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共鳴管内に形成される非対称な音響流

Takeru Yano

Division of Mechanical and Space Engineering, Hokkaido University, Sapporo, 060-8628, Japan

Abstract. Acoustic streaming caused by a large amplitude resonant oscillation of an ideal gas in a two-dimensional resonator is numerically studied by solving the system of Navier–Stokes equations with a finite-difference method, without the assumption of symmetry of the flow field. The sound field including shock waves is precisely determined, and then, the streaming velocity field is evaluated in terms of a time-averaged mass flux density vector. We shall demonstrate that, in the case where the amplitude of gas oscillation is moderately large, an asymmetric quasi-steady streaming is established after more than a thousand of oscillations of sound source.

Keywords: Numerical study, Acoustic streaming, Resonance, Shock wave
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INTRODUCTION

Streaming motions induced by acoustic standing waves are classical topics in physics [1, 2, 3]. Today, the active control of streaming in resonators becomes an important subject in various applications, in particular in thermoacoustic devices (see, e.g., [4] and Fig. 1). Some authors have recently carried out accurate measurements for slow streaming motions [5] in a resonator. However, its behavior in the case of large Reynolds number remains unresolved.

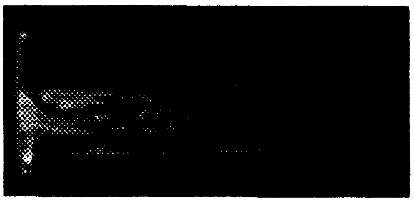

FIGURE 1. Example of asymmetric acoustic streaming in a standing-wave type thermoacoustic engine. Photograph courtesy of T. Yazaki.

Recently, the present author has numerically studied the resonant gas oscillation with a periodic shock wave in a closed tube by solving the system of compressible Navier–Stokes equations [6]. The result has suggested the occurrence of turbulent acoustic streaming when a streaming Reynolds number is sufficiently large. This is the first numerical evidence for the prediction based on the experiment [3]. Numerical studies of streaming motion with large Reynolds number have also been carried out by Alexeev and Gutfinger [7], Morris et al. [8], and Aktas and Farouk [9]. Furthermore, detailed gas motions in a vicinity of a stack plate in the thermoacoustic device have been computed by Besnoin and Knio [10] and Marx and Blanc-Benon [11]. Nevertheless, the direct numerical simulation of viscous compressible flow is an extraordinarily hard task if one

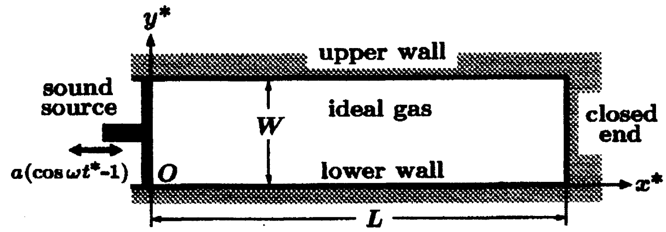

FIGURE 2. Schematic of model.

tries to resolve all phenomena from an initial state of uniform and at rest to an almost steady oscillation state throughout the entire flow field including the boundary layer. Therefore, our knowledge of streaming with large Reynolds number is still limited.

In previous papers [12, 13, 14], we have adopted a simple model based on the linear standing wave solution and a boundary layer analysis. This model employs the incompressible Navier–Stokes equations as the governing equations for the streaming velocity. As a result, we have numerically demonstrated the bifurcation and multiple existence of steady state solutions for a region of moderately large Reynolds number in a two-dimensional rectangular box. In the present paper, we shall investigate the problem of bifurcation of steady streaming, which is expected to occur before the transition to turbulent motions, not by utilizing the incompressible model, but by the direct simulation of compressible Navier–Stokes system. We treat the large Reynolds number acoustic streaming, but the Reynolds number is not so large that the turbulent streaming occurs.

PROBLEM

We shall consider the streaming motion induced by resonant gas oscillations in a two-dimensional rectangular box filled with an ideal gas (see Fig. 2). The box, whose length is L and width is W, is closed at one end by a solid plate and the other by a piston (sound source) oscillating harmonically with an amplitude a and angular frequency ω.

