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General Equilibrium Theory for Economy with Asymmetric Information: Basic Model

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Abstract

In this paper, a general equilibrium model under asymmetric information is constructed as a natural extension of the standard Arrow-Debreu general equilibrium model having "fictitious" technologies.

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1 INTRODUCTION

From the viewpoint of general equilibrium theory, the most important issue for economies under asymmetric information is the following *market viability problem.* For a certain commodity, commodity k, if the information about the (numerically measured) quality of k of sellers are finer than that of buyers, and if buyers are supposed to have expectations that thay will obtain the numerical average of the quality, then in the market of commodity k, only relatively low quality sellers are willing to join the market at any given prices, (the *adverse selection* arises,) so that may occurs the *market unraveling.* (Akerlof (1970), Rothschild-Stiglitz (1976)). Under the general equilibrium framework, it seems that the situation corresponds to the non existence of equilibrium, i.e., the market is not viable.

The above well-known story in partial equilibrium thory ignores the problem of budget constraints for buyers who received lower quality k than his/her expectations. (Under the general equilibrium settings, a situation like default may happen for such an agent.) Suppose that, for a while, in the market of commodity k, there exists a certain system of insurance pooling all the risk of uninformed agents, so that we may suppose every buyer to obtain an average quality in each trade of commodity k. (By this, as long as we are considering the rational expectation equilibrium, the default problem never occurs.) Under these circumstances, Dubey-Geanakoplos-Shubik (1994–2000) shows the existence of rational expectation equilibrium under the *anonymity* assumption for sellers in the asset market with strictly positive *utility punishment* for default actions. (They treats the economy with asymmetric information as a special case of default economy.) In the same way, Bisin-Gottardi (1999) shows the existence of equilibrium under the asset market anonymity condition with the special price system allowing *bid-ask spread.* In this paper, we show the existence of equilibrium for more natural and simpler *private* mechanism: (1) We do not use the asset market and asset market anonymity, (2) utility punishment is not necessary, and (3) the ordinary price mechanism is sufficient (bid-ask spread is not necessary.)

We formulate a general equilibrium model with asymmetric information as an extension of the standard Arrow-Debreu economy such that there may exist a *fictitious technology* which makes certain amounts of real commodities *virtually* into a certain amount of another commodity. Trade contracts of a producer or a consumer having such a technology may not properly becarried out, so that, in the market for each commodity, a buyer may not obtain the exact amount of each commodity for his purchasing contract. An owner of such a technology, (i.e., a seller), however, is supposed, in out model, to receive the exact price for the virtual output of the fictitious technology.

Hence, the asymmetric information we treat here is restricted merely on the information about commodities that is used by sellers (for buyers). It should be noted, however, that since we have looked at the asymmetric information superficially as technologies, we are able to treat an information as a special ability owned by an individual or a group of agents (a firm).

The equilibrium concept in this paper resembles to the concept used in Dubey-Geanakoplos-Shubik (1994). That is, under fictious technologies and trades (asymmetric information), each agent decides their action based on an expectation on what they will *really* receive from the market for each trade contract in each commodity market. As buyers, they expect to receive not an exact amount but an average in the market including the amount sold by sellers having the fictitious technologies. Of course, we are assuming that each commodity market is so 'matured' that all trade contracts are realized in such an aggregated (averaged) manner. Our treatment of the asymmetric information (as a sort of technology owned by agents) is more general than Dubey-Geanakoplos-Shubik's treatment (as a character based on a certain asset market), so that our model includes many partial equilibrium models of incomplete information. Not as Dubey-Geanakoplos-Shubik's model, we do not consider the structure of time periods (hence, states and asset markets) since it is not necessary for our purpose. But it is a routine task to take asset markets into our consideration and in such cases, the problem on defaults may also be treated as a part of the framework used in this paper.

