数理解析研究所講究録 1461 巻 2005 年 85-95

Error Bounds of P-matrix Linear Complementarity Problems[1]

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1 Introduction

The linear complementarity problem is to find a vector $x \in R^n$ such that

$$Mx + q \geq 0, \quad x \geq 0, \quad x^T(Mx+q) = 0,$$

or to show that no such vector exists, where $M \in R^{n\times n}$ and $q \in R^n$. We denote this problem by LCP(M,q). A matrix M is called a P-matrix if

$$\max_{1\leq i\leq n} x_i(Mx)_i > 0 \quad \text{for all} \quad x \neq 0.$$

It is well-known that M is a P-matrix if and only if the LCP(M,q) has a unique solution for any $q \in R^n$ [6]. Recall the following definitions for an $n\times n$ matrix.

M is called an M-matrix, if $M^{-1} \geq 0$ and $M_{ij} \leq 0$ $(i \neq j)$ for $i,j = 1,2,\ldots,n$.

M is called an H-matrix, if its comparison matrix is an M-matrix.

It is known that an H-matrix with positive diagonals is a P-matrix. Moreover, if M is a P-matrix, then there is a neighborhood $\mathcal{M}$ of M, such that all matrices in $\mathcal{M}$ are P-matrices. Hence, we can define a solution function $x(A,b) : \mathcal{M} \times R^n \to R^n_+$, where $x(A,b)$ is the solution of LCP(A,b) and $R^n_+ = \{x \in R^n \mid x \geq 0\}$.

It is easy to verify that x^* solves the LCP(M,q) if and only if x^* solves

$$r(x) := \min(x, Mx+q) = 0,$$

where the min operator denotes the componentwise minimum of two vectors. The function r is called the natural residual of the LCP(M,q), and often used in error analysis. Error bounds for the LCP(M,q) have been studied extensively, see [3, 6, 7, 11, 9, 12, 15].

[1]This work is partly supported by a Grant-in-Aid from Japan Society for the Promotion of Science.

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2 Global error bounds for P-matrix linear complementarity problems

For M being a P-matrix, Mathias and Pang [11] present the following error bound

$$\|x - x^*\|_\infty \le \frac{1 + \|M\|_\infty}{c(M)} \|r(x)\|_\infty, \tag{2.1}$$

for any $x \in R^n$, where

$$c(M) = \min_{\|x\|_\infty = 1} \left\{ \max_{1 \le i \le n} x_i (Mx)_i \right\}.$$

This error bound is well known and widely cited. However, the quantity $c(M)$ in (2.1) is not easy to find. For M being an H-matrix with positive diagonals, Mathias and Pang [11] gave a computable lower bound for $c(M)$,

$$c(M) \ge \frac{(\min_i b_i)(\min_i (\tilde{M}^{-1} b)_i)}{(\max_j (\tilde{M}^{-1} b)_j)^2} =: \tilde{c}(b), \tag{2.2}$$

for any vector $b > 0$, where $\tilde{M}$ is the comparison matrix of M, that is

$$\tilde{M}_{ii} = M_{ii} \qquad \tilde{M}_{ij} = -|M_{ij}| \quad \text{for} \quad i \ne j.$$

However, finding a large value of $\tilde{c}(b)$ is not easy. For some b, $\tilde{c}(b)$ can be very small, and thus the error coefficient

$$\mu(b) := \frac{1 + \|M\|_\infty}{\tilde{c}(b)} \tag{2.3}$$

can be very large.

Interval methods for validation of solution of the LCP(M, q) have been studied in [1, 14]. When a numerical validation condition for the existence of a solution holds, a numerical error bound is provided. However, the numerical validation condition is not ensured to be held at every point x.

