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An Improved Recursive Decomposition Ordering for Term Rewriting Systems Revisited

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Abstract

Simplification orderings, like the recursive path ordering and the improved recursive decomposition ordering, are widely used for proving the termination property of term rewriting systems. The improved recursive decomposition ordering is known as the most powerful simplification ordering.

In this paper, we investigate the improved recursive decomposition ordering for proving termination of term rewriting systems. We completely show that the improved recursive decomposition ordering is closed under substitutions.

Keywords: Term rewriting system, Termination, Improved recursive decomposition ordering, Simplification ordering

1 Introduction

Term rewriting systems (TRSs, for short) are regarded as a computation model that reduces terms by applying directed equations, called rewrite rules. TRSs are widely used as a model of functional and logic programming languages and as a basis of automated theorem proving, symbolic computation, algebraic specification and verification [1, 15, 23].

The terminating property is fundamental notion of TRSs as computation models [4]. Since the terminating property of TRS is undecidable in general [5], several sufficient conditions for proving this property have been successfully developed in particular cases. These techniques can be classified into two approaches: semantic methods and syntactic methods.

Simplification orderings are representatives of syntactic methods [18, 21]. Many simplification orderings (for instance, the recursive path ordering (with status) (RPO(S), for short) [2, 10], the recursive decomposition ordering (with status) (RDO(S), for short) [8, 13, 14], the improved recursive decomposition ordering (with status) (IRD(S), for short) [17, 19] and so on) have been defined on TRSs. IRDS is among the most powerful simplification orderings [19, 20].

First, Jouannaud, Lescanne and Reinig defined the recursive decomposition ordering with multiset status [8]. They said that the closure under substitutions of it is straightforward using definition of decomposition. However they did *not* give the formal proof of it.

The recursive decomposition ordering with arbitrary status (RDOS) was first described by Lescanne [13]. Complete proofs concerning the lexicographical status are given by Lescanne [14]. An implementation of recursive decomposition ordering with multiset status has been made in the first rewriting environment with tools for proving termination called REVE as it was a convenient tools for proposing extension of the precedence [12].

Rusinowitch [17] gave the definition of the improved recursive decomposition ordering (IRD) and investigated the relationship between several simplification orderings : the path of subterm ordering (PSO) [16], the recursive path ordering (RPO) and the recursive decomposition ordering (RDO). But they did *not* discuss that IRD is closed under substitutions.

Steinbach [19] gave the definition of the improved recursive decomposition ordering *with status* (IRDS) based on IRD defined by Rusinowitch [17] and compared of the power as well as the time behavior of all orderings suggested [18, 20, 22]. They showed that IRDS is a simplification ordering and IRDS is closed under substitutions [18, 19], however their proof was *not* complete. They used the proposition that for any substitution θ, $dec_p(s) \gg_{EL} dec_q(t)$ implies $dec_{p'}(s\theta) \gg_{EL} dec_{q'}(t\theta)$ as key idea in their proof without proving. But this proposition was not trivial. So, we need give the formal proof of it by induction on $|s| + |t|$ in this paper.

We proposed IRDS for higher-order rewrite systems, called the higher-order improved recursive decomposition ordering (HIRDS, for short) [6, 7]. Our method was inspired by Jouannaud and Rubio's idea for RPOS [9] and particular properties of IRDS. We showed that our ordering is a more powerful ordering than their ordering. Furthermore we showed that HIRDS is closed under substitutions. However our proof was very complicated and generalized, so we try to show that IRDS is closed under substitutions in this paper. Furthermore we review that IRDS is a simplification ordering.

In section 2 we give the basic notations. Section 3 presents the definition of the improved recursive decomposition ordering with status (IRDS) and we completely show that IRDS is closed under substitutions. Also, we review that the IRDS is a simplification ordering.

2 Preliminaries

We mainly follow the basic notations of [11, 19]. An *abstract reduction system* (ARS for short) is a pair $\langle A, \to \rangle$ consisting of a set A and a binary relation $\to \subseteq A \times A$. We say that ARS $\langle A, \to \rangle$ is *terminating* if there is no infinite sequence $a_0 \to a_1 \to a_2 \to \ldots$ of elements in A. A binary relation on a set A is called a *(strict) partial ordering* over A if it is a irreflexive and transitive on A. The partial ordering is usually denoted by $>$. A partial ordering $>$ on a set A is *well-founded* if $>$ has no infinite descending sequences, i.e., there is no sequence of the form $a_0 > a_1 > a_2 > \ldots$ of elements in A.