2 THE MODEL

There are m types of *consumers* and n types of *producers* indexed, respectively, by $i = 1, \ldots, m$ and $j = 1, \ldots, n$. In the market, there are ℓ types of *commodities* indexed by $k = 1, \ldots, \ell$.

Each agent $i = 1, \ldots, m+n$ has a private information about types of commodities which i supplies to the market. The problem of the asymmetric information about a certain kind of commodity is reduced here to the problem on differences in the types of commodities and the information about types of commodities. Then, we are able to describe the situation having such a private information as having a certain *fictitious technology* to make one or several commodities vertically into another. Of course, such an output is not available for the use of demanders. In the market, it is supposed that not the output but a part of the input (for the fictitious technology) is delivered to demanders. Fictitious technology (in the production function form), $F_i^k : R_+^\ell \to R_+$, $i = 1, \ldots, m+n$, $k = 1, \ldots, \ell$, is such that (1) it is a technology owned by i, (2) F_i^k is a technology to produce commodity k, and (3) $F_i^k(0) = 0$. For each $x \in R_+^\ell$, $y^k = F_i^k(x)$ means that consumer i using fictitious technology F_i^k could offer a certain amount $f_i^k(x) \in R_+^\ell$, $f_i^k(x) \leq x$, as for the output y^k to the market. The amount $f_i^k(x)$ is supposed to be 0 if $F_i^k(x) = 0$. In the market, such an amount, $f_i^k(x)$, is treated as the amount y^k of commodity k, and supposed to delivered (as the amount $y^k$of commodity k in an aggregated and averaged manner) to agents who demand commodity k through the market.

It follows that for each commodity k, agent i who demand commodity k through the market cannot expect to obtain the exact amount which i contracted to buy. They have to make some *expectations on their real receipts* for each of their contracts before they chose their actions. In our model, the expectation for each agent is supposed to be an identity function of an aggregate parameter given in the market. I.e., for each commodity k, if an aggregate amount $s_k \in R_{++}$ is contracted to supply to the market, and if $\hat{s}^k \in R_+^\ell$ is an aggregate amount that is really supplied, then for each one unit of demand contracts for commodity k, the amount $\hat{s}^k/s_k \in R_+^\ell$ will be delivered and the amount will exactly be expected by all members of the economy. (In this sense, we are considering a rational expectation equilibrium.)

Producer $j = m+1, \ldots, m+n$ has a production technology $Y_j \subset R^\ell$ such that Y_j is closed, Y_j is convex, and $0 \in Y_j$. Given a price $p \in \Delta = \{(p_1, \ldots, p_\ell) | p_1 \geq 0, \ldots, p_\ell \geq 0, \sum_{k=1}^\ell p_k = 1\}$, and expectations on their receipts for each commodity through the market, $\hat{s}^1/s_1 \in R_+^\ell, \ldots, \hat{s}^\ell/s_\ell \in R_+^\ell$, producer j chooses a production plan $y_j = y_j^+ - y_j^- \in Y_j$ and the input plan for pseudo-technology, $(w_j^{1^-}, \ldots, w_j^{\ell^-})$, together with transaction plans, $(y_j, w_j^{1^-}, \ldots, w_j^{\ell^-}, z_j^-)$, as a solution to the following maximization problem.[1]

$$\max \quad p \cdot (F_{j1}(w_j^{1^-}), \ldots, F_{j\ell}(w_j^{\ell^-})) + p \cdot y_j^+ - p \cdot z_j^- \tag{1}$$

sub.to

$$\sum_{k=1}^{\ell} w_j^{k^-} + y_j^- = z_{j1}^-(\hat{s}^1/s_1) + z_{j2}^-(\hat{s}^2/s_2) + \ldots + z_{j\ell}^-(\hat{s}^\ell/s_\ell), \tag{2}$$

$$y_j \in Y_j, \tag{3}$$

$$w_j^{1^-} \in R_+^\ell, \ldots, w_j^{\ell^-} \in R_+^\ell,\ z_j^- = (z_{j1}^-, \ldots, z_{j\ell}^-) \in R_+^\ell. \tag{4}$$

[1] If there are three variables denoted by x, x^+, and x^-, we always means that $x^+ = \sup\{x, 0\}$ and $x^- = \sup\{-x, 0\}$. It may happen that we use a notation x^+ (resp., x^-) without referring anything to x and/or x^- (resp., x^+). Even in such cases, however, a careful reader may find appropriate amounts in the model for the rest of two variables.