In [4], we observed that for every $x, y \in R^n$,

$$\min(x_i, y_i) - \min(x_i^*, y_i^*) = (1 - d_i)(x_i - x_i^*) + d_i(y_i - y_i^*), \qquad i \in N \tag{2.4}$$

where

$$d_i = \begin{cases} 0 & \text{if } y_i \ge x_i,\ y_i^* \ge x_i^* \\ 1 & \text{if } y_i \le x_i,\ y_i^* \le x_i^* \\ \dfrac{\min(x_i, y_i) - \min(x_i^*, y_i^*) + x_i^* - x_i}{y_i - y_i^* + x_i^* - x_i} & \text{otherwise.} \end{cases}$$

Moreover, we have $d_i \in [0,1]$. Hence putting $y = Mx + q$ and $y^* = Mx^* + q$ in (2.4), we obtain

$$r(x) = (I - D + DM)(x - x^*), \tag{2.5}$$

where D is a diagonal matrix whose diagonal elements are $d = (d_1, d_2, \ldots, d_n) \in [0,1]^n$.

It is known that M is a P-matrix if and only if $I - D + DM$ is nonsingular for any diagonal matrix D =diag(d) with $0 \le d_i \le 1$ [10]. This together with (2.5) yields upper and lower error bounds,

$$\frac{\|r(x)\|}{\max\limits_{d\in[0,1]^n} \|I - D + DM\|} \le \|x - x^*\| \le \max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\| \|r(x)\|. \tag{2.6}$$

Moreover, it is not difficult to verify that if M is a P-matrix and D =diag(d) with $d \in [0,1]^n$, we have

$$\max_{1\le i\le n} x_i((I - D + DM)x)_i > 0, \quad \text{for all } x \ne 0,$$

that is, $(I - D + DM)$ is a P-matrix. Therefore, computation of rigorous error bounds can be turned into $\|\cdot\|$ optimization problems over a P-matrix interval set, which is related to linear P-matrix interval systems.

The linear interval system has been studied intensively and some highly efficient numerical methods have been developed, see [13, 14] for references. In the rest part of this section, we give some simple upper bounds for

$$\max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\|.$$

Theorem 2.1 *[4] Suppose that M is an H-matrix with positive diagonals. Then we have*

$$\max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\| \le \|\tilde{M}^{-1} \max(\Lambda, I)\|. \tag{2.7}$$

Remark 1. Since $\tilde{M}^{-1} \max(\Lambda, I) \ge 0$, we have

$$\|\tilde{M}^{-1} \max(\Lambda, I)\|_\infty = \|\tilde{M}^{-1} \max(\Lambda, I)e\|_\infty$$

and

$$\|\tilde{M}^{-1} \max(\Lambda, I)\|_1 = \|(e^T \tilde{M}^{-1} \max(\Lambda, I))^T\|_\infty.$$

The upper error bound in (2.7) with $\|\cdot\|_\infty$ or $\|\cdot\|_1$ can be computed by solving a linear system of equations $\min(\Lambda^{-1}, I)\tilde{M}x = e$ or $\tilde{M}^T \min(\Lambda^{-1}, I)x = e$.

Theorem 2.2 *[4] Suppose that M is an M-matrix. Let $V = \{v \mid M^T v \leq e, v \geq 0\}$ and $f(v) = \max\limits_{1 \leq i \leq n} (e + v - M^T v)_i$. Then we have*

$$\max_{d \in [0,1]^n} \|(I - D + DM)^{-1}\|_1 = \max_{v \in V} f(v). \tag{2.8}$$

Theorem 2.3 *[4] If M is a P-matrix, then for any $x \in R^n$, the following inequalities hold.*

$$\begin{aligned}
&\frac{1}{1 + \|M\|_\infty} \|r(x)\|_\infty \qquad (\textit{Mathias-Pang [11]}) \\
&\leq \frac{1}{\max(1, \|M\|_\infty)} \|r(x)\|_\infty \qquad (\textit{Cottle-Pang-Stone [6]}) \\
&= \frac{1}{\max\limits_{d \in [0,1]^n} \|I - D + DM\|_\infty} \|r(x)\|_\infty \\
&\leq \|x - x^*\|_\infty \\
&\leq \max_{d \in [0,1]^n} \|(I - D + DM)^{-1}\|_\infty \|r(x)\|_\infty \\
&\leq \frac{\max(1, \|M\|_\infty)}{c(M)} \|r(x)\|_\infty \\
&= \frac{1 + \|M\|_\infty}{c(M)} \|r(x)\|_\infty - \frac{\min(1, \|M\|_\infty)}{c(M)} \|r(x)\|_\infty \\
&\leq \frac{1 + \|M\|_\infty}{c(M)} \|r(x)\|_\infty \qquad (\textit{Mathias-Pang [11]})).
\end{aligned}$$

