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Bossiness and Implementability in Pure Exchange Economies*

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Abstract

We explore the relationships between bossiness and implementability in pure exchange economies with free disposal. First, we show that non-bossiness together with individual monotonicity is necessary and sufficient for Nash implementation. This is an alternative version of the characterization of Nash implementable social choice functions, which is given by Maskin (1999). Next, we provide a characterization of securely implementable social choice functions, which is an alternative to the characterization provided by Saijo et al. (2004), by proving that strong-non-bossiness coupled with strategy-proofness and the weak rectangular property is necessary and sufficient for secure implementation.

1 Introduction

The mechanism design literature has dealt with a very large number of allocation rules (or direct revelation mechanisms).[1] The following is an

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[1]An allocation rule is a function that assigns a feasible allocation to each list of agents' types.

example of the allocation rules in a pure exchange economy: an allocation rule where there is an agent, called a *boss*, who can change another agent's consumption bundle by changing her type without changing her own bundle. This type of allocation rule was called *bossy* by Satterthwaite and Sonnenschein (1981); but the idea of the bossy allocation rules had already been known, since the well-known Vickrey–Clarke–Groves type of allocation rule (Vickrey (1961), Clarke (1971), and Groves (1973)) was bossy. So, bossy allocation rules can be regarded as acceptable if the Vickrey–Clarke–Groves type of allocation rule seems attractive.

The standard economic theory teaches us that agents are assumed to be selfish. This means that bosses do not care about consumption bundles of the other agents; so bosses will not deliberately change another agent's consumption bundle by changing her type even if her own bundle is kept unchanged. This is a key to making the Vickrey–Clarke–Groves type of allocation rule work well.

Nevertheless, Satterthwaite and Sonnenschein (1981) thought of bossy allocation rules as undesirable at least in terms of simplicity; so they introduced the notion of *non-bossiness*, which requires that there should be no boss who can affect another agent's consumption bundle by changing her type without affecting her own bundle. Subsequently, Ritz (1983) introduced *strong-non-bossiness*,[2] a stronger version of non-bossiness, which requires that there should be no boss who can influence another agent's consumption bundle by changing her type without influencing her own *utility*. Non-bossiness, strong-non-bossiness, and other notions that rule out the existence of "bosses" have since been widely used in the literature on strategy-proofness.[3]

But, almost all of the literature has not explained non-bossiness's reasonableness and desirability. Non-bossiness has often been imposed for technical convenience with the following exceptions: Barberà and Jackson (1995) showed that non-bossiness plus strategy-proofness implies weak

[2] Ritz (1983) called the term *non-corruptibility*.

[3] A partial list of such literature includes Barberà and Jackson (1995), Ju (2004), and Serizawa (2005) for pure exchange economies; Svensson (1999), Takamiya (2001), Miyagawa (2002), and Pàpai (2003) for the housing market of Shapley and Scarf (1974); Miyagawa (2001) and Svensson and Larsson (2002) for the Shapley–Scarf housing market with compensation; Svensson (1999), Pàpai (2000a), Pàpai (2000b), and Pàpai (2001a) for the house allocation problem; Schummer (2000a) and Svensson and Larsson (2002) for the house allocation problem with compensation; Barberà et al. (1997), Klaus (2001a), and Klaus (2001b) for allotment economies with single-peaked or single-dipped preferences; Serizawa (1996) for public good economies; Deb and Razzolini (1999), Dearden and Einolf (2003), and Jackson and Nicolò (2004) for excludable public good economies; and Pàpai (2001b) for others.

coalitional strategy-proofness in pure exchange economies,[4] and Pàpai (2000a) and Takamiya (2001) showed that non-bossiness together with strategy-proofness is equivalent to coalitional strategy-proofness in the house allocation problem and in the Shapley–Scarf housing market with strict preferences, respectively. These results tell us that non-bossiness is desirable in the sense that, when combined with strategy-proofness, it prevents manipulation by coalitions of agents.