Consumer $i = 1, \ldots, m$ has an initial endowment, $\omega_i \in R^\ell_+$, of real commodities, and a consumption set, $X_i \subset R^\ell$. We assume that X_i is a closed convex subset bounded from below such that $X_i = X_i + R^\ell_+$ for each i. Given a price $p \in \Delta$, and expectations on their receipts for each commodity through the market, $\hat{s}^1/s_1 \in R^\ell_+, \ldots, \hat{s}^\ell/s_\ell \in R^\ell_+$, consumer i chooses a consumption plan x_i together with transaction plans and inputs for fictitious technologies, $(x_i, v_i^+, z_i^-, w_i^{1-}, \ldots, w_i^{\ell-})$, as a solution to the following maximization problem.

$$\max \quad u_i(x_i) - \lambda_i \sum_{k=1}^{\ell} \|w_i^{k-}\| \tag{5}$$

sub.to

$$\sum_{k=1}^{\ell} \hat{w}_i^{k-} + \hat{x}_i + v_i^+ = \omega_i, \tag{6}$$

$$\sum_{k=1}^{\ell} \hat{\hat{w}}_i^{k-} + \hat{\hat{x}}_i = z_{i1}^-(\hat{s}^1/s_1) + z_{i2}^-(\hat{s}^2/s_2) + \ldots + z_{i\ell}^-(\hat{s}^\ell/s_\ell), \tag{7}$$

$$x_i \in X_i, \tag{8}$$

$$p \cdot z_i^- \leq p \cdot v_i^+ + p \cdot (F_i^1(w_i^{1-}), F_i^2(w_i^{2-}), \ldots, F_i^\ell(w_i^{\ell-})) + \sum_{j=1}^{n} \theta_{ij}\pi_j(p), \tag{9}$$

$$x_i = \hat{x}_i + \hat{\hat{x}}_i, \hat{x}_i \in R^\ell_+, \hat{\hat{x}}_i \in R^\ell_+, \tag{10}$$

$$\forall k = 1, \ldots, \ell, w_i^{k-} = \hat{w}_i^{k-} + \hat{\hat{w}}_i^{k-}, \ \hat{w}_i^{k-} \in R^\ell_+, \ \hat{\hat{w}}_i^{k-} \in R^\ell_+, \tag{11}$$

$$v_i^+ \in R^\ell_+, \ z_i^- = (z_{i1}^-, \ldots, z_{i\ell}^-) \in R^\ell_+, \tag{12}$$

where u_i is a continuous concave utility function of i, $\lambda_i \in R_+$ denotes the level of utility punishment, $\pi_j(p)$ is the profit of j under price p, and θ_{ij} denotes i's share for the profit of j.

Denote by $\mathcal{E} = ((X_i, \omega_i, u_i, \lambda_i, (F_{ik}, f_i^k)_{k=1}^\ell, (\theta_{ij})_{j=1}^n)_{i=1}^m, (Y_j, (F_{jk}, f_j^k)_{k=1}^\ell)_{j=m+1}^{m+n})$ the economy just stated. An *equilibrium* for $\mathcal{E}$ is $((x_i, v_i^+, z_i^-, w_i^{1-}, \ldots, w_i^{\ell-})_{i=1}^m, (y_j, z_j^-, w_j^{1-}, \ldots, w_j^{\ell-})_{j=1}^n, p) \in \prod_{i=1}^m (X_i \times (R^\ell_+) \times (R^\ell_+) \times (R^\ell_+)^\ell) \times \prod_{j=1}^n (Y_j \times R^\ell_+ \times (R^\ell_+)^\ell) \times \Delta$ satisfying (1) – (12) and the *market clearing condition* together with the *specification of an expectation*, $(\hat{s}^1/s_1, \ldots, \hat{s}^\ell/s_\ell)$, such that for each $k = 1, \ldots, \ell$,

