数理解析研究所講究録
1480 巻 2006 年 49-58

UFSCARF-TH-05-22

Integrable Vertex Models with General Twists

M.J. Martins

Universidade Federal de São Carlos
Departamento de Física
C.P. 676, 13565-905 São Carlos(SP), Brasil

Abstract

We review recent progress towards the solution of exactly solved isotropic vertex models with arbitrary toroidal boundary conditions. Quantum space transformations make it possible the diagonalization of the corresponding transfer matrices by means of the quantum inverse scattering method. Explicit expressions for the eigenvalues and Bethe ansatz equations of the twisted isotropic spin chains based on the B_n, D_n and C_n Lie algebras are presented. The applicability of this approach to the eight vertex model with non-diagonal twists is also discussed.

Solvable Lattice Models 2004, Kyoto

March 2005

A vertex model is a classical statistical system defined on a lattice whose geometry is given by a possibly infinite set of straight lines on the plane [1]. The intersections of these lines are called vertices or sites and here we consider the situation where not more than two lines meet at every vertex. Clearly, the square lattice of size $L \times L$ is the simplest one and from now on we shall restrict ourselves to it. A physical state is then defined by assigning to each lattice edge a discrete variable having one out of q possible values.

Next, we suppose that the corresponding row-to-row transfer matrix can be constructed from elementary local i-th site Boltzmann weights $\mathcal{L}_{\mathcal{A}i}(\lambda)$ where λ denotes a spectral parameter. This operator is best viewed as a $q \times q$ matrix on the auxiliary space $\mathcal{A} = \mathcal{C}^{\mathrm{u}}$ whose elements are operators acting on the $\prod_{i=1}^{L} \otimes \mathcal{C}_i^q$ Hilbert space. Considering toroidal boundary conditions on the square lattice, the transfer matrix $T(\lambda)$ can be written in terms of the trace over $\mathcal{A}$ of the following ordered product of operator [2, 3, 4]

$$T(\lambda) = \mathrm{Tr}_{\mathcal{A}}\left[\mathcal{G}_{\mathcal{A}}\mathcal{L}_{\mathcal{A}L}(\lambda)\mathcal{L}_{\mathcal{A}L-1}(\lambda)\dots\mathcal{L}_{\mathcal{A}1}(\lambda)\right] \tag{1}$$

where $\mathcal{G}_{\mathcal{A}}$ are $q \times q$ c-number matrices, representing generalized periodic boundary conditions.

A sufficient condition for integrability, i.e $[T(\lambda), T(\mu)] = 0$ for arbitrary values of λ and μ, is the existence of an invertible matrix $\check{R}(\lambda, \mu)$ satisfying the property [2, 3]

$$\check{R}(\lambda,\mu)\mathcal{L}_{\mathcal{A}i}(\lambda) \otimes \mathcal{L}_{\mathcal{A}i}(\mu) = \mathcal{L}_{\mathcal{A}i}(\mu) \otimes \mathcal{L}_{\mathcal{A}i}(\lambda)\check{R}(\lambda,\mu) \tag{2}$$

and that the matrix $\mathcal{G}_{\mathcal{A}}$ is a possible representation, without spectral parameter dependence, of the quadratic algebra (2), namely [4]

$$\left[\check{R}(\lambda,\mu), \mathcal{G}_{\mathcal{A}} \otimes \mathcal{G}_{\mathcal{A}}\right] = 0, \tag{3}$$

As usual the R-matrix $\check{R}(\lambda, \mu)$ is required to satisfy the famous Yang-Baxter equation

$$\check{R}_{23}(\lambda_1,\lambda_2)\check{R}_{12}(\lambda_1,\lambda_3)\check{R}_{23}(\lambda_2,\lambda_3) = \check{R}_{12}(\lambda_2,\lambda_3)\check{R}_{23}(\lambda_1,\lambda_3)\check{R}_{12}(\lambda_1,\lambda_2), \tag{4}$$

In this paper we will consider integrable models whose corresponding R-matrices are additive with respect the spectral parameters, $\check{R}(\lambda, \mu) = \check{R}(\lambda - \mu)$. In this case, the simplest spectral parameter dependent representation of the Yang-Baxter algebra (2) turns out to be

$$\mathcal{L}_{\mathcal{A}i}(\lambda) = P_{\mathcal{A}i}\check{R}(\lambda), \tag{5}$$

where $P_{\mathcal{A}i}$ is the exchange operator on the space $\mathcal{A} \otimes \mathcal{C}_i^q$.