However, coalitional strategy-proofness is too demanding in general, because it prevents not only self-enforcing coalitional manipulations but also non-self-enforcing coalitional manipulations. The standard economic theory also teaches us that in non-cooperative environments, agents can freely discuss their actions but cannot make binding commitments. This indicates that there is no need to rule out coalitional manipulations that are not self-enforcing unless an additional assumption that agents can sign binding agreements is imposed.[5] So, when coupled with strategy-proofness, non-bossiness appears strong without the additional assumption.[6]

Thus, the issue concerning reasonableness and desirability of ruling out "bosses" seems still open. This paper examines the desirability of non-bossiness and strong-non-bossiness by exploring the relationships between bossiness and implementability in pure exchange economies with free disposal.[7]

First, in Theorem 1, we provide a characterization of Nash implementable social choice functions, which is an alternative to the characterization provided by Maskin (1999), by showing that non-bossiness plus *individual monotonicity* (Takamiya (2001)), a individual version of monotonicity (Maskin (1999)), is necessary and sufficient for Nash implementation in pure exchange economies with three or more agents. This is a very

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[4]The converse of the result of Barberà and Jackson (1995) does not hold in pure exchange economies, since *inversely dictatorial rules* (Zhou (1991)) satisfy weak coalitional strategy-proofness but fail to satisfy non-bossiness.

[5]Serizawa (2005) introduced the notion of *effective pairwise strategy-proofness*, which rules out only unilateral manipulation and self-enforcing pairwise manipulation.

[6]Although coalitional strategy-proofness rules out manipulation by very large coalitions, it might not be necessary to worry about manipulation by such coalitions because it is difficult to coordinate actions of agents in such coalitions, as pointed out by Schummer (2000b) and Serizawa (2005). So, non-bossiness together with strategy-proofness might still seem strong, even if the additional assumption is imposed.

[7]In the Shapley–Scarf housing market with strict preferences, Takamiya (2001) has already shown that non-bossiness has relationships to Nash implementability. In pure exchange economies, however, the relationship of non-bossiness to Nash implementability is not yet known.

important result, because it indicates that bossy social choice functions are never Nash implementable in pure exchange economies with three or more agents.[8]

When combined with the result of Mizukami and Wakayama (2005a), Theorem 1 leads to Corollary 1: The non-bossy, individual monotonic, and strategy-proof social choice function can be implemented both in dominant strategy equilibria and in Nash equilibria. However, the corollary does not guarantee that the social choice function that is dominant strategy implemented by some mechanism can be Nash implemented by the *same* mechanism; it says only that the social choice function that is implemented in dominant strategy equilibria by a mechanism can be Nash implemented by *some* mechanism. So, we next look for social choice functions that can be *simultaneously* implemented in dominant strategy equilibria and in Nash equilibria.

In Theorem 2, we prove that strong-non-bossiness together with strategy-proofness and the weak rectangular property is necessary and sufficient for secure implementation, i.e., double implementation in dominant strategy equilibria and in Nash equilibria (see Saijo et al. (2004) for the robustness of secure implementation). This is an alternative to the characterization provided by Saijo et al. (2004). A *voluntary trading rule* (Barberà (2001)) is an example of social choice functions that are both dominant strategy implementable and Nash implementable, but which are not securely implementable.

To summarize, it turns out that non-bossiness and strong-non-bossiness each have close relationships to implementability. Theorem 1 implies that non-bossiness has a critical role in determining whether or not a social choice function is Nash implementable in pure exchange economies. Theorem 2 suggests that strong-non-bossiness plays an important role in deciding whether or not a social choice function is securely implementable in pure exchange economies. These would support the use of non-bossiness or strong-non-bossiness from the point of view of implementability.

Before concluding the Introduction, we reexamine the result of Repullo (1985), which asserts that if a social choice function can be dominant strategy implemented by an indirect mechanism, but it cannot be dominant strategy implemented by the associated direct revelation mechanism, then the original indirect mechanism cannot Nash implement it. Theorem 2 implies that if a social choice function violates either strong-non-bossiness or the weak rectangular property, then the mechanism that dominant strategy

[8]We use the terms "allocation rule" and "social choice function" interchangeably throughout this paper.

implements the social choice function cannot simultaneously Nash implement it. This yields the result of Repullo (1985), because every social choice function that is dominant strategy implementable but cannot be dominant strategy implemented by the associated direct revelation mechanism must violate strong-non-bossiness by the revelation principle and by the contrapositive of Lemma 6. Thus, strong-non-bossiness plays a key role in accounting for why the result of Repullo (1985) holds.

