The Effects of Learning on the Existence of Stable International Environmental Agreements

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Abstract

We extend a three-country game model of international environmental agreements (IEAs) by Na and Shin (1998) to an n-country model, and examine the effects of learning (resolution of uncertainty) on the stability of agreements. An agreement is said to be stable if no country has an incentive to defect from it and free ride. We have shown that whether negotiations are conducted before or after learning does not have a significant effect, which is quite different from Na and Shin's result, and the difference is caused by the assumption on the availability of side payments. Also shown is the fact that it is necessary to include the rule that countries in a coalition should respond to countries defecting from the agreement by individually reducing the amount of their abatements to Nash equilibrium levels.

Keywords: stability of an agreement, uncertainty, learning, cooperative game, core.
JEL classification: C71, D62, Q58

1 Introduction

One remarkable feature of global environmental issues is that there is no organization with supranational power to control anthropogenic pollutants. Hence international cooperation is essential to developing measures to protect the global environment. International conferences have been held frequently, and negotiations have been conducted. Hereafter, the agreements determined by negotiations among nations regarding each country's abatement of pollutants, and the rules added to them if necessary, are called *international environmental agreements* (IEAs). Examples of such agreements include the Helsinki Protocol for acid rain, the Montreal Protocol for the depletion of the ozone layer, and the Kyoto Protocol for global warming.

The group of countries concluding an agreement is called a *coalition*. If the IEA in which all countries participate was reached and each country complied with it, a globally efficient level of abatement would be attained. A coalition consisted of all countries is called the *grand coalition*. Although formation of the grand coalition is most desirable, there may be an incentive to free ride in some countries. In other words, they may refuse to comply the agreement and thus defect from it. An agreement is said to be *stable* if no country has an incentive to free ride. It is hoped that the design of the IEA would prevent any free ride and thus realize the stable grand coalition.

The role of *uncertainty* is also significant as a reason for difficulty in dealing with global environmental issues. For many problems the impact on society caused by pollutants is not completely clear at this time. Pollutants must be reduced because there is a concern that the society will suffer damage from environmental changes caused by these pollutants. Unfortunately, there is not enough information available at present regarding the degree of damage. The uncertainty, however, is expected to diminish as time passes. This process is called *learning*. It has been noted in various preceding studies that the existence of uncertainty and learning has a significant effect on national decision-making and welfare.[1] We must decide whether to implement policy measures now or to implement suitable measures only after the extent of the damage is well known.

The purpose of our paper is to consider the relation between the uncertainty regarding the scale of damage and the stability of IEAs. To be more specific, we use a game model regarding nations as players to examine if there is a change in the stability of the grand coalition brought about by the elimination of uncertainty and the changes in the rules.

Many game theoretic analyses of IEAs have been conducted to date. Carraro and Siniscalco (1993) define the concept of stability. They show that it is impossible to expand the size of a stable coalition by self-financed transfers to countries outside the coalition and proposed some commitments for realizing it. For our study, we modify their concept of stability slightly and call it the "CS-stability." A set of stable imputations is called the "CS-core."[2] The concept of a core is explained in detail in Section 2. Barrett (1994) emphasizes the importance of a self-enforcing agreement, which is an agreement that is concluded even when cooperation is not forced. When the grand coalition reaches a self-enforcing agreement, it is equivalent to being CS-stable. Barrett (1994) specifies the payoff function and showed that the size of a stable coalition is small in various cases.

Since then a number of studies using various factors have been actively done within a similar framework. Petrakis and Xepapadeas (1996) take into account factors such as asymmetric information and a moral hazard in the analysis. They define environmentally conscious countries

[1] For example, see Kolstad (1996), Ulph and Ulph (1997), Helm (1998), Kelly and Kolstad (1999), and Pindyck (2000). Among these articles, Kolstad (1996) and Pindyck (2000) explore the relation between uncertainty and irreversibilities associated with the investment for pollution abatement.

[2] Our concept of CS-stability can only be applied to the grand coalition, as it does not involve external stability as defined by Carraro and Siniscalco (1993), whereas their concept can be applied to any coalition.

