Let $h(y,n)=Y \setminus f(X \setminus g(f^{-1}(y),n))$ for each $(y,n) \in Y \times N$, then Y has a g-function h by the closedness of f. We will show that h satisfies the condition (α_0) and (wN). Let $z \in h(y,n)$, then for each $u \in f^{-1}(z)$ there exists $x_u \in f^{-1}(y)$ such that $u \in g(x_n,n)$. Since g satisfies the condition (α_0), $g(u,n) \subseteq g(x_u,n) \subseteq g(f^{-1}(y),n)$. Then $g(f^{-1}(z),n) \subseteq g(f^{-1}(y),n)$. It follows that $h(z,n) \subseteq h(y,n)$ holds. Next to verify that h satisfies the condition (wN), let $z_n \in h(y,n) \cap h(y_n,n)$ for each $n \in N$. Let $n \in N$, there exist u_n and w_n such that $u_n \in f^{-1}(y)$ and $w_n \in f^{-1}(z_n) \cap g(u_n,n)$, and since $z_n \in h(y_n,n)$, there is an $x_n \in f^{-1}(y_n)$ such that $w_n \in g(x_n,n)$. The sequence $\langle u_n \rangle$ has a cluster point p in $f^{-1}(y)$ by the countable compactness of $f^{-1}(y)$. For each $k \in N$, there is a $u_{n(k)} \in g(p,k)$ with $n(k)<n(k+1)$. Since g satisfies the condition (α_0), $g(u_{n(k)}, n(k)) \subseteq g(p,n(k)) \subseteq g(p,k)$ hold. Then $w_{n(k)} \in g(p,k) \cap g(x_{n(k)},k)$ for each $k \in N$. Since g satisfies the condition (wN), the sequence $\langle x_{n(k)} \rangle$ clusters, so $\langle x_n \rangle$ has a cluster point. Thus $\langle f(x_n) \rangle$ has a cluster point, that is, $\langle y_n \rangle$ has a cluster point.

(2): Let $f : X \to Y$ be a quasi-perfect map, and X an α–wN-space. The space X has a g-function g satisfying the conditions (α) and (wN). Let $h(x,n)=Y \setminus f(X \setminus g(f^{-1}(y),n))$ for each $n \in N$, then Y has a g-function h by the closedness of f. We will show that h satisfies the conditions (α) and (wN).From the proof of (1), we only show that $\cap\{h_n(y) \mid n \in N\}=\{y\}$ for each $y \in Y$. Suppose that there is a $y \in Y$ such that $h_n(y) \neq \{y\}$. Then there is an $x \in \cap\{g(f^{-1}(y),n) \mid n \in N\} \setminus f^{-1}(y)$. For each $n \in N$, there exists $x_n \in f^{-1}(y)$ such that $x \in g(x_n,n)$. The sequence $\langle x_n \rangle$ has a cluster point p in $f^{-1}(y)$ by the countable compactness of $f^{-1}(y)$. For each $k \in N$, there is an $n(k) \in N$ such that $x_{n(k)} \in g(y,k)$ with $n(k)<n(k+1)$. Then $x \in g(x_{n(k)},n(k)) \subseteq g(x_{n(k)},k) \subseteq g(p,k)$ hold. Thus we have $x \in \cap\{g(p,n) \mid n \in N\}=\{p\}$, so $x=p$. This is a contradiction.

Remark 5.2. The fact that the quasi-perfect image of an α-spaces is also α can be shown in the same manner as the proof of Theorem 5.1.

Remark 5.3. In [8], Ishi showed that the quasi-perfect image of a wM-space is also a wM-space.

Remark 5.4. As stated in section1, Lutzer showed in [10, Example4.3] there exists a perfect map $f:X \to Y$ where X is a Nagata space and Y is not q-space. We can find this space X is not wM by Proposition 4.3 and Theorem 5.1(1). Thus, this space X is a wN space which is not wM.

ところで、metrizability が closed map で保存されないことはよく知られている。

Example 5.5 (see[11, Example 10.1]). A closed image of a metrizable space is not a q-space.

この例は、sα-wN, α-wN そして α_0-wN のそれぞれの性質は closed map で保存されないことをも示している。

6. Nagata space の場合

これまでは wN-spaces に関して述べてきた。同様な事柄を Nagata-space について調べてみる。

Definition 6.1. For a space (X, τ),
X is called an *α_0-Nagata space*, if there exists a *g*-function g: X × N→ τ satisfying the conditions (α_0) and (N).

Theorem 6.2. *For a space X, the following conditions are equivalent:*

(1) *X is a metrizable space.*
(2) *X is an α_0-Nagta space.*

Proof. (1) ⇒(2):In the proof of Theorem 4.4(1) ⇒(2), we have shown this implication.
(2) ⇒(1):Let g be a *g*-function satisfying the condition (α_0) and (N). We will show that g satisfies the condition (γ). Let $x_n \in g(y_n,n)$ and $y_n \in g(x,n)$ for each $n \in N$, then $x_n \in g(y_n,n) \subseteq g(x,n)$, because g satisfies ($\alpha_0$). Since $x_n \in g(x,n) \cap g(x_n,n)$ and g satisfies the condition (N), the sequence $\langle x_n \rangle$ clusters at x. Thus, X is γ. By [6;Theorem 4.7], X is metrizable.