If the matrix $\mathcal{G}_{\mathcal{A}}$ is non-singular it is possible to derive an integrable quantum spin chain from $T(\lambda)$. Assuming that the operator $\mathcal{L}_{\mathcal{A}i}(\lambda)$ is proportional to the permutator $P_{\mathcal{A}i}$, say at certain special point $\lambda = 0$, the corresponding one-dimensional Hamiltonian reads [5],

$$\mathcal{H} = \sum_{i=1}^{L-1} P_{\mathcal{A}i} \frac{d\mathcal{L}_{\mathcal{A}i}(\lambda)}{d\lambda} |_{\lambda=0} + \mathcal{G}_L^{-1} P_{L1} \frac{d\mathcal{L}_{L1}(\lambda)}{d\lambda} |_{\lambda=0} \mathcal{G}_L \tag{6}$$

When the boundary matrix $\mathcal{G}_{\mathcal{A}}$ is non-diagonal, the diagonalization of either the transfer matrix (1) or the Hamiltonian (6) is indeed a highly non-trivial problem in the field of integrable models. The main difficulty is concerned with the apparent lack of simple references states to start the Bethe ansatz analysis. Here we would like to present the steps towards the direction of solving integrable isotropic vertex models with non-diagonal toroidal boundary conditions. As concrete examples we will consider those systems whose rational R-matrices are invariant by the B_n, D_n and C_n symmetries. One way to construct rational solutions of the Yang-Baxter equation (4) is by means of the braid-monoid algebra [6] at its degenerated point [7]. This algebra is generated by the identity I_i, by a braid b_i and a Temperley Lieb operator E_i acting on sites i of a chain of length L. On the degenerate point the braid operator becomes a generator of the symmetric group, namely

$$b_i = \sum_{a,b=1}^{q} \hat{e}_{ab}^{(i)} \otimes \hat{e}_{ba}^{(i+1)} \tag{7}$$

where $\hat{e}_{ab}^{(i)}$ are the $q \times q$ Weyl matrices acting on the space $\mathcal{C}_i^q$.

The monoid turns out to be represented by the following expression [8]

$$E_i = \sum_{a,b,c,d=1}^{q} \alpha_{ab}\alpha_{cd}^{-1} \hat{e}_{ac}^{(i)} \otimes \hat{e}_{bd}^{(i+1)} \tag{8}$$

where the matrix α for the models B_n, D_n and C_n are given by

$$\alpha_{B_n} = \mathcal{I}_{2n+1 \times 2n+1}, \quad \alpha_{D_n} = \mathcal{I}_{2n \times 2n}, \quad \alpha_{C_n} = \begin{pmatrix} \mathcal{O}_{n \times n} & \mathcal{I}_{n \times n} \\ -\mathcal{I}_{n \times n} & \mathcal{O}_{n \times n} \end{pmatrix} \tag{9}$$

such that $\mathcal{I}_{k\times k}$ is defined as a $k \times k$ anti-diagonal matrix.

The set of algebraic relations satisfied by the braid and monoid at its generated point can be "Baxterized" in terms of rational functions. More specifically, the solution $\check{R}(\lambda)$ in terms of combinations of the identity, braid and monoid is given by [7, 8]

$$\check{R}_{i,i+1}(\lambda) = I_i + \lambda b_i - \frac{\lambda}{\lambda - \delta} E_i \tag{10}$$

where the values of parameter δ are

$$\delta_{B_n} = -n + \frac{1}{2} \quad \delta_{C_n} = -n - 1 \quad \delta_{D_n} = -n + 1 \tag{11}$$