Theorem 2.4 *[4] If M is an H-matrix with positive diagonals, then for any $x, b \in R^n$, $b > 0$, the following inequalities hold.*

$$\begin{aligned}
&\|x - x^*\|_\infty \\
&\leq \max_{d \in [0,1]^n} \|(I - D + DM)^{-1}\|_\infty \|r(x)\|_\infty \\
&\leq \|\tilde{M}^{-1} \max(\Lambda, I)\|_\infty \|r(x)\|_\infty \\
&\leq (\mu(b) - \|\tilde{M}^{-1} \min(\Lambda, I)\|_\infty) \|r(x)\|_\infty \\
&\leq \mu(b) \|r(x)\|_\infty \qquad (\textit{Mathias-Pang [11]}).
\end{aligned}$$

In addition, if M is an M-matrix, then for any $x \in R^n$, the following inequalities hold.

$$\begin{aligned}
&\|x - x^*\|_\infty \\
&\leq \|M^{-1} \max(\Lambda, I)\|_\infty \|r(x)\|_\infty \\
&\leq \left(\frac{1 + \|M\|_\infty}{c(M)} - \|M^{-1} \min(\Lambda, I)\|\right) \|r(x)\|_\infty \\
&\leq \frac{1 + \|M\|_\infty}{c(M)} \|r(x)\|_\infty \qquad (\textit{Mathias-Pang [11]}).
\end{aligned}$$

Applying Theorem 2.1, we obtain the following relative error bounds

Corollary 2.1 *[4] Suppose M is an H-matrix with positive diagonals. For any $x \in R^n$, we have*

$$\frac{\|r(x)\|}{(1+\|M\|)\|\tilde{M}^{-1}\max(\Lambda, I)\|\|(-q)_+\|} \leq \frac{\|x-x^*\|}{\|x^*\|} \leq \frac{\|M\|\|\tilde{M}^{-1}\max(\Lambda, I)\|\|r(x)\|}{\|(-q)_+\|}.$$

3 Perturbation bounds of P-matrix linear complementarity problems

In [6], Cottle, Pang and Stone introduced the following Lemma which has been widely applied in perturbation bounds based on the fundamental quantity associated with a P-matrix,

$$c(M) = \min_{\|x\|_\infty=1} \max_{1\leq i\leq n} \{x_i(Mx)_i\}.$$

Lemma 3.1 *[6] Let $M \in R^{n\times n}$ be a P-matrix. The following statements hold:*

(i) for any two vectors q and p in R^n,

$$\|x(M,q) - x(M,p)\|_\infty \leq \frac{1}{c(M)}\|q-p\|_\infty$$

(ii) for each vector $q \in R^n$, there exist a neighborhood $\mathcal{U}$ of the pair (M,q) and a constant $c_0 > 0$ such that for any $(A,b),(B,p) \in \mathcal{U}$, A, B are P-matrices and

$$\|x(A,b) - x(B,p)\|_\infty \leq c_0(\|A-B\|_\infty + \|b-p\|_\infty).$$

Lemma 3.1 shows that when M is a P-matrix, for each q, $x(A,b)$ is a locally Lipschitzian function of (A,b) in a neighborhood of (M,q), and $x(M,b)$ is a globally Lipschitzian function of b. This property plays a very important rule in the study of the LCP and mathematical programs with LCP constraints [8]. However, the constant $c(M)$ is difficult to compute, and c_0 is not specified. It is hard to use this lemma for verifying accuracy of a computed solution of the LCP when the data (M,q) contain errors.

For M being a P-matrix, we [5] introduce the following constant

$$\beta(M) = \max_{d\in[0,1]^n} \|(I-D+DM)^{-1}D\|.$$

In the follows, we compare $\beta(M)$ with $c(M)^{-1}$ in $\|\cdot\|_\infty$ and give a simple version of $\beta(M)$ for M being an M-matrix, a symmetric positive definite matrix, and positive definite matrix.