This paper is organized as follows. Section 2 describes the model and gives some definitions. We explore the relationships between bossiness and implementability in Section 3. Section 4 contains some concluding remarks.

2 The Model

Consider a pure exchange economy *with* free disposal. Let $N := \{1, 2, \ldots, n\}$ be the set of *agents*, where $2 \leq n < +\infty$. Let $L := \{1, 2, \ldots, l\}$ be the set of *goods*, where $2 \leq l < +\infty$. The set of *feasible allocations* is $A := \left\{(a_1, a_2, \ldots, a_n) \in \mathbb{R}^l_+ \times \mathbb{R}^l_+ \times \cdots \times \mathbb{R}^l_+ \,\middle|\, \sum_{i \in N} a_i \leq \sum_{i \in N} \omega_i\right\}$,[9] where $a_i \in \mathbb{R}^l_+$ denotes agent i's *consumption bundle* and $\omega_i \in \mathbb{R}^l_{++}$ denotes agent i's *initial endowment*. Let $A_i := \left\{a_i \in \mathbb{R}^l_+ \,\middle|\, a_i \leq \sum_{i \in N} \omega_i\right\}$ denote the feasible set of agent i's consumption bundle.

We assume that each agent cares only about her own consumption bundle. For each agent $i \in N$, let Θ_i be the set of her *types*: $\theta_i \in \Theta_i$ if and only if $u_i(\cdot; \theta_i)$ is (i) *continuous*, (ii) *strictly increasing*, and (iii) *strictly quasi-concave*, where $u_i \colon \mathbb{R}^l_+ \times \Theta_i \to \mathbb{R}$ denotes her *utility function*. A *type profile* is a list $\theta = (\theta_1, \theta_2, \ldots, \theta_n) \in \Theta$, where $\Theta := \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$.

A *social choice function* is a single-valued function $f \colon \Theta \to A$ that assigns a feasible allocation $a \in A$ to each type profile $\theta \in \Theta$. Let f_i denote agent i's consumption bundle assigned by f.

Let M_i denote a *message space* of agent $i \in N$. We call $m_i \in M_i$ a *message* of agent $i \in N$. A *mechanism* is a pair $\Gamma = (M, g)$, where $M := M_1 \times M_2 \times \cdots \times M_n$ and $g \colon M \to A$ is an *outcome function*. Let g_i denote agent i's *outcome* of (M, g). Given a social choice function f, the mechanism $\Gamma^D := (\Theta, f)$ is called the *associated direct revelation mechanism*. A *message profile* is denoted by $m = (m_1, m_2, \ldots, m_n) \in M$.

A message profile $m^* = (m^*_1, m^*_2, \ldots, m^*_n) \in M$ is a *dominant strategy equilibrium* of (M, g) at $\theta \in \Theta$ if, for any $i \in N$, $u_i(g_i(m^*_i, m_{-i}); \theta_i) \geq u_i(g_i(m'_i, m_{-i}); \theta_i)$ for any $m'_i \in M_i$ and any $m_{-i} \in M_{-i}$. Let $DSE^{\Gamma}(\theta) \subseteq M$ be the set of

[9] The vector inequalities are $\gg$, $>$, and $\geq$.

dominant strategy equilibria of $\Gamma = (M, g)$ at $\theta \in \Theta$. Let $g(DSE^{\Gamma}(\theta)) := \{a \in A \mid a = g(m) \text{ for some } m \in DSE^{\Gamma}(\theta)\}$ be the set of *dominant strategy equilibrium outcomes* of $\Gamma = (M, g)$ at $\theta \in \Theta$.

A message profile $m^* = (m_1^*, m_2^*, \ldots, m_n^*) \in M$ is a *Nash equilibrium* of (M, g) at $\theta \in \Theta$ if, for any $i \in N$, $u_i(g_i(m_i^*, m_{-i}^*); \theta_i) \geq u_i(g_i(m_i', m_{-i}^*); \theta_i)$ for any $m_i' \in M_i$. Let $NE^{\Gamma}(\theta) \subseteq M$ be the set of Nash equilibria of $\Gamma = (M, g)$ at $\theta \in \Theta$. Let $g(NE^{\Gamma}(\theta)) := \{a \in A \mid a = g(m) \text{ for some } m \in NE^{\Gamma}(\theta)\}$ be the set of *Nash equilibrium outcomes* of $\Gamma = (M, g)$ at $\theta \in \Theta$.