We assume that the sound excitation is moderately weak, the thickness of the boundary layer is sufficiently thin compared with the width of the box, and the wavelength of the excited sound is comparable with the width of the box,

$$M = \frac{a\omega}{c_0} \ll 1, \quad \epsilon = \frac{\sqrt{\nu_0 \omega}}{c_0} \ll w, \quad w = \frac{W\omega}{c_0} = O(1), \tag{1}$$

where M is the acoustic Mach number at the sound source (c_0 is the speed of sound in the initial undisturbed state), ϵ is a measure of the ratio of the thickness of acoustic boundary layer to the wavelength $\lambda = 2\pi c_0/\omega$, and w is the normalized width of the box. Furthermore, we assume the second-mode resonance, which is prescribed by

$$b = 2\pi, \tag{2}$$

where $b = L\omega/c_0$ is the normalized box length. The wave motion in the bulk of the gas can then be a resonant gas oscillation including shock waves [15].

Governing equations, initial and boundary conditions

We obtain the wave and streaming motions by solving the initial and boundary value problem for the system of compressible Navier–Stokes equations,

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u_j}{\partial x_j}=0, \tag{3}$$

$$\frac{\partial \rho u_i}{\partial t}+\frac{\partial p\delta_{ij}+\rho u_i u_j}{\partial x_j}=\epsilon^2\frac{\partial \sigma_{ij}}{\partial x_j}, \quad \sigma_{ij}=\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}-\frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right), \tag{4}$$

$$\frac{\partial E}{\partial t}+\frac{\partial (E+p)u_j}{\partial x_j}=\epsilon^2\left(\frac{\partial \sigma_{ij}u_i}{\partial x_j}+\frac{\partial q_j}{\partial x_j}\right), \quad E=\frac{1}{2}\rho u_i^2+\frac{p}{\gamma-1}, \quad q_j=\frac{\mu}{(\gamma-1)Pr}\frac{\partial T}{\partial x_j}, \tag{5}$$

where $x_i=x_i^*\omega/c_0$ and $t=\omega t^*$ are the nondimensional space coordinates and time; $\rho=\rho^*/\rho_0$ and $p=p^*/(\rho_0 c_0^2)$ are the nondimensionalized gas density and pressure; $u_i=u_i^*/c_0$ is the nondimensional gas velocity; E is the nondimensional total gas energy per unit volume; σ_{ij} and q_i are the nondimensional viscous tensor and heat flux vector; $\mu=\mu^*/\mu_0$ is the nondimensional viscosity coefficient; γ is the ratio of specific heats; Pr is the Prandtl number. The equation of state for ideal gas $\gamma p=\rho T$ is used to close the system.

The initial condition is given as

$$u_i=0, \quad p=\gamma^{-1}, \quad \rho=1, \quad T=1. \tag{6}$$

The boundary conditions on the oscillating piston face are

$$u=-M\sin t, \quad v=0, \quad T=1 \quad \text{at} \quad x=M(\cos t-1) \text{ and } 0\leqq y\leqq w, \tag{7}$$

and the boundary condition except for the piston face are

$$u=v=0, \quad T=1, \tag{8}$$

where $x=x_1$, $y=x_2$, $u=u_1$, $v=u_2$, and the isothermal condition is imposed on the gas temperature on the wall.

The viscosity coefficient is assumed to obey the Sutherland's law and the Prandtl number Pr to be a constant (0.7).

The results presented in the following are the cases where the acoustic Mach number at the sound source $M=0.01$ and 0.001, the normalized box width $w=2\pi/5$ and $\pi/5$, and the ratio of the boundary layer thickness to the wavelength $\epsilon=0.00316$, which corresponds to $\omega/(2\pi)=12.5$ kHz in the air in the standard state.

NUMERICAL METHOD

The initial and boundary value problem (3)–(8) is solved with the high-resolution upwind finite-difference TVD scheme [16]. The method has been used for various nonlinear acoustics problems by the present author (e.g., see [6, 17]), and the details are omitted here.