Definition 3.2 (2),(3) と同様に、α-Nagata space と sα-Nagata space を定義することは可能である。しかし、Theorem 4.4 の証明より、α-Nagata space および sα-Nagata space は metrizable であることがわかる。

Nagata space と wN-space の関係について述べられていた Theorem 3.1 と同様な定理が成り立つ。

Theorem 6.3. A *space is a Nagata space if and only if it is a strong α, wN-space.*

Proof. Let X be a Nagata space. We will show that X is strong α. Since every Nagata space is semi-stratifiable and paracompact T_2, X has a G_δ -diagonal sequnce $\langle \delta_n \rangle$. And for each $n \in N$, there is a locally finite closed refinement ζ_n of δ_n. For each $(x,n) \in X \times N$, let $g(x,n)=X \setminus \cup\{F \in \zeta_n \mid x \notin F\}$. Then it is easily verified that if $y \in g(x,n)$, then $g(y,n) \subseteq g(x,n)$ holds. We will show that $x \in \cap\{Cl(g(x,n)) \mid n \in N\}=\{x\}$ for each $x \in X$. For any $y \in X$ with $y \neq x$, there is an $m \in N$ such that $y \notin st(x, \zeta_m)$. Then we have $g(x,m) \cap g(y,m)=\phi$. Suppose that there is a $z \in g(x,m) \cap g(y,m)$, then $z \in F \in \zeta_m$. Since ζ_m is a refinement of δ_m, we have $x, y \in F \subseteq G$ for some $G \in \delta_m$. Then we have that $y \in st(x, \delta_m)$, this is a contradiction. Thus, X is a strongly α, wN-sapce.

Conversely, Let X be a strongly α, wN-space. Let h be a g-function satisfying condition (sα), and k g-function satisfying condition (wN). Let $g(x,n)=h(x,n) \cap k(x,n)$ for each $(x,n) \in X \times N$, then g is a g-function. We will verify that X is regular. Suppose that U be an open neighbourhood of x and $x_n \in Cl(g(x,n)) \setminus U$ for each $n \in N$. Since $k(x,n) \cap k(x_n,n) \neq \phi$, the sequence $\langle x_n \rangle$ has a cluster point $p \notin X \setminus U$. And $p \in cl(\{xk \mid k \geq n\} \subseteq Cl(g(x,n))$ for each $n \in N$. On the other hand, $\cap\{Cl(h(x,n)) \mid n \in N\}=\{x\}$, since h satisfies the condition (sα). Then, $p \in \cap\{Cl(g(x,n)) \mid n \in N\} \subseteq \cap\{Cl(h(x,n)) \mid n \in N\}=\{x\}$, so we have x=p. This is a contradiction. Thus, X is a regular α, wN-space. By Theorem 3.1, X is Nagata.

7. 関連する問題

Proposition4.1(1) で countably compact space は α_0-wN-space であることを述べた。countably compact の一般化された空間としてよく知られているものとして Morita が定義した M-space がある。さらに、M-space の一般化として、Siwiec and Nagata は[13]において $M^\#$-space を定義した。

Definition 7.1. A space X is called an $M^\#$-space if it has a sequence $\{\zeta_n\}$ of closure-preserving closed covers of X such that whenever $x_n \in st(x, \zeta_n)$ for each $n \in N$, then the sequence $\langle x_n \rangle$ has a cluster point.

$M^\#$-space に対して次の命題が成り立つ。

Proposition 7.2. *Every $M^\#$-space is an α_0-wN-space.*

Proof. Let $\langle \zeta_n \rangle$ be a sequence of closure-preserving closed covers of X such that whenever $x_n \in st(x, \zeta_n)$ for each $n \in N$, then the sequence $\langle x_n \rangle$ has a cluster point, where we may assume that ζ_{n+1} is a refinement of ζ_n. Let $g(x,n)=X \setminus \cup\{F \in \zeta_n \mid n \in N\}$ for each $n \in N$, then the g-function g satisfies condition (α_0). And let $g(x,n) \cap g(x_n,n) \neq \phi$ for each $n \in N$, then $x_n \in st(x, \zeta_n)$ for each $n \in N$. So the sequence $\langle x_n \rangle$ has a cluster point. Thus X is α_0-wN.

上の命題について、逆が成り立つかどうか不明である。

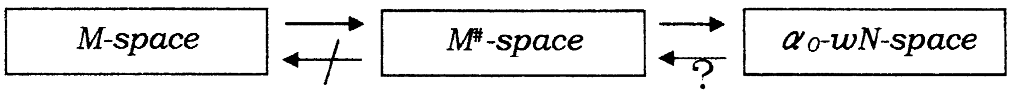
Question 7.3. *Does there exist an α_0-wN-space which is not $M^{\#}$?*

なお、α_0-wN-space と $M^{\#}$-space の関係については、次が示される。

Proposition 7.4. Every paracompact T_2, wM-space is an $M^{\#}$-space.

Proof. Let X be a paracompact T_2, wM-space. Let $\{\gamma_n\}$ be a sequence of open covers of X such that $x_n \in st^2(x, \gamma_n)$ for each $n \in N$, then $\langle x_n \rangle$ has a cluster point. For each $n \in N$, there is a locally finite closed refinement ζ_n of γ_n. Then the sequence $\{\zeta_n\}$ is the one of closure-preserving closed covers of X such that whenever $x_n \in st(x, \zeta_n)$ for each $n \in N$, then the sequence $\langle x_n \rangle$ has a cluster point. Thus X is an $M^{\#}$-space.

M-space, $M^{\#}$-space そして α_0-wN-space の関係は次の通りである。



$M^{\#}$-space であって M-space でない例は[12]で示されている。

最後に、wN-space および Nagata space の距離化について、Hodel が示した定理に着目する。

Theorem 7.5. *The following statements hold:*

(1)[6;Theorem 4.3] *Every T_2 γ, wN-space is metrizable.*

(2)[6;Theorem 4.7] *Every $w\gamma$, Nagata space is metrizable.*

この定理より、次の2つの問題は Question 4.6 と同値な問題となる。

Question 7.6. Is every T_2 α-wN-space, Nagata ?

Question 7.7. Is every T_2 α-wN-space, γ ?

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