Theorem 3.1 *[5] Let M be a P-matrix. Then*

$$\beta_\infty(M) := \max_{d\in[0,1]^n} \|(I-D+DM)^{-1}D\|_\infty \le \frac{1}{c(M)}.$$

It is known that an H-matrix with positive diagonals is a P-matrix, and a positive definite matrix is a P-matrix [6]. Now, we consider the two subclasses of P-matrix.

Theorem 3.2 *[5] Let M be an H-matrix with positive diagonals. Then*

$$\beta(M) \le \|\tilde{M}^{-1}\|,$$

where $\tilde{M}$ is the comparison matrix of M. In particular, if M is an M-matrix, then the equality holds.

Theorem 3.3 *[5] Let M be a symmetric positive definite matrix. Then*

$$\beta_2(M) := \max_{d\in[0,1]^n} \|(I-D+DM)^{-1}D\|_2 = \|M^{-1}\|_2.$$

In comparison to Lemma 3.1, the following theorem gives sharp perturbation error estimates for the P-matrix LCP

Theorem 3.4 *[5] Let $M \in R^{n\times n}$ be a P-matrix. Then the following statements hold:*

(i) For any two vectors q and p in R^n,

$$\|x(M,q)-x(M,p)\| \le \beta(M)\|q-p\|.$$

(ii) Every matrix $A \in \mathcal{M} := \{A \mid \beta(M)\|M-A\| \le \eta < 1\}$ is a P-matrix. Let

$$\alpha(M) = \frac{1}{1-\eta}\beta(M).$$

Then for any $A, B \in \mathcal{M}$ and $q, p \in R^n$

$$\|x(A,q)-x(B,p)\| \le \alpha(M)^2\|(-p)_+\|\|A-B\| + \alpha(M)\|q-p\|.$$

From Theorem 3.2 and Theorem 3.3, the Lipschitz constants $\beta(M)$ and $\alpha(M)$ can be estimated by matrix norms, if M is an H-matrix with positive diagonals or a symmetric positive definite matrix. In particular, we have the following two corollaries.

Corollary 3.1 *[5] Let $M \in R^{n\times n}$ be an H-matrix with positive diagonals. Then the following statements hold:*

(i) For any two vectors q and p in R^n,

$$\|x(M,q) - x(M,p)\|_\infty \le \|\tilde{M}^{-1}\|_\infty \|q - p\|_\infty$$

(ii) Every matrix $A \in \mathcal{M}_\infty := \{A \mid \|\tilde{M}^{-1}\|_\infty \|M - A\|_\infty \le \eta < 1\}$ is an H-matrix with positive diagonals. Let

$$\alpha_\infty(M) = \frac{1}{1-\eta}\|\tilde{M}^{-1}\|_\infty.$$

Then for any $A, B \in \mathcal{M}_\infty$ and $q, p \in R^n$

$$\|x(A,q) - x(B,p)\|_\infty \le \alpha_\infty(M)^2 \|(-p)_+\|_\infty \|A - B\|_\infty + \alpha_\infty(M)\|q - p\|_\infty.$$

Corollary 3.2 *[5] Let $M \in R^{n\times n}$ be a symmetric positive definite matrix. Then the following statements hold:*

(i) For any two vectors q and p in R^n,

$$\|x(M,q) - x(M,p)\|_2 \le \|M^{-1}\|_2 \|q - p\|_2$$

(ii) Every matrix $A \in \mathcal{M}_2 := \{A \mid \|M^{-1}\|_2 \|M - A\|_2 \le \eta < 1\}$ is a P-matrix. Let

$$\alpha_2(M) = \frac{1}{1-\eta}\|M^{-1}\|_2.$$

Then for any $A, B \in \mathcal{M}_2$ and $q, p \in R^n$

$$\|x(A,q) - x(B,p)\|_2 \le \alpha_2(M)^2 \|(-p)_+\|_2 \|A - B\|_2 + \alpha_2(M)\|q - p\|_2.$$

A matrix A is called positive definite if

$$x^T A x > 0, \qquad 0 \ne x \in R^n.$$

Since $x^T A x = x^T \frac{A + A^T}{2} x$, A is positive definite if and only if $\frac{A + A^T}{2}$ is symmetric positive definite. Note that a positive definite matrix is not necessarily symmetric. Such asymmetric matrices frequently appear in the context of the LCP.