Let us now turn our attention to the diagonalization of the transfer matrix (1) for the above B_n, D_n and C_n vertex models, [1] considering the most general admissible boundary matrix satisfying the condition (3), i.e $[E_i, \mathcal{G}_{\mathcal{A}} \otimes \mathcal{G}_{\mathcal{A}}] = 0$. Denoting by $M_{\mathcal{A}}$ the matrix that diagonalize the boundary matrix $\mathcal{G}_{\mathcal{A}}$ and by inserting the terms $M_{\mathcal{A}} M_{\mathcal{A}}^{-1}$ all over the trace (1) one we can write $T(\lambda)$ as

$$\begin{aligned} T(\lambda) &= \mathrm{Tr}_{\mathcal{A}}\left[M_{\mathcal{A}} D_{\mathcal{A}} M_{\mathcal{A}}^{-1} M_{\mathcal{A}} \left(M_{\mathcal{A}}^{-1} \mathcal{L}_{\mathcal{A}L}(\lambda) M_{\mathcal{A}}\right) \dots \left(M_{\mathcal{A}}^{-1} \mathcal{L}_{\mathcal{A}1}(\lambda) M_{\mathcal{A}}\right) M_{\mathcal{A}}^{-1}\right], & (12)\\ &= \mathrm{Tr}_{\mathcal{A}}\left[D_{\mathcal{A}} \widetilde{\mathcal{L}}_{\mathcal{A}L}(\lambda) \widetilde{\mathcal{L}}_{\mathcal{A}L-1}(\lambda) \dots \widetilde{\mathcal{L}}_{\mathcal{A}1}(\lambda)\right], & (13)\end{aligned}$$

where $D_{\mathcal{A}}$ is diagonal matrix whose entries are the eigenvalues of $\mathcal{G}_{\mathcal{A}}$ and the $\widetilde{\mathcal{L}}$-operators are given by

$$\widetilde{\mathcal{L}}_{\mathcal{A}i}(\lambda) = M_{\mathcal{A}}^{-1} \mathcal{L}_{\mathcal{A}i}(\lambda) M_{\mathcal{A}}. \tag{14}$$

The next step in our approach is to note that it is always possible to choose an invertible transformation U_i on the space C_i^q such that

$$U_i^{-1} \widetilde{\mathcal{L}}_{\mathcal{A}i}(\lambda) U_i = \mathcal{L}_{\mathcal{A}i}(\lambda) \tag{15}$$

and therefore to undo the modifications on the Lax operators (14) by means of quantum space transformations.

[1]We recall that similar problem for the A_n model has been tackled in ref.[9].

Now one can take advantage of this remarkable property by defining a new transfer matrix $T'(\lambda)$

$$T'(\lambda) = \prod_{j=1}^{L} \otimes U_j^{-1} T(\lambda) \prod_{j=1}^{L} \otimes U_j = \mathrm{Tr}_{\mathcal{A}} \left[D_{\mathcal{A}} \mathcal{L}_{\mathcal{A}L}(\lambda) \dots \mathcal{L}_{\mathcal{A}1}(\lambda) \right], \tag{16}$$

which is precisely the transfer matrix of the vertex model we have started with diagonal twist $D_{\mathcal{A}}$.

Because the boundary $D_{\mathcal{A}}$ is diagonal the transfer matrix $T'(\lambda)$ can be diagonalized with very little difference from the standard periodic case [8]. Furthermore, the operators $T(\lambda)$ and $T'(\lambda)$ share the same eigenvalues and if $|\psi'\rangle$ is an eigenstate of $T'(\lambda)$ the corresponding eigenvector $|\psi\rangle$ of $T(\lambda)$ is then $\prod_{j=1}^{L} \otimes U_j \, |\psi'\rangle$. Considering that the algebraic framework to diagonalize $T'(\lambda)$ has already been described in ref.[8], there is no need to repeat it here, and in what follows we shall present only the final results concerning the Bethe ansatz equations and eigenvalues of the related Hamiltonian (6). The expression for the latter can be written in a compact form in terms of the underlying Cartan matrix C_{ab} and the normalized length η_a of the roots. To each a-ath root we associate a set of rapidities $\lambda_j^{(a)}$ that satisfy the following non-linear coupled equations,

$$\left[\frac{\lambda_j^{(a)} - \frac{\delta_{a,1}}{2\eta_a}}{\lambda_j^{(a)} + \frac{\delta_{a,1}}{2\eta_a}} \right]^L = \frac{g_a}{g_{a+1}} \prod_{b=1}^{n} \prod_{k=1,\, k\neq j}^{m_b} \frac{\lambda_j^{(a)} - \lambda_k^{(b)} - \frac{C_{a,b}}{2\eta_a}}{\lambda_j^{(a)} - \lambda_k^{(b)} + \frac{C_{a,b}}{2\eta_a}}, \quad j = 1, \dots, m_a; \quad a = 1, \dots, n \tag{17}$$

where g_a is the a-th eigenvalue of the matrix $D_{\mathcal{A}}$.