A mechanism $\Gamma = (M, g)$ *dominant strategy implements* a social choice function f if $g(DSE^{\Gamma}(\theta)) = f(\theta)$ for any $\theta \in \Theta$.[10] A social choice function f is *dominant strategy implementable* if there exists a mechanism $\Gamma = (M, g)$ such that $g(DSE^{\Gamma}(\theta)) = f(\theta)$ for all $\theta \in \Theta$.

A mechanism $\Gamma = (M, g)$ *Nash implements* a social choice function f if $g(NE^{\Gamma}(\theta)) = f(\theta)$ for any $\theta \in \Theta$. A social choice function f is *Nash implementable* if there exists a mechanism $\Gamma = (M, g)$ such that $g(NE^{\Gamma}(\theta)) = f(\theta)$ for all $\theta \in \Theta$.

A mechanism $\Gamma = (M, g)$ *securely implements* a social choice function f if $g(DSE^{\Gamma}(\theta)) = g(NE^{\Gamma}(\theta)) = f(\theta)$ for any $\theta \in \Theta$. A social choice function f is *securely implementable* if there exists a mechanism $\Gamma = (M, g)$ such that $g(DSE^{\Gamma}(\theta)) = g(NE^{\Gamma}(\theta)) = f(\theta)$ for all $\theta \in \Theta$.

Let $LC_i(a; \theta_i) := \{x \in A \mid u_i(a_i; \theta_i) \geq u_i(x_i; \theta_i)\}$ be agent i's *lower contour set* of $a \in A$ at $\theta_i \in \Theta_i$.

Now we introduce six properties of social choice functions. The first two properties relate to implementability. *Strategy-proofness* is closely related to dominant strategy implementability in pure exchange economies.

Definition 1 (Strategy-Proofness). A social choice function f satisfies *strategy-proofness* if, for all $\theta \in \Theta$ and all $i \in N$, there is no $\theta_i' \in \Theta_i$ such that $u_i(f_i(\theta_i', \theta_{-i}); \theta_i) > u_i(f_i(\theta); \theta_i)$.

Monotonicity (Maskin (1999)) is both necessary and sufficient for Nash implementation in pure exchange economies with three or more agents.

Definition 2 (Monotonicity). A social choice function f satisfies *monotonicity* if, for all $\theta, \theta' \in \Theta$, if $LC_i(f(\theta); \theta_i) \subseteq LC_i(f(\theta); \theta_i')$ for all $i \in N$, then $f(\theta') = f(\theta)$.

The following is a individual version of monotonicity, which is due to Takamiya (2001).

[10]To simplify notation, we write $\{f(\theta)\}$ as $f(\theta)$.

Definition 3 (Individual Monotonicity). A social choice function f satisfies *individual monotonicity* if, for all $\theta \in \Theta$, all $i \in N$, and all $\theta'_i \in \Theta_i$, if $LC_i(f(\theta); \theta_i) \subseteq LC_i(f(\theta); \theta'_i)$, then $f_i(\theta'_i, \theta_{-i}) = f_i(\theta)$.

The last three properties concern bossiness. *Non-bossiness*, which was introduced by Satterthwaite and Sonnenschein (1981), requires that if an agent changes her type but her consumption bundle is unchanged, then the bundle of each agent should be unchanged.

Definition 4 (Non-Bossiness). A social choice function f satisfies *non-bossiness* if, for all $\theta \in \Theta$, all $i \in N$, and all $\theta'_i \in \Theta_i$, if $f_i(\theta) = f_i(\theta'_i, \theta_{-i})$, then $f(\theta) = f(\theta'_i, \theta_{-i})$.

Strong-non-bossiness (Ritz (1983)) requires that if an agent changes her type but her *utility* is kept unchanged, then the consumption bundle of each agent should be unchanged.

Definition 5 (Strong-Non-Bossiness). A social choice function f satisfies *strong-non-bossiness* if, for all $\theta \in \Theta$, all $i \in N$, and all $\theta'_i \in \Theta_i$, if $u_i(f_i(\theta); \theta_i) = u_i(f_i(\theta'_i, \theta_{-i}); \theta_i)$, then $f(\theta) = f(\theta'_i, \theta_{-i})$.