We shall remark that in the present computation we don't assume any symmetry of flow pattern of acoustic streaming, and therefore we solve the entire field in the box $0 \leqq x \leqq b$ and $0 \leqq y \leqq w$. The entire field is subdivided into a boundary-fitted 700×300 nonuniform mesh, where the mech points are clustered near the boundary. The minimum grid size is 0.0002 in the vicinity of the wall and this is so small compared with $\epsilon = 0.00316$ that we can resolve the acoustic boundary layer. The resolution of boundary layer is crucially important because the acoustic streaming in the bulk of the gas is mainly induced by the streaming motion in the boundary layer or the so-called limiting velocity of the inner streaming.

The time step is $2\pi/50000$, and the CFL number is about 0.5. The CPU time for computation of one oscillation cycle of piston is about 4.4 hours on a state-of-the-art PC (dual-cpu machine).

The streaming velocity $\boldsymbol{u}_S$ is evaluated by the time-averaged mass flux vector,

$$\boldsymbol{u}_S = \begin{pmatrix} u_S \\ v_S \end{pmatrix} = \int_t^{t+2\pi} \begin{pmatrix} \rho u \\ \rho v \end{pmatrix} dt. \tag{9}$$

We shall further remark that we don't give any artificial seed of asymmetry in the computation. The numerical code is written symmetrically in the algebraic sense. The asymmetry inherent in the numerical operations of finite figures spontaneously grows due to the instability of the system concerned.

RESULTS

Evolution of resonant gas oscillation

At the initial instant $t = 0$, the gas in the box is uniform and at rest. After the beginning of oscillation of the piston, the wave amplitude grows in proportion to Mt. At $t = O(1/\sqrt{M})$, the wave amplitude reaches the maximum value of $O(\sqrt{M})$, where two shock waves are formed since the excitation at the sound source is the second mode. Figure 3 shows the temporal evolution of pressure amplitude at the closed end. At almost $t/(2\pi) = 15$, the wave amplitude reaches its maximum value, and thereafter a quasi-steady oscillation state continues.

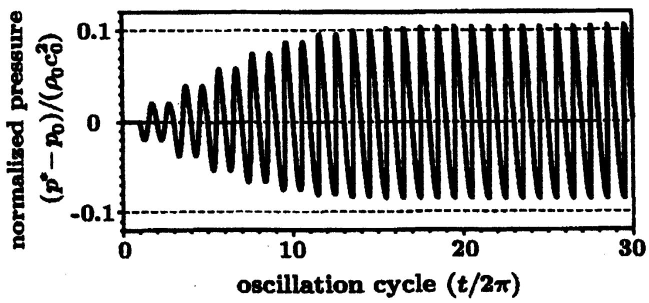

FIGURE 3. Initial evolution of pressure amplitude at the closed end.

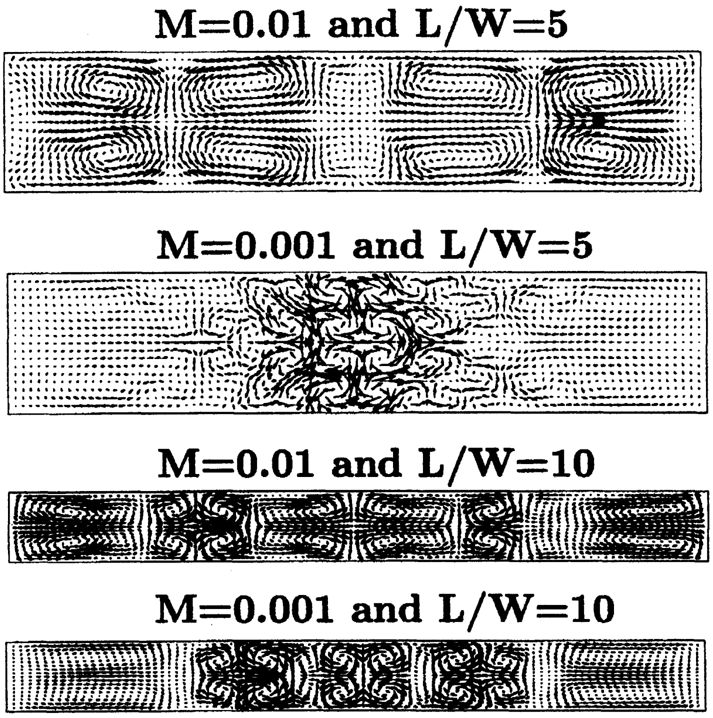

FIGURE 4. Streaming velocity field at $t/(2\pi) = 300$.