A *signature* is a set of *function symbols*. Associated with $f \in \mathcal{F}$ is a natural number denoting its arity. Function symbols of arity 0 are called *constants*. Let $\mathcal{T}(\mathcal{F}, \mathcal{V})$ be the set of all terms built from $\mathcal{F}$ and a countably infinite set $\mathcal{V}$ of *variables*, disjoint from $\mathcal{F}$. The set of variables occurring in a term t is denoted by $\mathcal{V}(t)$. The *root symbol* of a term t is defined as follows: $root(t) = t$ if t is a variable and $root(t) = f$ if $t = f(t_1, \cdots, t_n)$.

A *substitution* is a map θ from $\mathcal{V}$ to $\mathcal{T}(\mathcal{F}, \mathcal{V})$ with the property that the set $\{x \in \mathcal{V} \mid \theta(x) \neq x\}$ is finite. If θ is a substitution and t a term then $t\theta$ denotes the result of applying θ to t. We call $t\theta$ an *instance* of t.

We introduce a fresh constant symbol $\square$, named *hole*. A *context* $C[\,]$ is a term in $\mathcal{T}(\mathcal{F} \cup \{\square\}, \mathcal{V})$ containing precisely one hole. If $C[\,]$ is a context and t a term then $C[t]$ denotes the result of replacing the hole in $C[\,]$ by t.

A binary relation R on terms is *closed under substitutions* if $s\ R\ t$ implies $s\theta\ R\ t\theta$, for any substitution θ. And a binary relation R on terms is *closed under contexts* if $s\ R\ t$ implies $C[s]\ R\ C[t]$, for any context $C[\,]$. $|t|$ denotes the size of term t, i.e., the total number of function symbols and variables occurring in t. Terms are identified with finite labeled trees. A *position* in a term can be viewed as a finite sequence of natural numbers, pointing out a path from the root of this tree. $\mathcal{P}(t)$ denotes the set of all positions of a term t. $\mathcal{P}_\mathcal{T}(t)$ denotes the set of all *terminal positions* (positions of all leaves) of the term t. The letter ϵ denotes *root positions*. We write $w \preceq z$ if w is a prefix of z. The *subterms* of t at position p is denoted by $t|_p$, and we write $t \trianglerighteq t|_p$. If $t \trianglerighteq t|_p$ and $t \neq t|_p$ then $t|_p$ is called the *proper subterm* of t, denoted by $t \triangleright t|_p$.

A *rewrite rule* on $\mathcal{T}(\mathcal{F}, \mathcal{V})$ is a pair of terms $l \to r$ such that $l \notin \mathcal{V}$ and $\mathcal{V}(r) \subseteq \mathcal{V}(l)$. A *term rewriting system* (TRS, for short) is a pair $(\mathcal{F}, \mathcal{R})$ where $\mathcal{F}$ is a set of function symbols and $\mathcal{R}$ is a set of rewrite rules on $\mathcal{T}(\mathcal{F}, \mathcal{V})$. $(\mathcal{F}, \mathcal{R})$ is often abbreviated as $\mathcal{R}$ and in that case $\mathcal{F}$ is defined to be the set of function symbols that appear in $\mathcal{R}$. We often present a TRS as a set of rewrite rules, without making explicit its signature, assuming that the signature consists of the function symbols occurring in the rewrite rules. The smallest rewrite relation on $\mathcal{T}(\mathcal{F}, \mathcal{V})$ that contains $\mathcal{R}$ is denoted by $\to_\mathcal{R}$. So $s \to_\mathcal{R} t$ if there exists a rewrite rule $l \to r$ in $\mathcal{R}$, a substitution θ, and a context C such that $s = C[l\theta]$ and $t = C[r\theta]$. The subterm $l\theta$ of s is called a *redex* and we say that s rewrites to t by *contracting* redex $l\theta$. We call $s \to_\mathcal{R} t$ a *rewrite* or *reduction step*.