Quasi-strong-non-bossiness (Mizukami and Wakayama (2005a)) requires that if an agent changes her type but her *utility* remains unchanged *irrespective of the other agents' types*, then the consumption bundle of each agent should be unchanged.

Definition 6 (Quasi-Strong-Non-Bossiness). A social choice function f satisfies *quasi-strong-non-bossiness* if, for all $\theta \in \Theta$, all $i \in N$, and all $\theta'_i \in \Theta_i$, if $u_i(f_i(\theta); \theta_i) = u_i(f_i(\theta'_i, \theta_{-i}); \theta_i)$ for all $\theta_{-i} \in \Theta_{-i}$, then $f(\theta) = f(\theta'_i, \theta_{-i})$.

Note that a social choice function is called bossy, strong-bossy, and quasi-strong-bossy if it violates non-bossiness, strong-non-bossiness, and quasi-strong-non-bossiness, respectively.

Remark 1. By definition, quasi-strong-non-bossiness is weaker than strong-non-bossiness, which is stronger than non-bossiness.

3 Results

In this section, we examine the relationships among non-bossiness, strong-non-bossiness, and three notions of implementability—dominant strategy implementability, Nash implementability, and secure implementability. Before proceeding, we provide four results that have already been known.

The first two results concern dominant strategy implementability in pure exchange economies with free disposal.

Lemma 1 (Mizukami and Wakayama (2005a)). *A social choice function is dominant strategy implementable if and only if it satisfies strategy-proofness.*

Lemma 2 (Mizukami and Wakayama (2005a)). *A social choice function is dominant strategy implemented by the associated direct revelation mechanism if and only if it satisfies both quasi-strong-non-bossiness and strategy-proofness.*

The last two results are related to the *rectangular property* (Saijo et al. (2004)),[11] which is a necessary condition for secure implementation, i.e., double implementation in dominant strategy equilibria and in Nash equilibria (see Saijo et al. (2004) for the importance of secure implementation).

Lemma 3 (Saijo et al. (2004)). *If a social choice function is securely implementable, then it satisfies the rectangular property.*

Lemma 4 (Saijo et al. (2004)). *If a social choice function satisfies the rectangular property, then it satisfies strong-non-bossiness.*

3.1 Implementation of Bossy Social Choice Functions

In this subsection, we analyze the implementability of bossy social choice functions. We begin by looking at the relationships between non-bossiness and monotonicity, which is a necessary and sufficient condition for Nash implementation in pure exchange economies with three or more agents.

Lemma 5. *A social choice function f satisfies both non-bossiness and individual monotonicity if and only if it satisfies monotonicity.*

Proof. See Mizukami and Wakayama (2005b). □

We are now ready to provide a characterization of Nash implementation in pure exchange economies with three or more agents, which is an alternative version given by Maskin (1999).

Theorem 1. *Suppose $n \geq 3$. A social choice function is Nash implementable if and only if it satisfies both non-bossiness and individual monotonicity.*

Proof. Since monotonicity is both necessary and sufficient for Nash implementation in pure exchange economies with three or more agents, Theorem 1 follows directly from Lemma 5. □

[11] A social choice function f satisfies the *rectangular property* if, for all $\theta, \theta' \in \Theta$, if $u_i(f_i(\theta'); \theta_i) = u_i(f_i(\theta_i, \theta'_{-i}); \theta_i)$ for all $i \in N$, then $f(\theta') = f(\theta)$.

Theorem 1 implies that every social choice function that violates non-bossiness or individual monotonicity is never Nash implementable. The theorem also indicates that non-bossiness has close relationships to Nash implementability, in the sense that non-bossiness is a necessary condition for Nash implementation and is part of the sufficient condition for Nash implementation. Thus, bossiness is undesirable at least from the point of view of Nash implementability.

3.2 Implementation of Non-Bossy Social Choice Functions

In this subsection, we study the implementability of non-bossy social choice functions. Together with Lemma 1, Theorem 1 gives us the following corollary.

Corollary 1. *Suppose $n \geq 3$. A social choice function is implementable both in dominant strategy equilibria and in Nash equilibria if and only if it satisfies non-bossiness, individual monotonicity, and strategy-proofness.*

Corollary 1 tells us that only the non-bossy, individual monotonic, and strategy-proof social choice function is both dominant strategy implementable and Nash implementable. However, the corollary does not necessarily imply that the mechanism which dominant strategy implements the social choice function can Nash implement it. This is because the corollary asserts only that there exists some mechanism that Nash implements the social choice function; the corollary does not guarantee that the mechanism *simultaneously* implements the social choice function in dominant strategy equilibria and in Nash equilibria.