(ENCCs) as countries that increase their profits by cooperation. If ENCCs commit to cooperation, the stable grand coalition is obtained by side payments from ENCCs to other countries, and they show an enforcement mechanism in the agreement. Hoel and Schneider (1997) analyze the effect of side payments, considering the non-environmental costs. They conclude that the size of a stable coalition increased rapidly under the consideration of non-environmental costs; they also observe the phenomenon in which the incentive to participate in an agreement becomes low under conditions where side payments are permitted. Chander and Tulkens (1997) define the "γ-core" of the cooperative games with environmental externalities. Barrett (2001) shows when there is strong asymmetry among countries, the rule of the game changes so that countries that gain much from the agreement (developed countries) first make decisions to join an agreement and then offer side payments to countries that gain less (developing countries) to let them sign in. This change of the rule would expand the size of stable coalition substantially, thus Barrett (2001) insists that the key feature about the stability of coalition was an asymmetry among countries. He also stresses that his theory is consistent with what actually happened through the process of conclusion of the Montereal Protocol. Lange and Vogt (2003) explain the cooperation in environmental negotiations by the preferences for equity. They show that in a two-stage game on coalition formation, the presence of equity-interested countries increases the coalition size.

The analyses mentioned above, however, assume a game with perfect information. Few studies have analyzed the role of uncertainty in the design of stable IEAs. To my knowledge, Na and Shin (1998) is the first study in this field. They develop a three-country model and the CS-stability of the grand coalition is analyzed in the two cases where countries negotiate before and after learning. As a result, they point out that it would be more desirable to negotiate before learning. This is because there are countries that would become losers by participating in an agreement and an agreement is no longer stable if the difference in the payoff structure for each country becomes clear.

In what follows we try to extend Na and Shin's analysis. The effects of uncertainty on the stability of an IEA in a general model for n countries are considered, and the robustness of their findings is examined. We focus on the stability of the grand coalition. Moreover, Na and Shin considered the CS-stability only, whereas we consider the γ-stability as well. The basic model is explained in Section 2. We define a coalition and a characteristic function and introduce the concept of a core. Sections 3 and 4 are concerned with analyses of negotiation before and after uncertainty is eliminated. Section 5 concludes. Our main conclusion is that whether negotiations are conducted before or after learning does not have a significant effect on the stability of an agreement, which means that the results of Na and Shin cannot be extended to wider class of models.

2 The model

2.1 Payoff and coalition

Described below is a simple game model of the pollutants abatement that extends the model of Na and Shin (1998) to n countries. Each country $i(\in N = \{1, \cdots, n\})$ is a *player*. The game consists of two stages. In the first stage, the contents of an IEA (i.e. the coalition structure and the additional rules) are determined by negotiation. In the second stage, each country plays a noncooperative game to choose its *strategy*, that is the amount of abatement of a specific anthropogenic pollutant $x_i(\geq 0)$ during a certain period. The final payoff to country i is

$$\pi_i = \theta_i \sum_{j \in N} x_j - \frac{x_i^2}{2}. \tag{1}$$

The first term of the right-hand side of Eq. (1) is the benefit to country i, and the second term is the abatement cost; θ_i is a random variable, a parameter representing the benefit to country i from the abatement. Assume $\theta_i \in \{z_1, \cdots, z_n\}$ $(z_1 \geq \cdots \geq z_n \geq 0,\ z_1 > 0)$. Learning occurs at a certain time, and the value of θ_i is determined. Let $\{\theta_1, \cdots, \theta_n\}$ be a permutation of $\{z_1, \cdots, z_n\}$. This means that there exists unique $j \in N$, which satisfies $\theta_j = z_i$ for each z_i. We can consider that the value of θ_i is allocated by chance moves at the time of learning. The probability that each θ_i equals $z_1, \cdots, z_n$ is $1/n$ respectively. For simplicity, we normalize these parameter values so that $\sum_{i \in N} z_i = \sum_{i \in N} \theta_i = 1$. When the first stage occurs before learning, it is termed *ex ante* negotiation, and when it occurs after learning it is termed *ex post* negotiation. Because all countries have the same expected value of the benefit parameter in the case of *ex ante* negotiation, each country is a virtually identical player. In many long-term environmental issues such as climate change, the magnitude of damage to each country is thought to vary, but we do not know how it will actually be distributed. We set the assumption above as a simplification of these facts.