Before proceeding it should be remarked that due to the constraint $[E_i, \mathcal{G}_{\mathcal{A}} \otimes \mathcal{G}_{\mathcal{A}}] = 0$ not all the eigenvalues g_a are independent. It turns out that only the first n ratios $\frac{g_a}{g_{a+1}}$ are indeed arbitrary. The eigenvalues $E(L)$ of the Hamiltonian (6) are parameterized by the variables $\lambda_j^{(1)}$ by

$$E(L) = -\sum_{i=1}^{m_1} \frac{1}{[\lambda_i^{(1)}]^2 - 1/4} + L \tag{18}$$

We expect that these results extend to all isotropic integrable vertex models invariant by the discrete representations of Lie algebras as well as to superalgebras. Note that such systems possess a broader class of possible non-diagonal boundary matrices as compared with their trigonometric counterparts . This means that isotropic vertex models with the most general twisted boundary conditions are in fact genuine systems that deserve to be studied independently. They are also of potential physical interest, since suitable combinations between non-diagonal boundaries $\mathcal{G}_{\mathcal{A}}$ and rational $\mathcal{L}$-operators can described interesting solvable atom-fields models [10].

A natural question to be asked is whether or not this approach can also be of utility for non-rational vertex models. A tantalizing problem would be the solution of the eight-vertex model in the presence of non-diagonal toroidal boundary conditions. The symmetrical eigth-vertex model [1] possesses four different Boltzmann weights $a(\lambda)$, $b(\lambda)$, $c(\lambda)$ and $d(\lambda)$ whose local operators $\mathcal{L}_{\mathcal{A}i}(\lambda)$ which are given by the following 2×2 matrix

$$\mathcal{L}_{\mathcal{A}i}(\lambda) = \begin{pmatrix} a(\lambda)\sigma_i^+\sigma_i^- + b(\lambda)\sigma_i^-\sigma_i^+ & d(\lambda)\sigma_i^+ + c(\lambda)\sigma_i^- \\ c(\lambda)\sigma_i^+ + d(\lambda)\sigma_i^- & b(\lambda)\sigma_i^+\sigma_i^- + a(\lambda)\sigma_i^-\sigma_i^+ \end{pmatrix} \tag{19}$$

and $\sigma_i^{\pm}$ are Pauli matrices acting on the i-th sites of an one-dimensional lattice of size L.

This vertex model is known to be solvable in the manifold

$$2\Delta = \frac{a^2(\lambda) + b^2(\lambda) - c^2(\lambda) - d^2(\lambda)}{a(\lambda)b(\lambda) + c(\lambda)d(\lambda)} \quad \Gamma = \frac{a(\lambda)b(\lambda) - c(\lambda)d(\lambda)}{a(\lambda)b(\lambda) + c(\lambda)d(\lambda)} \tag{20}$$

where Δ and Γ are arbitrary constants. A possible non-diagonal twist compatible with integrability is given by

$$\mathcal{G}_{\mathcal{A}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{21}$$

One can now follow the same steps discussed above. Though we could not undo completely the modifications carried out on the auxiliary space due to the manipulations (13) we find out that the following quantum space transformation

$$\bar{\mathcal{L}}_{\mathcal{A}i}(\lambda) = U_i^{-1}\widetilde{\mathcal{L}}_{\mathcal{A}i}(\lambda)U_i \tag{22}$$

where the matrix U_i is given by

$$U_i = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \tag{23}$$

This transformation leads us to an operator $\bar{\mathcal{L}}_{\mathcal{A}i}(\lambda)$ that preserves the eight vertex form (19),

$$\bar{\mathcal{L}}_{\mathcal{A}i}(\lambda) = \begin{pmatrix} \bar{a}(\lambda)\sigma_i^+\sigma_i^- + \bar{b}(\lambda)\sigma_i^-\sigma_i^+ & \bar{d}(\lambda)\sigma_i^+ + \bar{c}(\lambda)\sigma_i^- \\ \bar{c}(\lambda)\sigma_i^+ + \bar{d}(\lambda)\sigma_i^- & \bar{b}(\lambda)\sigma_i^+\sigma_i^- + \bar{a}(\lambda)\sigma_i^-\sigma_i^+ \end{pmatrix} \tag{24}$$