An example of social choice functions satisfying non-bossiness, individual monotonicity, and strategy-proofness is the *voluntary trading rule* (Barberà (2001)). So, the voluntary trading rule is both dominant strategy implementable and Nash implementable. However, the mechanism that implements the voluntary trading rule in dominant strategy equilibria might not be able to simultaneously implement it in Nash equilibria. Thus, we next seek to characterize social choice functions that are securely implementable, i.e., doubly implementable in dominant strategy equilibria and in Nash equilibria.

The following is a weaker version of the rectangular property, a necessary condition of secure implementation.

Definition 7 (The Weak Rectangular Property). A social choice function f satisfies the *weak rectangular property* if, for all $i, j \in N$ and all

$(\theta'_i, \theta'_j, \theta_{-i,j}), \theta \in \Theta$, if $f(\theta) = f(\theta'_i, \theta_{-i}) = f(\theta'_j, \theta_{-j})$, then $u_i(f_i(\theta); \theta'_i) \geq u_i(f_i(\theta'_i, \theta'_j, \theta_{-i,j}); \theta'_i)$ and $u_j(f_j(\theta); \theta'_j) \geq u_j(f_j(\theta'_i, \theta'_j, \theta_{-i,j}); \theta'_j)$.

The weak rectangular property requires the following. Suppose a change in type profile from $(\theta'_i, \theta'_j, \theta_{-i,j})$ to θ. Then, if, for agents $i, j \in N$, the unilateral changes in her type from θ to (θ'_i, θ_{-i}) and from θ to (θ'_j, θ_{-j}) leave the consumption bundles of everyone unchanged, then agents $i, j \in N$ should weakly prefer $f_i(\theta)$ to $f_i(\theta'_i, \theta'_j, \theta_{-i,j})$ and $f_j(\theta)$ to $f_j(\theta'_i, \theta'_j, \theta_{-i,j})$ at the original types θ'_i and θ'_j, respectively.

We are now ready to characterize the class of securely implementable social choice functions.

Theorem 2. *A social choice function f is securely implementable if and only if it satisfies strong-non-bossiness, strategy-proofness, and the weak rectangular property.*

Theorem 2 says that only social choice functions that satisfy strong-non-bossiness, strategy-proofness, and the weak rectangular property are securely implementable. So, in contrast to Corollary 1, the theorem indicates that whenever a social choice function satisfies strong-non-bossiness, strategy-proofness, and the weak rectangular property, the mechanism that dominant strategy implements the social choice function can simultaneously Nash implement it. Theorem 2 also shows that there are close relationships between strong-non-bossiness and secure implementability, in the sense that strong-non-bossiness is necessary for secure implementation and is part of the sufficient condition for secure implementation. So, strong-bossiness is unacceptable at least in terms of secure implementability.

Remark 2. It is an open question whether it is possible to construct in pure exchange economies *without* free disposal an example which demonstrates that in the absence of strategy-proofness, the rectangular property implies strong-non-bossiness and the weak rectangular property, but not vice versa. So, Theorem 2 is merely an alternative version of the characterization given by Saijo et al. (2004), which shows that a social choice function is securely implementable if and only if it satisfies strategy-proofness and the rectangular property. As demonstrated in Example 1 below, however, it is possible to do so in pure exchange economies *with* free disposal. Thus, in pure exchange economies with free disposal, Theorem 2 strengthens the characterization of Saijo et al. (2004) by weakening the rectangular property to strong-non-bossiness and the weak rectangular property.

Example 1. The following social choice function f satisfies both strong-non-bossiness and the weak rectangular property but violates the rectangular property: $f(\theta) = 0$ for some $\theta \in \Theta$ and $f(\theta') = \bar{a} > 0$ for all $\theta' \in \Theta \setminus \{\theta\}$.

Before proceeding to the proof of Theorem 2, we provide two lemmas that are useful in the proof of the if part of Theorem 2.