Combining the ideas of Mathias and Pang [11] and Corollary 3.2, we present perturbation bounds for the positive definite matrix LCP.

Theorem 3.5 *[5] Let $M \in R^{n\times n}$ be a positive definite matrix. Then the following statements hold:*

(i) For any two vectors q and p in R^n,

$$\|x(M,q)-x(M,p)\|_2 \le \|(\frac{M+M^T}{2})^{-1}\|_2\|q-p\|_2.$$

(ii) Every matrix $A \in \mathcal{M}_2 := \{A \mid \|(\frac{M+M^T}{2})^{-1}\|_2\|M-A\|_2 \le \eta < 1\}$ is positive definite. Let

$$\alpha_2(M) = \frac{1}{1-\eta}\|(\frac{M+M^T}{2})^{-1}\|_2.$$

Then for any $A, B \in \mathcal{M}_2$ and $q, p \in R^n$

$$\|x(A,q)-x(B,p)\|_2 \le \alpha_2(M)^2\|(-p)_+\|_2\|A-B\|_2 + \alpha_2(M)\|q-p\|_2.$$

Example 3.1 Theorem 3.1 shows that for every P-matrix, $\beta_\infty(M) \le c(M)^{-1}$. Now we show that $\beta_\infty(M)$ can be much smaller than $c(M)^{-1}$ in some case. Consider

$$M = \begin{pmatrix} 1 & -t \\ 0 & t \end{pmatrix}.$$

For $t \ge 1$, M is an M-matrix. By Theorem 3.2, $\beta_\infty(M) = \|M^{-1}\|_\infty = 2$. For $\bar{x} = (1, t^{-1})$, we have

$$c(M) \le \max_{i\in N} \bar{x}_i(M\bar{x})_i = \frac{1}{t}.$$

Hence, $c(M)^{-1} \ge t \to \infty$, as $t \to \infty$.

Using the results in the last section, we derive relative perturbation bounds expressed in the term of $\beta(M)\|M\|$.

For the system of linear equations, A is nonsingular if and only if $Ax = b$ has a unique solution for any vector b. A system of linear equations is considered to be well-conditioned (ill-conditioned) if small changes in A or b result in small (large) changes in the solution x. The condition number of A is a measure of sensitivity of the solution of $Ax = b$ for A being a nonsingular matrix. For the linear complementarity problem, M is a P-matrix if and only if LCP(M, q) has a unique solution for any vector q. A linear complementarity problem is considered to be well-conditioned (ill-conditioned) if small changes in M or q result in small (large) changes in the solution x. Based on the preceding analysis, we are able to give a perturbation theorem for the P-matrix LCP, and define a measure of sensitivity of the solution of LCP(M, q) for M being a P-matrix.

Theorem 3.6 *[5] Suppose*

$$\begin{aligned} \min(x, Mx+q) &= 0 \qquad M \in R^{n\times n},\ \ 0 \neq (-q)_+ \in R^n \\ \min(y, (M+\Delta M)y + q + \Delta q) &= 0 \qquad \Delta M \in R^{n\times n},\ \ \Delta q \in R^n. \end{aligned}$$

with

$$\|\Delta M\| \le \epsilon\|M\|, \qquad \|\Delta q\| \le \epsilon \max(\|(-q)_+\|, \|q\| - \|Mx+q\|).$$

If M *is a P-matrix and* $\epsilon\beta(M)\|M\| = \eta < 1$, *then* $M + \Delta M$ *is a P-matrix and*

$$\frac{\|y-x\|}{\|x\|} \le \frac{2\epsilon}{1-\eta}\beta(M)\|M\|.$$

Theorem 3.6 indicates that $\beta(M)\|M\|$ is a measure of sensitivity of the solution of the LCP(M,q) for M being a P-matrix. Application of Theorem 3.6 with Corollary 3.1, Corollary 3.2 and Theorem 3.5 gives $\beta(M)\|M\|$ in the term of condition number for the H-matrix LCP, symmetric positive definite LCP and positive definite LCP.