A coalition is defined as a nonempty subset of N. During the second stage, each country chooses the strategy to maximize its expected payoff and a member of a coalition S must choose its strategy to maximize the expected value of $\sum_{i \in S} \pi_i = \sum_{i \in S} \theta_i \sum_{j \in N} x_j - \sum_{i \in S} x_i^2/2$. Therefore, the strategy of country i belonging to coalition S is $\sum_{i \in S} \theta_i$. Note that this strategy does not depend on the strategy of any country outside a coalition, as the benefit is assumed to be a linear function of the total amount of abatement. Put differently, strategic substitutability does not exist.[3] The transfers of payoff (*side payments*) within a coalition are possible without restriction. Each country decides whether to enter a coalition with other countries during the first stage, considering the payoff and the strategy during the second stage. Figure 1 illustrates the two cases, where time passes from left to right. Payoffs to countries depend on the coalition formation and learning.

Now consider two coalitions, S and T ($S \cap T = \emptyset$), and compare the total payoff to n countries when they act separately in the second stage to that when they cooperate as a coalition $S \cup T$ in the second stage. Let $|S| = s$ and $|T| = t$. The payoff to country i belonging to $S \cup T$ is expressed as π_i when each coalition acts separately, and π_i' when two coalitions cooperate. Through simple calculations, we get:

$$\sum_{j \in S \cup T} \pi_j = (\theta_S + \theta_T)\left(s\theta_S + t\theta_T + \sum_{j \notin S \cup T} x_j\right) - \frac{s}{2}\theta_S^2 - \frac{t}{2}\theta_T^2,$$

$$\sum_{j \in S \cup T} \pi_j' = (\theta_S + \theta_T)\left\{(s+t)(\theta_S + \theta_T) + \sum_{j \notin S \cup T} x_j\right\} - \frac{s+t}{2}(\theta_S + \theta_T)^2,$$

therefore,

$$\sum_{j \in S \cup T} (\pi_j' - \pi_j) = \frac{1}{2}(t\theta_S^2 + s\theta_T^2) \geq 0.$$

As for the countries which do not belong to $S \cup T$, the abatement cost is the same regardless of whether S and T cooperate or not because of no strategic substitutability, and the benefit is the same or greater when S and T cooperate. This clearly indicates that the payoff is the same

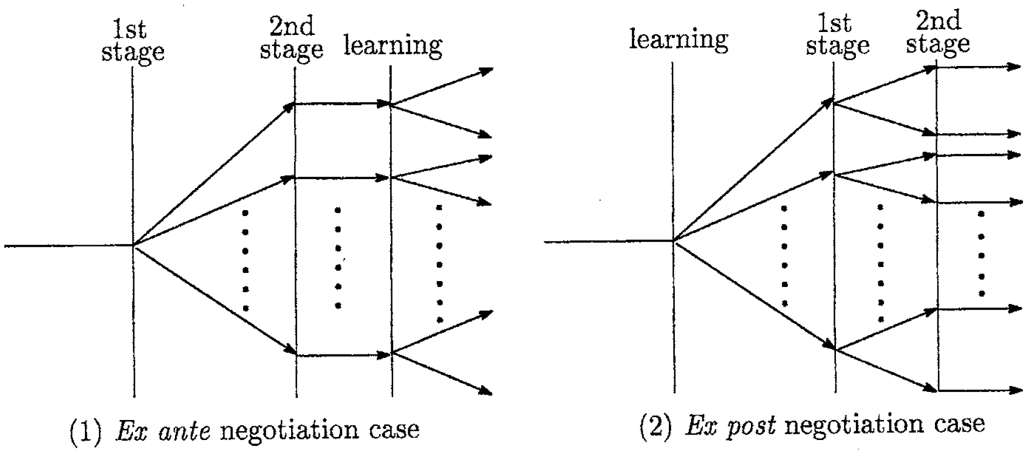

Fig. 1. Two cases

[3]This assumption means that there is nothing like "carbon leakage" for global warming.