Given a binary relation $>$, the *multiset extension* $\gg$ is defined as the transitive closure of the following relation $\Rightarrow$ on multisets. $M \cup \{s\} \Rightarrow M \cup \{t_1, \ldots, t_n\}$ where $n \geq 0$ and $s > t_i$ for any $i \in \{1, \ldots, n\}$. Assume $>$ is a well-founded ordering on a set A. Then $\gg$ is a well-founded ordering on the multisets of elements in A [3]. We say that a binary relation R on terms has the *subterm property* if $C[t]Rt$ for any context $C[\,] \neq \square$ and term t.

3 Improved Recursive Decomposition Ordering Revisited

Throughout this section we are dealing with *finite* signatures only.

Definition 3.1 *([2, 4, 11]) A* simplification ordering *on $\mathcal{T}(\mathcal{F},\mathcal{V})$ is a partial ordering that is closed under substitutions, contexts and has the subterm property.*

Since we are dealing with finite signatures only, we obtain the following result.

Theorem 3.2 *([2, 4, 11]) Simplification orderings are well-founded.*

We obtain the following theorem from the result of Dershowitz [2].

Theorem 3.3 *Let $\mathcal{R}$ be a TRS and let $>$ be a simplification ordering on $\mathcal{T}(\mathcal{F},\mathcal{V})$. If $l > r$ for any $l \to r \in \mathcal{R}$ then $\mathcal{R}$ is terminating.*

Proof. Assume that $s \to_{\mathcal{R}} t$, where s and t are terms. There exists a rewrite rule $l \to r$ in $\mathcal{R}$, a substitution θ and a context $C[\,]$ such that $s = C[l\theta]$ and $t = C[r\theta]$. By the assumption $l > r$ and definition 3.1, hence $s = C[l\theta] > C[r\theta] = t$ holds. Since $>$ is well-founded on $\mathcal{T}(\mathcal{F},\mathcal{V})$ by theorem 3.2, $\mathcal{R}$ is terminating. □

The improved recursive decomposition ordering depends on a partial ordering $>_{\mathcal{F}}$ on the signature $\mathcal{F}$, the so-called *precedence.*

A *status* function τ is assumed, mapping every $f \in \mathcal{F}$ to either mul or lex_π for some permutation π on n elements, where n is arity of f. For a partial ordering $>$ on $\mathcal{T}(\mathcal{F},\mathcal{V})$ the partial ordering $>^{\tau(f)}$ is defined on sequences of length n : $\tau(f) = mul$ describes multiset extension and $\tau(f) = lex_\pi$ describes lexicographic comparison according to the permutation π. The results of an application of the function $args$ to a term $t = f(t_1,\ldots,t_n)$ depend on the status of f : If $\tau(f) = mul$, then $args(t)$ is the multiset $\{t_1,\ldots,t_n\}$ and otherwise, $args(t)$ is the tuple $(t_1,\ldots,t_n)$.

For $u \in \mathcal{P}_{\mathcal{T}}(t)$, a *path-decomposition* $dec_u(t) = \{t|_v \mid u \succeq v\}$ is a set of subterms of t. Note that $i.v \in \mathcal{P}_{\mathcal{T}}(f(t_1,\ldots,t_n))$ implies $v \in \mathcal{P}_{\mathcal{T}}(t_i)$. We also define a *decomposition dec* $(\{t_1,\ldots,t_n\}) = \{dec_u(t_i) \mid i \in \{1,\ldots,n\}, u \in \mathcal{P}_{\mathcal{T}}(t_i)\}$. A decomposition is a multiset of all path-decomposition of the terms $t_1,\ldots,t_n$. For the path-decomposition $dec_u(t)$, a set of subterms of $dec_u(t)$, $sub(dec_u(t),s) = \{s' \in dec_u(t) \mid s \rhd s'\}$.

We give the improved recursive decomposition ordering with status (IRDS) defined by Steinbach [19, 20] as following.

Definition 3.4 (IRDS) *Let s and t be terms. For a precedence $>_{\mathcal{F}}$ and a status τ the* improved recursive decomposition ordering with status *(IRDS) on $\mathcal{T}(\mathcal{F},\mathcal{V})$ is defined as follows:*

$s >_{IRDS} t \iff dec(\{s\}) \ggg_{EL} dec(\{t\})$ where $\ggg_{EL}$ is the multiset extension of $\gg_{EL}$.