Lemma 6. *If a social choice function satisfies strong-non-bossiness and strategy-proofness, then it is dominant strategy implemented by the associated direct revelation mechanism.*

Proof. Lemma 6 follows from Remark 1 and Lemma 2. □

Lemma 7. *If a social choice function f satisfies strong-non-bossiness, strategy-proofness, and the weak rectangular property, then it is Nash implemented by the associated direct revelation mechanism.*

Proof. Since f satisfies strong-non-bossiness and strategy-proofness, Lemma 6 implies that $f(DSE^{\Gamma^D}(\theta)) = f(\theta)$ for all $\theta \in \Theta$. So, to prove that f is Nash implementable by Γ^D, it suffices to show that $f(NE^{\Gamma^D}(\theta)) = f(DSE^{\Gamma^D}(\theta))$ for all $\theta \in \Theta$.

Suppose to the contrary that $f(NE^{\Gamma^D}(\bar{\theta})) \neq f(DSE^{\Gamma^D}(\bar{\theta}))$ for some $\bar{\theta} \in \Theta$. Then, there exists $\hat{a} \in A$ such that $\hat{a} \in f(NE^{\Gamma^D}(\bar{\theta})) \setminus f(DSE^{\Gamma^D}(\bar{\theta}))$. Let $\hat{\theta} \in NE^{\Gamma^D}(\bar{\theta})$ be such that $f(\hat{\theta}) = \hat{a}$.

Step 1: $f(\bar{\theta}_i, \hat{\theta}_{-i}) = f(\hat{\theta})$ for all $i \in N$.

Pick any $i \in N$. Since $\hat{\theta} \in NE^{\Gamma^D}(\bar{\theta})$, $u_i(f_i(\bar{\theta}_i, \hat{\theta}_{-i}); \bar{\theta}_i) \leq u_i(f_i(\hat{\theta}); \bar{\theta}_i)$. By strategy-proofness, $u_i(f_i(\bar{\theta}_i, \hat{\theta}_{-i}); \bar{\theta}_i) \geq u_i(f_i(\hat{\theta}); \bar{\theta}_i)$. So, $u_i(f_i(\bar{\theta}_i, \hat{\theta}_{-i}); \bar{\theta}_i) = u_i(f_i(\hat{\theta}); \bar{\theta}_i)$. Hence, strong-non-bossiness implies $f(\bar{\theta}_i, \hat{\theta}_{-i}) = f(\hat{\theta})$.

Step 2: $f(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}) = f(\bar{\theta}_i, \hat{\theta}_{-i})$ for all $j \in N$.

Pick any $j \in N$. Consider $(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}), \hat{\theta} \in \Theta$. By Step 1, $f(\hat{\theta}) = f(\bar{\theta}_i, \hat{\theta}_{-i}) = f(\bar{\theta}_j, \hat{\theta}_{-j})$. So, the weak rectangular property implies $u_j(f_j(\hat{\theta}); \bar{\theta}_j) \geq u_j(f_j(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}); \bar{\theta}_j)$. Since $f(\hat{\theta}) = f(\bar{\theta}_i, \hat{\theta}_{-i})$ by Step 1, this implies $u_j(f_j(\bar{\theta}_i, \hat{\theta}_{-i}); \bar{\theta}_j) \geq u_j(f_j(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}); \bar{\theta}_j)$. By strategy-proofness, $u_j(f_j(\bar{\theta}_i, \hat{\theta}_{-i}); \bar{\theta}_j) \leq u_j(f_j(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}); \bar{\theta}_j)$. Therefore, $u_j(f_j(\bar{\theta}_i, \hat{\theta}_{-i}); \bar{\theta}_j) = u_j(f_j(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}); \bar{\theta}_j)$. Thus, strong-non-bossiness implies $f(\bar{\theta}_i, \hat{\theta}_{-i}) = f(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j})$.

Step 3: $f(\bar{\theta}_i, \bar{\theta}_j, \bar{\theta}_k, \hat{\theta}_{-i,j,k}) = f(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j})$ for all $k \in N$.