or greater when there are cooperative relations. In general, the total payoff becomes greater when coalitions S and T cooperate than when they act separately. As this fact holds for any two coalitions with no intersection, it is shown that a more efficient result is obtained when a larger coalition is formed. Therefore, in the following we focus on whether the stable grand coalition is formed in the first stage in various cases.

2.2 Characteristic function and core

Next, we introduce the concept of a *characteristic function*, which determines the property of a game. The value of the characteristic function of coalition S, $v(S)$ is the payoff that S can realize by itself using a joint strategy. In this game,

$$v(S) = \sum_{i \in S} \left\{ E(\theta_i) \left(\sum_{j \in S} x_j + \sum_{j \notin S} x_j \right) \right\} - \frac{1}{2} \sum_{i \in S} x_i^2, \tag{2}$$

where $E(\cdot)$ denotes an expected value.

Consider the strategy of each country. First, the strategy of country i which belongs to S in the second stage is $\sum_{i \in S} E(\theta_i)$, which maximizes the expected total payoff to S.[4] As understood from Eq. (2), $v(S)$ is also dependent on the strategies of the countries outside S. We set the following assumptions regarding the actions of countries outside a coalition.

1. Take the optimal response strategy to the strategy of S individually.
2. Take the optimal response strategy as the coalition $N \backslash S$.

In assumption 1, coalition S and each country outside S are decision-making agents and follow the Nash equilibrium strategy of an $(n - s + 1)$-player game at the second stage. Here, $s = |S|$. This type of characteristic function is defined as a γ-*characteristic function* by Chander and Tulkens (1997). Assumption 2 means that coalition S has an optimistic prediction for the actions of countries outside the coalition. It is posited that if coalition S escaped from the grand coalition N, the remaining countries would form a coalition of $N \backslash S$ and undertake a joint strategy. Although the characteristic function of this type is not found in the existing literature, it can be defined. In this study we call it a *CS-characteristic function*, in reference to Carraro and Siniscalco (1993), who create the framework that analyzes an IEA according to assumption 2 above.

Having specified a characteristic function, a *core* is found as the solution of the game. If a vector $y = (y_1, \cdots, y_n)$ satisfies $\sum_{i \in N} y_i = v(N)$, y is called an *imputation*. It describes how the

[4] We assume that the inner solution always exists in the domain. These strategies are dominant strategies and Nash equilibrium strategies.

payoff obtained by the grand coalition is divided among players. A core is a set of imputation y that satisfies

$$\sum_{i \in S} y_i \geq v(S), \quad \forall S \subset N. \tag{3}$$

If a core is not empty and imputation y belongs to the core, an agreement forming the grand coalition is reached in which country i obtains y_i by the payoff transfers. This agreement is stable, and it is not refused because any coalition S cannot obtain a payoff greater than $\sum_{i \in S} y_i$ by acting on its own, according to Eq. (3).

Needless to say, the concept of a core depends on the form of the characteristic function. We term it a γ-core when the γ-characteristic function is assumed and a CS-core when the CS-characteristic function is assumed. Moreover, when the γ-core is nonempty, the agreement is γ-stable — and the same for the CS-core. In the context of environmental topics, γ-core is a solution under the rule that should any coalition defect from an agreement the entire agreement would be void; thus, all countries outside the coalition would reduce their abatement to Nash equilibrium levels. On the other hand, the CS-core is a solution under the assumption that should such a defection occur remaining countries would keep acting collectively to take joint abatement strategies.