$$\sum_{i=1}^{m} v_{ik}^+ + \sum_{i=1}^{m} F_i^k(w_i^{k-}) + \sum_{j=1}^{n} y_{jk}^+ = \sum_{i=1}^{m+n} z_{ik}^- \text{ and} \tag{13}$$

$$\frac{\sum_{i=1}^m v_{ik}^+ e^k + \sum_{i=1}^{m+n} f_i^k(w_i^{k-}) + \sum_{j=m+1}^{m+n} y_{jk}^+ e^k}{\sum_{i=1}^m v_{ik}^+ + \sum_{i=1}^{m+n} F_i^k(w_i^{k-}) + \sum_{j=m+1}^{m+n} y_{jk}^+ + \sum_{j=1}^{n} v_{jk}^+} = \frac{\hat{s}^k}{s_k}, \tag{14}$$

where e^k is the unit vector in R^ℓ whose t-th coordinate is 1 if $t = k$ and 0 if $t \neq k$ and we are considering equation (14) only when $\sum_{i=1}^m v_{ik}^+ + \sum_{i=1}^{m+n} F_i^k(w_i^{k-}) + \sum_{j=m+1}^{m+n} y_{jk}^+ > 0$.[2] Of course, in the above equations we use the notation $v_i^- = (v_{i1}^-, \ldots, v_{i\ell}^-) \in R^\ell_+$ for $i = 1, \ldots, m$, $y_j^+ = (y_{j1}^+, \ldots, y_{j\ell}^+) \in R^\ell_+$ for $j = m+1, \ldots, m+n$, and $w_i^{h-} = (w_{i1}^{h-}, \ldots, w_{i\ell}^{h-}) \in R^\ell_+$ for $h = 1, \ldots, \ell$ and for $i = 1, \ldots, m+n$.

[2] Hence, if $\sum_{i=1}^m v_{ik}^+ + \sum_{i=1}^{m+n} F_i^k(w_i^{k-}) + \sum_{j=m+1}^{m+n} y_{jk}^+ = 0$, we have no restriction on the specification of expectations.

3 Existence of Equilibrium

We need several additional assumptions to assure the existence of equilibria for the economy, $\mathcal{E} = ((X_i, \omega_i, u_i, \lambda_i, (F_{ik}, f_i^k)_{k=1}^{\ell}, (\theta_{ij})_{j=1}^{n})_{i=1}^{m}, (Y_j, (F_{jk}, f_j^k)_{k=1}^{\ell})_{j=m+1}^{m+n})$. All conditions, however, are standard except for the special assumption on the fictitious technology stated in 3.3 and 3.4. In the following, we shall give a list for the assumptions together with a brief sketch for the proof of equilibrium existence.

3.1 Producers

As stated in the previous section, each real technology, $Y_j \subset R^{\ell}$, $j = m+1, \ldots, m+n$, is assumed to be closed, convex, and having 0 as its element. Therefore, if we suppose the continuity and concavity of the fictitious production function, F_{jk}, for each $j = m+1, \ldots, m+n$, (see 3.3), it is easy to check that the set of solutions to the maximization problem, (1) subject to (2),(3), and (4), is closed and convex as long as the maximization problem has a solution. To assure the existence of solutions, we may restrict the domain for the maximization problem on a sufficiently large cube since the condition for f_j^k in 3.3 makes the feasible set including fictitious outputs to be bounded.

3.2 Consumers

The set solutions for the maximization problem ((5) subject to (6) - (12)) is closed if each F_i^k is continuous. It is convex if each F_i^k is concave. To assure the existence of solutions, the same sort of truncation argument we have used for the production case enable us to restrict the domain for the maximization problem on a compact set.