Development of acoustic streaming

Figure 4 shows the streaming velocity fields at $t/(2\pi) = 300$. Except for the case of $M = 0.01$ and $L/W = 5$, the streaming patterns are considerably different from that of the classical Rayleigh streaming, which consists of the regular arrangement of four vortex pairs and the flow pattern is symmetric with respect to $x = b/2$ and $y = w/2$. Although the flow pattern for $M = 0.01$ and $L/W = 5$ is apparently similar to that of Rayleigh streaming, the flow directions of its major vortexes are opposite to those of Rayleigh streaming. This has been reported by Alexeev and Gutfinger [7], although their computation assumes the symmetry of flow field with respect to $y^* = W/2$ and they have truncated the computations at one hundred cycles.

The streaming velocity fields at $t/(2\pi) = 500$ are shown in Fig. 5. An origin of asymmetry appears near $x = 0$ and $y = w/2$ in the case of $M = 0.01$ and $L/W = 5$. The flow patterns in the other cases constantly change from those shown in Fig. 4. This clearly means that computations for a few hundreds of cycles are insufficient for the analysis of high Reynolds number acoustic streaming.

We therefore continue the computations for the two cases of $L/W = 5$ over a thousand of cycles. Figure 6 shows the development of asymmetric streaming pattern for the case of $M = 0.01$ and $L/W = 5$, and Fig. 7 shows the case of $M = 0.001$ and $L/W = 5$. The time from the beginning of oscillation and the maximum of streaming velocity U_{max} are shown in each plot in Fig. 6. Note that, as can be seen from Figs. 6 and 7, the maximum velocity U_{max} is of the order of M, because the maximum wave amplitude is of $O(\sqrt{M})$

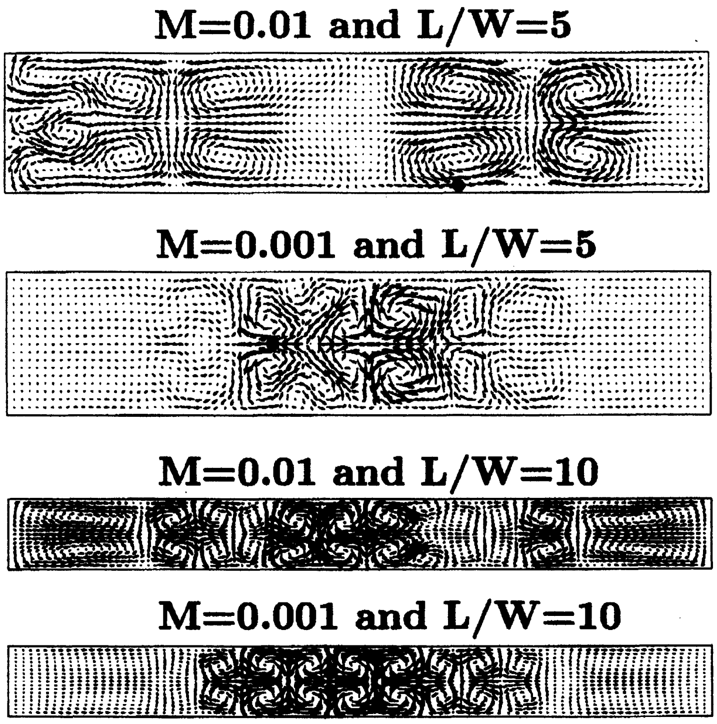

FIGURE 5. Streaming velocity field at $t/(2\pi) = 500$.

and the streaming motion is a second-order nonlinear phenomenon.