3 *Ex ante* negotiation case

We consider the case where negotiations among countries are performed and all countries decide their strategies before learning. From Eq. (1), the expected payoff to country i in the second stage is

$$E(\pi_i) = E(\theta_i) \sum_{j \in N} x_j - \frac{x_i^2}{2}.$$

From the assumptions of Section 2.1, $E(\theta_i) = (z_1 + \cdots + z_n)/n = 1/n$. In this situation all players are symmetric, or more precisely, identical.

3.1 γ-core

The strategy of each country belonging to coalition S is $\sum_{i \in S} E(\theta_i) = s/n$, which maximizes $\sum_{i \in S} E(\pi_i)$ and strategy of each country not belonging to S is $1/n$. Hence, the value of the γ-characteristic function for S is

$$v^{\gamma}(S) = s\left\{\frac{1}{n}\left(s \cdot \frac{s}{n} + \frac{n-s}{n}\right) - \frac{1}{2}\left(\frac{s}{n}\right)^2\right\} = \frac{s}{n^2}\left(\frac{s^2}{2} + n - s\right). \tag{4}$$

Substituting $s = n$ into Eq. (4), $v^{\gamma}(N) = n/2$.

Proposition 1 *In the case of ex ante negotiation, the γ-core is nonempty, and the grand coalition is stable irrespective of n.*

Proof. Suppose $y_i = 1/2\ \forall i \in N$. Then $\sum_{i\in N} y_i = v^\gamma(N)$, and for all $S \subset N$, $\sum_{i\in S} y_i - v^\gamma(S) = \frac{s}{2} - \frac{s}{n^2}\left(\frac{s^2}{2} + n - s\right) = \frac{s}{2n^2}(n-s)(n+s-2) \geq 0$. □

Note that y_i is the same as the payoff to country i when each country adopts a joint strategy "1" and there is no need for side payments.

3.2 CS-core

If countries that do not belong to coalition S take the joint strategy as coalition $N \backslash S$, each strategy of $n-s$ countries is $(n-s)/n$. Therefore, the CS-characteristic function is

$$v^{\mathrm{CS}}(S) = \frac{s}{n^2}\left(\frac{3}{2}s^2 + n^2 - 2ns\right). \tag{5}$$

Calculation from Eqs. (4) and (5) leads to: $v^{\mathrm{CS}}(S) - v^\gamma(S) = (n-s)(n-s-1)/n^2$. We can see $v^{\mathrm{CS}}(S) > v^\gamma(S)$ if $|S| < n-1$, and $v^{\mathrm{CS}}(S) = v^\gamma(S)$ if $|S|$ is n or $n-1$, which show that generally the CS-characteristic function of S takes a greater value than the γ-characteristic function. When $|S| = n$ $(S = N)$ there is no country outside S; when $|S| = n-1$ there is only one country outside S, and the only strategy it can take is to act individually. Hence in these cases, they take the same value.

Substituting $s = n$ and $s = 1$ into Eq. (5), we have $v^{\mathrm{CS}}(N) = n/2, \forall i \in N$, and $v^{\mathrm{CS}}(\{i\}) = (2n^2 - 4n + 3)/2n^2$. The payoff obtained when country i withdraws from an agreement independently and acts as a one-country coalition is expressed as $v^{\mathrm{CS}}(\{i\})$.

Proposition 2 *In the case of ex ante negotiation the CS-core is empty when $n \geq 4$.*

Proof. $\sum_{i\in N} v^{\mathrm{CS}}(\{i\}) - v^{\mathrm{CS}}(N) = (n-1)(n-3)/2n$, so when $n \geq 4$, $\sum_{i\in N} v^{\mathrm{CS}}(\{i\}) > v^{\mathrm{CS}}(N)$. If we suppose that the CS-core is nonempty and that there exists an imputation $y = (y_1, \cdots, y_n)$ belonging to the CS-core, $y_i \geq v^{\mathrm{CS}}(\{i\})\ \forall i \in N$. Adding each side of this inequality for all $i \in N$, we obtain $v^{\mathrm{CS}}(N) \geq \sum_{i\in N} v^{\mathrm{CS}}(\{i\})$. This leads to a contradiction. □

Na and Shin (1998) prove that the imputation $(9/2n^2, 9/2n^2, 9/2n^2)$ belonging to the CS-core is achieved without side payments in a three-country model. It is easily checked that a core is nonempty when $n \leq 3$. Unfortunately, when there are four or more countries, the profit to any country from taking a free ride becomes large, and a free ride cannot be prevented by means of side payments within the coalition. This result shows the difficulty of concluding IEAs explicitly.