$dec_p(u) \ni u' >_{EL} v' \in dec_q(v)$ is defined by the following (a), (b) and (c).

(a) *$root(u') >_{\mathcal{F}} root(v')$, or*

(b) *$root(u') = root(v')$, $\tau(root(u')) = mul$ and either*

- *$sub(dec_p(u),u') \gg_{EL} sub(dec_q(v),v')$, or*
- *$sub(dec_p(u),u') = sub(dec_q(v),v')$ and $dec(args(u')) \ggg_{EL} dec(args(v'))$, or*

(c) *$root(u') = root(v')$, $\tau(root(u')) \neq mul$, $args(u') >_{IRDS}^{\tau(root(u'))} args(v')$ and $\{u'\} \gg_{IRDS} args(v')$.*

Next, we give the example of comparison using IRDS.

Example 3.5 *We consider the term $s = \neg X \supset (Y \supset Z)$ and $t = Y \supset (X \vee Z)$ where $X, Y, Z \in \mathcal{V}$, $\mathcal{F} = \{\neg, \supset, \vee\}$ and $\tau(f) = mul$ for any $f \in \mathcal{F}$. We give the precedence as follows: $\neg >_{\mathcal{F}} \supset >_{\mathcal{F}} \vee$.*

We have $\mathcal{P}_{\mathcal{T}}(s) = \{11, 21, 22\}$ and $\mathcal{P}_{\mathcal{T}}(t) = \{1, 21, 22\}$. See figure 1.

Then $dec(\{s\}) = \{dec_{11}(s), dec_{21}(s), dec_{22}(s)\}$ where $dec_{11}(s) = \{s, \neg X, X\}$, $dec_{21}(s) = \{s, Y \supset Z, Y\}$ and $dec_{22}(s) = \{s, Y \supset Z, Z\}$.

Then $dec(\{t\}) = \{dec_1(t), dec_{21}(t), dec_{22}(t)\}$ where $dec_1(t) = \{t, Y\}$, $dec_{21}(t) = \{t, X \vee Z, X\}$ and $dec_{22}(t) = \{t, X \vee Z, Z\}$.

By the following cases (1), (2) and (3), $dec(\{s\}) \ggg_{EL} dec(\{t\})$ holds. Then $s >_{IRDS} t$ holds by definition of IRDS.

(1) $dec_{11}(s) \gg_{EL} dec_{21}(t)$ *holds.* $s >_{EL} t$ *and* $\neg X >_{EL} X \vee Z$ *since* $sub(dec_{11}(s), s) \gg_{EL} sub(dec_{21}(t), t)$ *and* $\neg >_{\mathcal{F}} \vee$.

(2) $dec_{21}(s) \gg_{EL} dec_{1}(t)$ *holds.* $s >_{EL} t$ *since* $sub(dec_{21}(s), s) \gg_{EL} sub(dec_{1}(t), t)$.

(3) $dec_{22}(s) \gg_{EL} dec_{22}(t)$ *holds.* $s >_{EL} t$ *and* $Y \supset Z >_{EL} X \vee Z$ *since* $sub(dec_{22}(s), s) \gg_{EL} sub(dec_{22}(t), t)$ *and* $\supset >_{\mathcal{F}} \vee$.

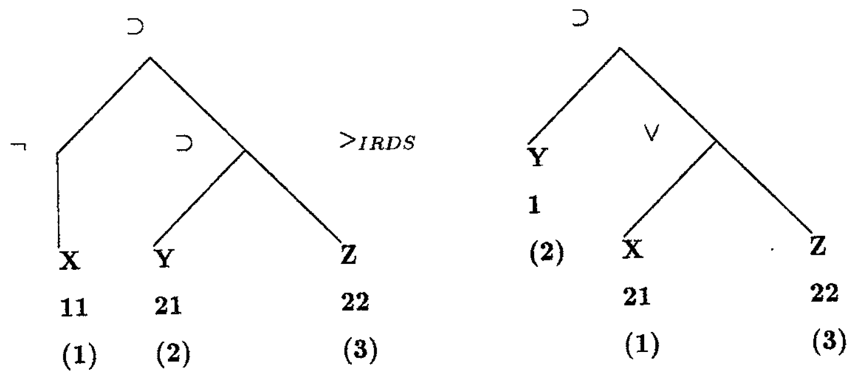

Figure 1: $\neg X \supset (Y \supset Z) >_{IRDS} Y \supset (X \vee Z)$.