where the new Boltzmann weights $\bar{a}(\lambda)$, $\bar{b}(\lambda)$, $\bar{c}(\lambda)$ and $\bar{d}(\lambda)$ are given by

$$\bar{a}(\lambda) = \frac{a(\lambda) + b(\lambda) + c(\lambda) + d(\lambda)}{2} \tag{25}$$

$$\bar{b}(\lambda) = \frac{a(\lambda) + b(\lambda) - c(\lambda) - d(\lambda)}{2} \tag{26}$$

$$\bar{c}(\lambda) = \frac{a(\lambda) - b(\lambda) + c(\lambda) - d(\lambda)}{2} \tag{27}$$

$$\bar{d}(\lambda) = \frac{a(\lambda) - b(\lambda) - c(\lambda) + d(\lambda)}{2} \tag{28}$$

whose invariants are $\bar{\Delta} = \frac{1}{\Delta}$ and $\bar{\Gamma} = \frac{\Gamma}{\Delta}$ [2].

As a consequence of that our remaining task now consists in diagonalizing the following transfer matrix,

$$\bar{T}(\lambda) = \mathrm{Tr}_{\mathcal{A}}\left[\mathcal{D}_{\mathcal{A}}\bar{\mathcal{L}}_{\mathcal{A}L}(\lambda)\bar{\mathcal{L}}_{\mathcal{A}L-1}(\lambda)\dots\bar{\mathcal{L}}_{\mathcal{A}1}(\lambda)\right] \tag{29}$$

where the diagonal boundary matrix is

$$\mathcal{D}_{\mathcal{A}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{30}$$

By construction $\bar{T}(\lambda)$ (29) and $T(\lambda)$ given by Eqs.(1,19,21) share the same eigenvalues while the eigenvectors are related by the similarity transformation (23). Though this procedure brings some simplification in the eigenvalue problem, it is not enough to make the diagonalization of the transfer matrix $\bar{T}(\lambda)$ amenable by Bethe ansatz analysis. This is because the operator $\bar{\mathcal{L}}_{\mathcal{A}i}(\lambda)$ has no simple local pseudovacuum that annihilate one of its off-diagonal matrix elements for arbitrary values of the spectral parameter. The standard way of solving this problem is by means of the so-called Baxter's gauge transformations [1, 2] which unfortunately does not work

[2]This then reemphasize why the isotropic limit $\Delta = 1$ is special under both auxiliary and quantum transformations.

here since the diagonal boundary $\mathcal{D}_{\mathcal{A}}$ is not an identity matrix. In other words, the problem of finding gauge transformations $M_i(\lambda)$ with the conditions that both the transformed operator $M_{i+1}^{-1}(\lambda)\bar{\mathcal{L}}_{\mathcal{A}i}(\lambda)M_i(\lambda)$ has a local vacuum independent of λ and that does not spoil the diagonal property of $\mathcal{D}_{\mathcal{A}}$ has eluded us so far. An advantage of this approach, however, is that we can easily identify the existence of at least one case in each the eigenvalue problem for $\bar{T}(\lambda)$ (29) can be solved by standard algebraic Bethe ansatz. This clearly occurs when the Boltzmann weight $\bar{d}(\lambda)$ (28) is null. Direct inspection reveals us that this happens at the point in which the modulus $\mathcal{K}$ of the elliptic functions parameterizing the eight vertex Boltzmann weights becomes unity. More specifically, at the value $\mathcal{K}=1$ the weights $a(\lambda)$, $b(\lambda)$, $c(\lambda)$ and $d(\lambda)$ are given by the following expressions

$$a(\lambda)=\tanh[\lambda+\gamma] \quad b(\lambda)=\tanh[\lambda] \tag{31}$$

$$c(\lambda)=\tanh[\gamma] \quad d(\lambda)=\tanh[\gamma]\tanh[\lambda]\tanh[\lambda+\gamma] \tag{32}$$

which due to (28) implies $\bar{d}(\lambda)=0$.