Pick any $k \in N$. Consider $(\bar{\theta}_i, \bar{\theta}_j, \bar{\theta}_k, \hat{\theta}_{-i,j,k}), (\bar{\theta}_i, \hat{\theta}_{-i}) \in \Theta$. By Step 2, $f(\bar{\theta}_i, \hat{\theta}_{-i}) = f(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}) = f(\bar{\theta}_i, \bar{\theta}_k, \hat{\theta}_{-i,k})$. So, the weak rectangular property implies $u_k(f_k(\bar{\theta}_i, \hat{\theta}_{-i}); \bar{\theta}_k) \geq u_k(f_k(\bar{\theta}_i, \bar{\theta}_j, \bar{\theta}_k, \hat{\theta}_{-i,j,k}); \bar{\theta}_k)$. Since $f(\bar{\theta}_i, \hat{\theta}_{-i}) =$

$f(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j})$ by Step 2, this implies $u_k(f_k(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}); \bar{\theta}_k) \geq u_k(f_k(\bar{\theta}_i, \bar{\theta}_j, \bar{\theta}_k, \hat{\theta}_{-i,j,k}); \bar{\theta}_k)$. By strategy-proofness, $u_k(f_k(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}); \bar{\theta}_k) \leq u_k(f_k(\bar{\theta}_i, \bar{\theta}_j, \bar{\theta}_k, \hat{\theta}_{-i,j,k}); \bar{\theta}_k)$. Therefore, $u_k(f_k(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}); \bar{\theta}_k) = u_k(f_k(\bar{\theta}_i, \bar{\theta}_j, \bar{\theta}_k, \hat{\theta}_{-i,j,k}); \bar{\theta}_k)$. Thus, strong-non-bossiness implies $f(\bar{\theta}_i, \bar{\theta}_j, \hat{\theta}_{-i,j}) = f(\bar{\theta}_i, \bar{\theta}_j, \bar{\theta}_k, \hat{\theta}_{-i,j,k})$.

Iterating the above step for further agents in N provides $f(\bar{\theta}) = f(\hat{\theta})$, which contradicts $f(\bar{\theta}) = f(\hat{\theta}) = \hat{a} \notin f(DSE^{\Gamma^D}(\bar{\theta}))$ because $f(\bar{\theta}) \in f(DSE^{\Gamma^D}(\bar{\theta}))$ by strategy-proofness. □

We are now ready to prove Theorem 2.

Proof of Theorem 2. The *if* part: Since f satisfies strong-non-bossiness and strategy-proofness, the associated direct revelation mechanism can dominant strategy implement f by Lemma 6. Since f satisfies strong-non-bossiness, strategy-proofness, and the weak rectangular property, the associated direct revelation mechanism can Nash implement f by Lemma 7. Thus, f is securely implementable.[12]

The *only if* part: Since f is securely implementable, f satisfies strategy-proofness and the rectangular property by the revelation principle and by Lemma 3, respectively. So, f satisfies strong-non-bossiness and the weak rectangular property by Lemma 4 and by definition, respectively. □

4 Conclusion

In this paper, we have shown that there are close relationships in pure exchange economies (i) between non-bossiness and Nash implementability and (ii) between strong-non-bossiness and secure implementability, respectively. These relationships tell us that non-bossiness and strong-non-bossiness are desirable from the point of view of implementability. This desirability of non-bossiness or strong-non-bossiness seems significant in terms of requiring no additional assumption, which is in contrast to the desirability mentioned in the Introduction. It would be an interesting topic for further research to see whether or not similar relationships between bossiness and implementability hold in other environments listed in the Introduction.

As pointed out by Satterthwaite and Sonnenschein (1981), non-bossiness is automatically satisfied in pure public goods economies. It might show

[12]It should be noted that a social choice function is securely implementable if and only if it is securely implemented by the associated direct revelation mechanism, as shown by Saijo et al. (2004).

the universality of non-bossiness. At the same time, however, it means that bossiness is characteristic of economies with excludable goods, such as private goods economies or excludable public goods economies. So, a negative aspect of non-bossiness is that non-bossiness rules out allocation rules inherent in economies with excludable goods to identify excludable goods economies with non-excludable goods economies. In taking account of this, Theorem 1 seems an impossibility theorem in the sense that the theorem indicates that it is impossible in pure exchange economies with three or more agents to implement bossy allocation rules in Nash equilibria, which embody the characteristics inherent in economies with excludable goods including pure exchange economies. However, this is not the case with dominant strategy implementation, because non-bossiness has no relationship to strategy-proofness, which is necessary and sufficient for dominant strategy implementation in pure exchange economies *with* free disposal, as shown by Mizukami and Wakayama (2005a).

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