We review that IRDS is a simplification ordering, i.e., IRDS is a partial ordering has the subterm property on $\mathcal{T}(\mathcal{F}, \mathcal{V})$ that is closed under substitutions and is closed under contexts. These properties are essential for applying IRDS to termination proof of TRS.

Lemma 3.6 *The IRDS is partial ordering on* $\mathcal{T}(\mathcal{F}, \mathcal{V})$.

Proof. Let $s' \in dec_p(s)$, $t' \in dec_q(t)$ and $u' \in dec_r(u)$. We can show that $dec_p(s) \ni s' >_{EL} t' \in dec_q(t)$ and $dec_q(t) \ni t' >_{EL} u' \in dec_r(u)$ imply $dec_p(s) \ni s' >_{EL} u' \in dec_r(u)$ by induction on $|s'| + |t'| + |u'|$. For any term s and s' in $dec_p(s)$, we can prove that $dec_p(s) \ni s' \not>_{EL} s' \in dec_p(s)$ by induction on $|s|$. □

Lemma 3.7 *The IRDS on* $\mathcal{T}(\mathcal{F}, \mathcal{V})$ *has the subterm property.*

Proof. Let s and t be terms such that $s \rhd t$. It is shown by induction on $|s|$ that $s >_{IRDS} t$. □

The following lemma is the *key* to prove the main result in this paper that IRDS is closed under substitutions.

Lemma 3.8 *Let* $dec_p(s) \gg_{EL} dec_q(t)$ *where* s *and* t *are terms and* $p \in \mathcal{P}_{\mathcal{T}}(s)$ *and* $q \in \mathcal{P}_{\mathcal{T}}(t)$. *Then for any substitution* θ, *the following two claims hold.*

(1) *If* $s|_p = t|_q \in \mathcal{V}$ *then* $dec_{p.i}(s\theta) \gg_{EL} dec_{q.i}(t\theta)$, *for any* $i \in N^*$ *such that* $p.i \in \mathcal{P}_{\mathcal{T}}(s\theta)$ *and* $q.i \in \mathcal{P}_{\mathcal{T}}(t\theta)$.

(2) *If* $t|_q \notin \mathcal{V}$ *then* $dec_{p.i}(s\theta) \gg_{EL} dec_q(t\theta)$, *for any* $i \in N^*$ *such that* $p.i \in \mathcal{P}_{\mathcal{T}}(s\theta)$.

□

Proof. See appendix A.

Lemma 3.9 *Let* s *and* t *be terms. Then* $dec(\{s\}) \gg\!\!\gg_{EL} dec(\{t\})$ *implies* $dec(\{s\}) \cap dec(\{t\}) = \emptyset$.

The following lemma is the *main result* in this paper. We completely show that IRDS is closed under substitutions.

Lemma 3.10 *The IRDS is closed under substitutions, i.e.,* $s >_{IRDS} t$ *implies* $s\theta >_{IRDS} t\theta$ *for any substitution* θ.

Proof. Assume that $s >_{IRDS} t$, i.e., $dec(\{s\}) \gg\!\gg_{EL} dec(\{t\})$ where s and t are terms. We show that $dec(\{s\theta\}) \gg\!\gg_{EL} dec(\{t\theta\})$, i.e., $s\theta >_{IRDS} t\theta$, holds for any substitution θ. Strictly speaking, we must prove: $(\forall q \in \mathcal{P}_{\mathcal{T}}(t), \exists p \in \mathcal{P}_{\mathcal{T}}(s)$ such that $dec_p(s) \gg_{EL} dec_q(t))$ implies $(\forall q' \in \mathcal{P}_{\mathcal{T}}(t\theta), \exists p' \in \mathcal{P}_{\mathcal{T}}(s\theta)$ such that $dec_{p'}(s\theta) \gg_{EL} dec_{q'}(t\theta))$. Let $q' \in \mathcal{P}_{\mathcal{T}}(t\theta)$, then $\exists i, q \in N^*$ such that $q' = q.i$ and $q \in \mathcal{P}_{\mathcal{T}}(t)$. Since $s >_{IRDS} t$ and by lemma 3.9, there exists $p \in \mathcal{P}_{\mathcal{T}}(s)$ such that $dec_p(s) \gg_{EL} dec_q(t)$. To prove that $\exists p' \in \mathcal{P}_{\mathcal{T}}(s\theta)$ such that $dec_{p'}(s\theta) \gg_{EL} dec_{q'}(t\theta)$, we have to distinguish two cases:

1. $t|_q \in \mathcal{V}$, i.e. $s|_p = t|_q$ (Otherwise $dec_p(s) \not\gg_{EL} dec_q(t)$).
   Since $q' = q.i$ for some $i \in N^*$, $dec_p(s) \gg_{EL} dec_q(t)$, $s\theta|_p = t\theta|_q = \theta(t|_q)$ and lemma 3.8, $\forall i \in \mathcal{P}_{\mathcal{T}}(\theta(t|_q))$, $dec_{p.i}(s\theta) \gg_{EL} dec_{q.i}(t\theta)$. Hence, $p' = p.i$ with $i \in \mathcal{P}_{\mathcal{T}}(\theta(t|_q))$.

2. $t|_q \notin \mathcal{V}$, i.e. $q' = q$.

   (1) $s|_p \notin \mathcal{V}$. Since $dec_p(s) \gg_{EL} dec_q(t)$ and lemma 3.8, $dec_p(s\theta) \gg_{EL} dec_q(t\theta)$. Hence, $p' = p$.

   (2) $s|_p \in \mathcal{V}$. Since $dec_p(s) \gg_{EL} dec_q(t)$ and lemma 3.8, $\forall i \in \mathcal{P}_{\mathcal{T}}(\theta(s|_p))$, $dec_{p.i}(s\theta) \gg_{EL} dec_q(t\theta)$. Hence, $p' = p.i$ with $i \in \mathcal{P}_{\mathcal{T}}(\theta(s|_p))$. □

Lemma 3.11 *The IRDS is closed under contexts.*

Proof. Let s and t be terms. We have to show that $s >_{IRDS} t$ implies $C[s] >_{IRDS} C[t]$ for any context $C[\,]$. It can be proved by induction on context $C[\,]$. □

Lemma 3.12 *The IRDS is a simplification ordering on $\mathcal{T}(\mathcal{F}, \mathcal{V})$.*

Proof. By lemmas 3.6, 3.7, 3.10 and 3.11, the IRDS is partial ordering on $\mathcal{T}(\mathcal{F}, \mathcal{V})$ that is closed under substitutions, contexts and has the subterm property. □

Example 3.13 ([18]) Given the following signature and TRS $\mathcal{R}$: $\mathcal{F} = \{\neg, \supset, \vee\}$,

$$R = \{\ \neg X \supset (Y \supset Z) \longrightarrow Y \supset (X \vee Z).$$

We give the following precedence and status: $\neg >_{\mathcal{F}} \supset >_{\mathcal{F}} \vee$ and $\tau(f) = mul$ for any $f \in \mathcal{F}$. Since $\neg X \supset (Y \supset Z) >_{IRDS} Y \supset (X \vee Z)$ by example 3.5 and theorem 3.3 and lemma 3.12, $\mathcal{R}$ is terminating.

4 Conclusion

We have investigated the improved recursive decomposition ordering to term rewriting systems for proving termination. We completely have shown that the improved recursive decomposition ordering is closed under substitutions as main result in this paper. Also we have reviewed the improved recursive decomposition ordering is a simplification ordering.

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A Proof of lemma 3.8

Definition A.1 *Let θ be a substitution. Let $\{s_1,\dots,s_n\}$ be a subset of $\mathcal{T}(\mathcal{F},\mathcal{V})$. $\{s_1,\dots,s_n\}\theta$ denotes $\{s_1\theta,\dots,s_n\theta\}$.*

Lemma A.2 *(Lemma 3.8) Let $dec_p(s) \gg_{EL} dec_q(t)$ where s and t be terms and $p \in \mathcal{P}_\mathcal{T}(s)$ and $q \in \mathcal{P}_\mathcal{T}(t)$. Then for any substitution θ, the following two claims hold.*