Corollary 3.3 *[5] Suppose*

$$\begin{aligned} \min(x, Mx+q) &= 0 \qquad M \in R^{n\times n},\ \ 0 \neq (-q)_+ \in R^n \\ \min(y, (M+\Delta M)y + q + \Delta q) &= 0 \qquad \Delta M \in R^{n\times n},\ \ \Delta q \in R^n. \end{aligned}$$

(i) If M *is an H-matrix with positive diagonals,* $\epsilon\kappa_\infty(\tilde{M}) = \eta < 1$, *and*

$$\|\Delta M\|_\infty \le \epsilon\|\tilde{M}\|_\infty, \qquad \|\Delta q\|_\infty \le \epsilon \max(\|(-q)_+\|_\infty, \|q\|_\infty - \|Mx+q\|_\infty)$$

then $M + \Delta M$ *is an H-matrix with positive diagonals and*

$$\frac{\|y-x\|_\infty}{\|x\|_\infty} \le \frac{2\epsilon}{1-\eta}\kappa_\infty(\tilde{M}).$$

(ii) If M *is a symmetric positive definite matrix,* $\epsilon\kappa_2(M) = \eta < 1$, *and*

$$\|\Delta M\|_2 \le \epsilon\|M\|_2, \qquad \|\Delta q\|_2 \le \epsilon \max(\|(-q)_+\|_2, \|q\|_2 - \|Mx+q\|_2),$$

then $M + \Delta M$ *is a P-matrix and*

$$\frac{\|y-x\|_2}{\|x\|_2} \le \frac{2\epsilon}{1-\eta}\kappa_2(M).$$

(iii) If M *is a positive definite matrix,* $\epsilon\kappa_2(\frac{M+M^T}{2}) = \eta < 1$, *and*

$$\|\Delta M\|_2 \le \epsilon\|\frac{M+M^T}{2}\|_2, \qquad \|\Delta q\|_2 \le \epsilon \max(\|(-q)_+\|_2, \|q\|_2 - \|Mx+q\|_2)\frac{\|M+M^T\|_2}{2\|M\|_2},$$

then $M + \Delta M$ *is a positive matrix, and*

$$\frac{\|x-y\|_2}{\|x\|_2} \le \frac{2\epsilon}{1-\eta}\kappa_2(\frac{M+M^T}{2}).$$

Remark 3.1. If $Mx+q=0$, then (i) of Corollary 3.3 for M being an M-matrix and (ii) of Corollary 3.3 reduce to the perturbation bounds for the system of linear equations.

For the H-matrix LCP, componentwise perturbation bounds based on the Skeel condition number $\| |\tilde{M}^{-1}||\tilde{M}| \|_\infty$ can be represented as follows.

Theorem 3.7 *[5] Suppose*

$$\begin{aligned} \min(x, Mx+q) &= 0 \qquad M \in R^{n\times n},\ \ 0 \neq (-q)_+ \in R^n \\ \min(y, (M+\triangle M)y + q + \triangle q) &= 0 \qquad \triangle M \in R^{n\times n},\ \ \triangle q \in R^n. \end{aligned}$$

with

$$|\triangle M| \leq \epsilon |M|, \qquad |\triangle q| \leq \epsilon \max((-q)_+, |q| - |Mx+q|). \tag{3.1}$$

If M is an H-matrix with positive diagonals and $\epsilon\kappa_\infty(\tilde{M}) = \eta < 1$, then $M+\triangle M$ is an H-matrix with positive diagonals and

$$\frac{\|y-x\|_\infty}{\|x\|_\infty} \leq \frac{2\epsilon}{1-\eta} \| |\tilde{M}^{-1}||\tilde{M}| \|_\infty. \tag{3.2}$$

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