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Commutative Closure of Languages [1]

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Abstract

In this paper, we provide a necessary and sufficient condition for the commutative closure of a special type of regular (context-free) language to be regular (context-free).

1 Introduction

Let X^* denote the free monoid generated by a nonempty finite alphabet X and let $X^+ = X^* \setminus \{\lambda\}$ where λ denotes the empty word of X^*. For the sake of simplicity, if $X = \{a\}$, then we write a^+ and a^* instead of $\{a\}^+$ and $\{a\}^*$, respectively. Let $L \subseteq X^*$. Then L is called a *language* over X. By $|L|$, we denote the cardinality of L. If $L \subseteq X^*$, then L^+ denotes the set of all concatenations of words in L and $L^* = L^+ \cup \{\lambda\}$. In particular, if $L = \{w\}$, then we write w^+ and w^* instead of $\{w\}^+$ and $\{w\}^*$, respectively. Let $u \in X^*$. Then u is called a *word* over X. Let $u \in X^*$. Then $alph(u)$ denotes $\{a \in X \mid u = vaw, v, w \in X^*\}$. We will deal with the commutative closures of some languages. The commutative closure of L means $\{a_{\sigma(1)}a_{\sigma(2)} \cdots a_{\sigma(n)} \mid a_i \in X, i = 1, 2, \ldots, n, a_1a_2 \cdots a_n \in L$ and σ is a permutation on $\{1, 2, \ldots, n\}\}$. By $com(L)$, we denote the commutative closure of $L \subseteq X^*$. In this paper, we give simple criterions for the following restricted classes of regular languages and context-free languages.

Let $L \subseteq X^*$ be a regular (context-free) language and let $z \in L$ be a word whose length is long enough. Then, by the well-known pumping lemma for regular (context-free) languages, z can be decomposed as $z = uvw$ ($z = uvwxy$) and $uv^+w \subseteq L$ ($\{uv^pwx^py \mid p \geq 1\} \subseteq L$) where the length of v (vwx) is bounded. Thus we will consider the commutative closure of a finite union of those languages.

[1] This is an abstract and the details will be published elsewhere.

2 Commutative closure of regular languages

In this section, we provide a necessary and sufficient condition for the commutative closure of a language $L = \bigcup_{i=1}^{k} u_i v_i^+ w_i$ to be regular.

Proposition 2.1 *Let $u_i, v_i, w_i \in X^*$ with $i = 1, 2, \ldots, k$ and let $L = \bigcup_{i=1}^{k} u_i v_i^+ w_i$. Then $com(L)$ is regular if and only if for any $i = 1, 2, \ldots, k$, we have $|alph(v_i)| \leq 1$.*

3 Commutative closure of context-free languages

In this section, we provide a necessary and sufficient condition for the commutative closure of a language $L = \bigcup_{i=1}^{k} \{u_i v_i^p w_i x_i^p y_i \mid p \geq 1\}$ to be regular.

Proposition 3.1 *Let $u_i, v_i, w_i, x_i, y_i \in X^*$ where $i = 1, 2, \ldots, k$ and let $L = \bigcup_{i=1}^{k} \{u_i v_i^p w_i x_i^p y_i \mid p \geq 1\}$. Then $com(L)$ is context-free if and only if for any $i = 1, 2, \ldots, k$, we have $|alph(v_i x_i)| \leq 2$.*

4 Commutative closure of other languages

In this section, we consider the commutative closure of a context-sensitive (recursively enumerable, recursive) language.

Proposition 4.1 *Let $L \subseteq X^*$ be a context-sensitive (recursively enumerable, recursive) language. Then $com(L)$ is context-sensitive (recursively enumerable, recursive), too.*