(1) *If $s|_p = t|_q \in \mathcal{V}$ then $dec_{p.i}(s\theta) \gg_{EL} dec_{q.i}(t\theta)$, for any $i \in N^*$ such that $p.i \in \mathcal{P}_\mathcal{T}(s\theta)$ and $q.i \in \mathcal{P}_\mathcal{T}(t\theta)$.*

(2) *If $t|_q \notin \mathcal{V}$ then $dec_{p.i}(s\theta) \gg_{EL} dec_q(t\theta)$, for any $i \in N^*$ such that $p.i \in \mathcal{P}_\mathcal{T}(s\theta)$.*

Proof. We show that the claim (1) $\wedge$ (2) by induction on $|s| + |t|$. Assume that $dec_p(s) \gg_{EL} dec_q(t)$.

(1) Consider the case $s|_p = t|_q \in \mathcal{V}$.

By the assumption $dec_p(s) \gg_{EL} dec_q(t)$ and definition of multiset extension, consider the cases that $dec_p(s) = M \cup \{s_1,\ldots,s_m\}$, $dec_q(t) = M \cup \{t_1,\ldots,t_n\}$, and for any $k \in \{1,\ldots,n\}$, there exists $l \in \{1,\ldots,m\}$ such that $dec_p(s) \ni s_l >_{EL} t_k \in dec_q(t)$.

For any $i \in N^*$ ($q.i \in \mathcal{P}_{\mathcal{T}}(t\theta)$), we can show that $dec_{p.i}(s\theta) = M\theta \cup \{s_1\theta, \ldots, s_m\theta\} \cup L$, $dec_{q.i}(t\theta) = M\theta \cup \{t_1\theta, \ldots, t_n\theta\} \cup L$ where $L = \{v \mid v \in sub(dec_i(s|_p\theta), s|_p\theta)\}$. Hence we have to show that $dec_p(s) \ni s_l >_{EL} t_k \in dec_q(t)$ implies $dec_{p.i}(s\theta) \ni s_l\theta >_{EL} t_k\theta \in dec_{q.i}(t\theta)$. We distinguish the cases with respect to the definition of $>_{EL}$.

(a) If $root(s_l) >_{\mathcal{F}} root(t_k)$ then $root(s_l\theta) >_{\mathcal{F}} root(t_k\theta)$ holds.

(b) If $root(s_l) = root(t_k)$, $\tau(root(s_l)) = mul$ and $sub\ (dec_p(s), s_l) \gg_{EL} sub\ (dec_q(t), t_k)$ then we can show $sub(dec_{p.i}(s\theta), s_l\theta) \gg_{EL} sub(dec_{q.i}(t\theta), t_k\theta)$ by induction hypothesis.

(c) In the case that $root(s_l) = root(t_k)$, $\tau(root(s_l)) = mul$, $sub(dec_p(s), s_l) = sub(dec_q(t), t_k)$ and $dec(args(s_l)) \gg\gg_{EL} dec(args(t_k))$, it follows that $dec(args(s_l\theta)) \gg\gg_{EL} dec(args(t_k\theta))$ from induction hypothesis.

(d) Consider the case that $root(s_l) = root(t_k)$, $\tau(root(s_l)) \neq mul$, $args(s_l) >_{IRDS}^{\tau(root(s_l))} args(t_k)$ and $\{s_l\} \gg_{IRDS} args\ (t_k)$. We can show $args(s_l\theta) >_{IRDS}^{\tau(root(s_l))} args(t_k\theta)$ and $\{s_l\theta\} \gg_{IRDS} args(t_k\theta)$ by induction hypothesis.

(2) In case of $t|_q \notin \mathcal{V}$, for any $i \in N^*$ ($p.i \in \mathcal{P}_{\mathcal{T}}(s\theta)$), we can show that $dec_{p.i}(s\theta) = M\theta \cup \{s_1\theta, \ldots, s_m\theta\} \cup L$ where $L = \{v \mid v \in sub(dec_i(s|_p\theta), s|_p\theta)\}$, $dec_q(t\theta) = M\theta \cup \{t_1\theta, \ldots, t_n\theta\}$. Hence we can show that $dec_p(s) \ni s_l >_{EL} t_k \in dec_q(t)$ implies $dec_{p.i}(s\theta) \ni s_l\theta >_{EL} t_k\theta \in dec_q(t\theta)$, in similar to the proof of (1). $\square$