Thanks to the above simplification it now remains only the diagonalization of a symmetric six vertex model, whose solution has appeared in many different contexts in the literature. The result for the eigenvalue $\Lambda(\lambda)$ of $T(\lambda)$ is therefore

$$\Lambda(\lambda)=[\bar{a}(\lambda)]^L\prod_{i=1}^{m}\frac{\bar{a}(\lambda_j-\lambda)}{\bar{b}(\lambda_j-\lambda)}-[\bar{b}(\lambda)]^L\prod_{i=1}^{m}\frac{\bar{a}(\lambda-\lambda_j)}{\bar{b}(\lambda-\lambda_j)} \tag{33}$$

where the weights $\bar{a}(\lambda)$, $\bar{b}(\lambda)$ and $\bar{c}(\lambda)$ are

$$\bar{a}(\lambda)=\frac{\sinh[\lambda+\gamma]}{\cosh[\lambda]\cosh[\gamma]} \quad \bar{b}(\lambda)=\frac{\sinh[\lambda]}{\cosh[\lambda+\gamma]\cosh[\gamma]} \quad \bar{c}(\lambda)=\frac{\sinh[\gamma]}{\cosh[\lambda+\gamma]\cosh[\lambda]} \tag{34}$$

The integers $m\le L$ parameterize the multiparticle state of $\bar{T}(\lambda)$ and the Bethe ansatz roots λ_j satisfy the equations

$$\left[\frac{\bar{a}(\lambda_j)}{\bar{b}(\lambda_j)}\right]^L=-\prod_{k\neq j}^{m}\frac{\bar{b}(\lambda_k-\lambda_j)}{\bar{a}(\lambda_k-\lambda_j)}\frac{\bar{a}(\lambda_j-\lambda_k)}{\bar{b}(\lambda_j-\lambda_k)}, \quad j=1,\ldots,m \tag{35}$$

It is conceivable that an adaptation of the above ideas might work for the general eight vertex model (19) with the boundary (21). In fact, the case $\mathcal{K} = 0$ [5] [3] was solved by means of certain functional relations even though the eigenvectors structure is not yet known. If this could be carried out, even for particular values of the modulus $\mathcal{K}$, it would be an important step toward the understanding of properties of the eight vertex.

Acknowledgements

I would like to thank J. Shiraishi for the invitation to participate in the workshop "Solvable Lattice Models 2004", the Kyoto University for the hospitality and W. Galleas and G.A.P. Ribeiro for collaborating on ref.[9]. This work is partially supported by Fapesp (Fundação de Amparo à Pesquisa do Estado de São Paulo) and the Brazilian Research Council-CNPq.

References

[1] R.J. Baxter, *Exactly Solved Models in Statistical Mechanics, Academic Press, New York, 1982*

[2] L.A. Takhtajan and L.D. Faddeev, *Russian Math. Surveys, 34 (1979) 11*

[3] V.E. Korepin, G. Izergin and N.M. Bogoliubov, *Quantum Inverse Scattering Method and Correlation Functions, Cambridge University Press, 1993*

[4] H.J. de Vega, *Nucl.Phys.B, 240 (1984) 495*

[5] C.M. Yung and M.T. Bachelor, *Nucl.Phys.B, 446 (1995) 461*; M.T. Batchelor, R.J. Baxter, M.J.O. Rourke and C.M. Yung, *J.Phys.A:Math.Gen. 28 (1995) 2759*

[3] In this case, the new Boltzmann weights $\bar{a}(\lambda)$, $\bar{b}(\lambda)$, $\bar{c}(\lambda)$ and $\bar{d}(\lambda)$ are double periodic as compared with the original six-vertex weights. This is one way to see why the Bethe ansatz phase-shift is expected to be half of that of the six-vertex model with diagonal toroidal conditions [5].

[6] M. Wadati, T. Deguchi and Y. Akutsu, *Phys.Rep. 180 (1987) 247*

[7] Y. Cheng, M.L. Ge and K. Xue, *Comm.Math.Phys. 136 (1991) 195*

[8] M.J. Martins and P.B. Ramos, *Nucl.Phys.B 500 (1997) 579*

[9] G.A.P. Ribeiro, M.J. Martins and W. Galleas, *Nucl. Phys.B, 675 (2003) 567*; G.A.P. Ribeiro and M.J. Martins, *Nucl.Phys.B 705 (2005) 521*

[10] L. Amico and K. Hikami, *Eur.Phys.J B (42) (2005) 387*