4 *Ex post* negotiation case

Next, consider the case where negotiations among countries are conducted after learning. The payoff to country i in the second stage is expressed by Eq. (1). As each country negotiates after learning, the strategy of a coalition in the second stage depends on θ_i.

4.1 γ-core

The strategy of s countries belonging to coalition S is θ_S, and the strategy of country j outside S is θ_j. The value of the γ-characteristic function is

$$v^\gamma(S) = \sum_{i\in S}\theta_i\left(s\theta_S + \sum_{j\notin S}\theta_j\right) - \frac{s}{2}\theta_S^2 = \left(\frac{s}{2}-1\right)\theta_S^2 + \theta_S. \tag{6}$$

Proposition 3 *In the case of ex post negotiation, the γ-core is nonempty, and an agreement by the grand coalition is attainable by appropriate side payments.*

Proof. From (6), $v^\gamma(N) = n/2$, and $v^\gamma(\{i\}) = -\theta_i^2/2+\theta_i$. Note that $v^\gamma(N) \geq \sum_{i\in N} v^\gamma(\{i\})$. Now let $y_i^* = v^\gamma(\{i\}) + \theta_i\left\{v^\gamma(N) - \sum_{j\in N} v^\gamma(\{j\})\right\} = \theta_i\left(n - \theta_i + \sum_{i\in N}\theta_i^2\right)/2$, following Chander and Tulkens (1997). We can easily see that $y^* = (y_1^*, \cdots, y_n^*)$ is an imputation. For any S ($\neq N$),

$$\begin{aligned}\sum_{i\in S} y_i^* - v^\gamma(S) &= \frac{n}{2}\theta_S - \frac{1}{2}\sum_{i\in S}\theta_i^2 + \frac{\theta_S}{2}\sum_{i\in N}\theta_i^2 - \left(\frac{s}{2}-1\right)\theta_S^2 - \theta_S \\ &\geq \frac{n}{2}\theta_S - \frac{\theta_S^2}{2} - \left(\frac{s}{2}-1\right)\theta_S^2 - \theta_S \\ &= \theta_S\left\{\left(\frac{n}{2}-1\right) - \frac{s-1}{2}\theta_S\right\} \geq \frac{n-s-1}{2}\theta_S^2 \geq 0. \end{aligned} \tag{7}$$

We have used an inequality $\theta_S^2 \geq \sum_{i\in S}\theta_i^2$ for calculating from the first line to the second line, and used $\theta_S \leq 1$ in the third line of Eq. (7). Equation (7) shows that there is always an imputation belonging to the γ-core. □

The following side payment:

$$T_i = \frac{1-\theta_i^2}{2} - \frac{\theta_i}{2}\left(n - \sum_{j\in N}\theta_j^2\right) \tag{8}$$

for country i is necessary to reach this imputation. The value of T_i is calculated as a difference between y_i^* and the payoff to country i when the grand coalition is formed. When $T_i > 0$, country i receives a side payment; when $T_i < 0$, country i pays it. From Eq. (8), we see that the *benefit principle* applies, that is, countries with more benefits from the agreement (with large values of θ) must pay larger side payments.

Example 1 If $n = 4, \theta_i = i/10\ \forall i \in N$, then $v^\gamma(N) = 2$ and $y^* = (0.21, 0.41, 0.6, 0.78)$ belongs to the γ-core. The vector of T_i becomes $T = (0.31, 0.11, -0.1, -0.32)$.