3.3 Attainable Set and Fictitious Technology

Since each X_i is bounded from below, a certain kind of conditions for upper bounds for (real) technologies, e.g., an appropriate condition for asymptotic cones of Y_j's, enables us to define a sufficiently large cube $\Gamma_0 = [-\gamma_0, \gamma_0]^{\ell}$, $\gamma_0 \in R_{++}$, such that every *real* feasible state for economy $\mathscr{E}$, $((x_i)_{i=1}^{m}, (y_j)_{j=m+1}^{m+n})$, belongs to $(\Gamma_0)^{m+n}$.

Then, the next condition for the fictitious technology makes all feasible states (including fictitious outputs and trade contracts) in a certain bounded cube, Γ.

(3.3.A) For all $i = 1, \ldots, m+n$ and $k = 1, \ldots, \ell$, $F_{ik}(x^{\nu}) \to \infty$ as $\nu \to \infty$ implies that $\|(x^{\nu} - (0, \ldots, 0, f_i^k(x^{\nu}), 0, \ldots, 0))^{+}\| \to \infty$,

where $F_{ik}(x^{\nu})$ is the k-th coordinate. The condition is quite natural. It says that even for fictitious technologies, if the real supply is bounded, the fictitious supply is also bounded.

For the fictitious technology, we also assume the following:

(3.3.B) Each f_i^k is continuous and non-decreasing.

(3.3.C) Each F_{ik} is continuous and concave.

3.4 0 neighbourhood Condition for Fictitious Technologies

We use the following condition to assure the existence of an upper bound for equilibrium values of $\hat{s}^k/s_k$ for each k.

(3.4.A) For all $k = 1, \ldots, \ell$, we assume that there exist 0 neighbourhood U^k in R^ℓ_+ and $\delta^k_0 \in R_+$ such that for all $i = 1, \ldots, m+n$ and $w_i^{k-} \in U^k \setminus \{0\}$,

$$\frac{f_i^k(w_i^{k-})}{F_i^k(w_i^{k-})} \leq (\delta_0^k, \ldots, \delta_0^k).$$

It is equation (14) which defines the expectation $(\hat{s}^1/s_1, \ldots, \hat{s}^\ell/s_\ell)$. Condition (3.4.A) assures the existence of upper bound $\delta_0 = \max\{\delta_0^1, \ldots, \delta_0^\ell\}$ for the equilibrium expectation throuth equation (14).

3.5 Fixed Points and Limit Arguments

We may prove the existence of equilibrium by ordinary truncation method together with limit arguments. Assumption (3.3.B), (3.3.C) and a certain minimum wealth condition assures the upper semicontinuity of demand and supply correspondences. Summation of equation (9) for all agents gives the Walras' law for the market clearing condition (13). Equation (14) gives the adjustment mechanism for expectations. Lastly, conditions (3.3.A) and (3.4.A) assures for a sufficiently large truncation, that the equilibrium for truncated economy is a real equilibrium. Urai-Yoshimachi (2004) deals with some details of the proof, implications for assumptions, and applications.

References

Akerlof, G.: "The Market for Lemons. Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics* , Vol. 84, pp.488–500, 1970.

Bisin, A. and P. Gottardi: "Competitive Equilibria with Asymmetric Information," *Journal of Economic Theory*, Vol. 87, pp.1–48, 1999.

Dubey, P., J. Geanakoplos, and M. Shubik: "Default in a General Equilibrium Model with Incomplete Markets," Cowles Commission Discussion Paper No.1247, 1990–2000.

Rothschild, M. and J. Stiglitz: "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, Vol. 90, pp.629–650, 1976.

Urai, K. and A. Yoshimachi: "A General Equilibrium Model for Economy with Asymmetric Information," *Osaka Economic Papers* (in Japanese), Vol. 54, pp.13–19, 2004.