4.2 CS-core

The strategy of countries inside S is θ_S, as in Section 4.1. The strategy of all $n-s$ countries outside S is $1-\theta_S$. The CS-characteristic function in this case is

$$v^{\mathrm{CS}}(S)=\sum_{i\in S}\theta_i\{s\theta_S+(n-s)(1-\theta_S)\}-\frac{s}{2}\theta_S^2=\left(\frac{3}{2}s-n\right)\theta_S^2+(n-s)\theta_S. \tag{9}$$

Note that from Eqs. (6) and (9): $v^{\mathrm{CS}}(S)-v^{\gamma}(S)=(n-s-1)\theta_S(1-\theta_S)$, so $v^{\mathrm{CS}}(S)>v^{\gamma}(S)$ if $|S|<n-1$, and $v^{\mathrm{CS}}(S)=v^{\gamma}(S)$ if $|S|$ is n or $n-1$, as in the *ex ante* negotiation case.

From Eq. (9),

$$\sum_{i\in N}v^{\mathrm{CS}}(\{i\})-v^{\mathrm{CS}}(N)=\left(\frac{3}{2}-n\right)\sum_{i\in N}\theta_i^2+\left(\frac{n}{2}-1\right). \tag{10}$$

Rearranging Eq. (10) gives

$$\sum_{i\in N}v^{\mathrm{CS}}(\{i\})>v^{\mathrm{CS}}(N)\Longleftrightarrow\sum_{i\in N}\theta_i^2<\frac{n-2}{2n-3}. \tag{11}$$

Proposition 4 *In the case of ex post negotiation, the CS-core is nonempty when $n\le 3$.*

Proof. When $n=2$ and $n=3$, (11) becomes $\sum_{i\in N}v^{\mathrm{CS}}(\{i\})>v^{\mathrm{CS}}(N)\Longleftrightarrow\sum_{i\in N}\theta_i^2<0$ and $\sum_{i\in N}v^{\mathrm{CS}}(\{i\})>v^{\mathrm{CS}}(N)\Longleftrightarrow\sum_{i\in N}\theta_i^2<1/3$ respectively, and we can see that $\sum_{i\in N}v^{\mathrm{CS}}(\{i\})\le v^{\mathrm{CS}}(N)$ holds in both cases. It is clear that CS-core is nonempty when $n=2$. Check the CS-stability when $n=3$. Define the imputation y^* in the same way as in Section 4.1:

$$y_i^*=v^{\mathrm{CS}}(\{i\})+\theta_i\left\{v^{\mathrm{CS}}(N)-\sum_{j\in N}v^{\mathrm{CS}}(\{j\})\right\}=\frac{3}{2}\theta_i\left(1-\theta_i+\sum_{j\in N}\theta_j^2\right). \tag{12}$$

If $|S|=1$, clearly $\sum_{i\in S}y_i^*\ge v^{\mathrm{CS}}(S)$. If $|S|=2$, (9) implies $v^{\mathrm{CS}}(S)=\theta_S$. From (12), we get

$$\begin{aligned}\sum_{i\in S}y_i^*-v^{\mathrm{CS}}(S)&=\frac{\theta_S}{2}-\frac{3}{2}\sum_{i\in S}\theta_i^2+\frac{3\theta_S}{2}\sum_{i\in N}\theta_i^2=\frac{\theta_S}{2}-\frac{3}{2}(1-\theta_S)\sum_{i\in S}\theta_i^2+\frac{3\theta_S}{2}\sum_{j\notin S}\theta_j^2\\&\ge\frac{\theta_S}{2}-\frac{3}{2}(1-\theta_S)\theta_S^2=\frac{\theta_S}{2}\left\{3\left(\theta_S-\frac{1}{2}\right)^2+\frac{1}{4}\right\}\ge 0.\end{aligned}$$

Thus it is shown that y^* belongs to the CS-core. □

The availability of side payments is not considered in the analysis of Na and Shin (1998). Their results show that if there is even a small difference in the value of the benefit parameter among three countries in the case of *ex post* negotiation, a free ride by a country enjoying only small benefit from abatement (e.g., a country assigned z_n) cannot be prevented, and the CS-core becomes empty. If side payments among countries are not allowed, the CS-core is empty for all

$n \geq 3$. On the other hand, we have seen from the above discussion that when side payments are allowed within a coalition, it is possible to maintain the cooperative relationship by resource transfer from a country with a large benefit to a country with a small benefit, relying on the characteristic differences of each country.

If $n \geq 4$, there is a case where the CS-core is nonempty, in contrast to the case of *ex ante* negotiation.

Proposition 5 *In the case of ex post negotiation, for any number of countries n, there exists a parameter set $(\theta_1, \cdots, \theta_n)$ for which the CS-core is nonempty.*

Proof. Consider an extreme case where only one country, say country 1 suffers from pollution: $\theta_1 = 1, \theta_j = 0$ $(\forall j \neq 1)$. Then the values of the CS-characteristic function are $v^{\text{CS}}(S) = s/2$ $(1 \in S)$ and $v^{\text{CS}}(S) = 0$ $(1 \notin S)$. Clearly $v^{\text{CS}}(N) = n/2$ and an imputation $y = (n/2, 0, \cdots, 0)$ belongs to the CS-core. □

Equation (11) shows that when the variance of $\{\theta_i\}$ is very small, the CS-core is undoubtedly empty. When it is relatively large, the CS-core is either empty or nonempty. It can be said that the larger the variance is the greater is the possibility that the CS-core is nonempty. Only in a case where the differences among countries are large is the stable grand coalition realized by suitable side payments. The following are examples of when the CS-core is empty and when it is nonempty, for a four-country model.

Example 2 If $n = 4, \theta_i = i/10$ $\forall i \in N$, as in Example 1, $v^{\text{CS}}(N) = 2, v^{\text{CS}}(\{1\}) = 0.275, v^{\text{CS}}(\{2\}) = 0.5, v^{\text{CS}}(\{3\}) = 0.675$, and $v^{\text{CS}}(\{4\}) = 0.8$. As $\sum_i v^{\text{CS}}(\{i\}) = 2.25 > 2$, the CS-core is empty.

Example 3 Considering the case of $\theta_1 = \theta_2 = 0.1, \theta_3 = 0.2$, and $\theta_4 = 0.6$, where the variance of the benefit parameter $\{\theta_i\}$ is large, the values of the CS-characteristic function are $v^{\text{CS}}(\{1\}) = v^{\text{CS}}(\{2\}) = 0.275, v^{\text{CS}}(\{3\}) = 0.5, v^{\text{CS}}(\{4\}) = 0.9, \cdots$, and $v^{\text{CS}}(N) = 2$. An imputation $y^* = (0.28, 0.28, 0.51, 0.93)$ obtained by Eq. (12) belongs to the CS-core.

5 Conclusions

We have examined the effects of uncertainty on the stability of the grand coalition. In the case of *ex ante* negotiation the γ-core is always nonempty irrespective of the number of countries (Proposition 1). The CS-core is nonempty when there are three or fewer countries, but it always becomes empty when there are four or more countries (Proposition 2).

In the case of *ex post* negotiation the γ-core is always nonempty, as it is in the case of *ex ante* negotiation (Proposition 3). The CS-core is nonempty when there are three or fewer countries, and the stable grand coalition is realized by side payments from countries with large benefits to those with small benefits (Proposition 4). We show that even when four or more countries are involved, the situation in which the CS-core is nonempty always exists (Proposition 5), and that whether a core is nonempty depends on the variance of the benefit parameter. A greater variance leads to a higher possibility of having a nonempty core.

These results indicate that whether negotiations are conducted before or after learning does not have a significant effect on the stability of the grand coalition. Thus the result of Na and Shin is questionable. More important is the fact that the γ-core always becomes nonempty. Hence, it is necessary to include "punishment" in the rules; that is, countries in a coalition should respond to a country defecting from the agreement by reducing their abatement to Nash equilibrium levels. It seems that this kind of punishment rule has not been seen in the context of global environmental agreements so far. Moreover, it should be noted that side payments are essential to the realization of a stable imputation.

Future research topics may include the numerical analyses of more complicated scenarios and the examination of dynamic factors.

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