The top figure in Fig. 6 shows that the streaming velocity field is fully asymmetric at 860th cycle. From the comparison of the streaming pattern at 1160th cycle and that at 1260th cycle, we may conclude that an asymmetric streaming almost reaches a quasi-steady state. On the other hand, the symmetry of streaming velocity field for the case of $M = 0.001$ and $L/W = 5$ is hardly destroyed up to 1020th cycle, as shown in the top figure in Fig. 7. Nevertheless, an origin of asymmetry appears at around the center of the resonator at 1120th cycle, and then the asymmetry grows and prevails in the entire field at 1320th cycle. At this stage, however, we cannot conclude that the streaming velocity field for the case of $M = 0.001$ and $L/W = 5$ reaches a quasi-steady state.

Here, we shall comment on the streaming Reynolds number. The streaming Reynolds number Rs may be defined by

$$\mathrm{Rs} = \frac{U_S L_S}{\nu_0}. \tag{10}$$

From the bottom figure in Fig. 6, we take the characteristic speed of streaming $U_S = 0.007c_0$. The characteristic length of streaming L_S may be taken as $W = (2\pi c_0)/(5\omega)$ (width of box). Consequently, we have $\mathrm{Rs} = 880$ for the case of $M = 0.01$ and $L/W = 5$.

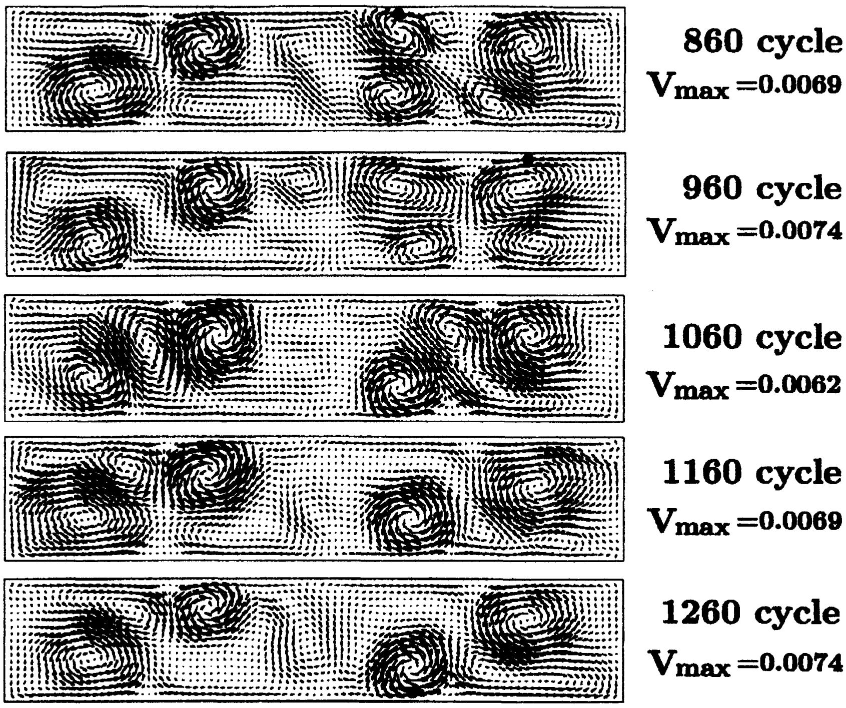

FIGURE 6. Development of asymmetric streaming pattern for $M = 0.01$ and $L/W = 5$.

CONCLUSIONS

We have demonstrated that acoustic streaming in a resonator develops into an asymmetric quasi-steady flow. The previous authors [7, 9] assumed the symmetry with respect to the centerline of resonator in their computations, and therefore, they couldn't find the asymmetric solutions. Furthermore, they truncated the computation at 100 or 200 cycles from the beginning of sound excitation, which is clearly insufficient for the analysis of streaming motion of moderately large Reynolds number, unless the strong nonlinearity rapidly excites a turbulent streaming motion as shown in [6].

The existence of the steady asymmetric flow regime prior to the transition to turbulent motions is important for understanding of acoustic streaming with large Reynolds number in resonators.

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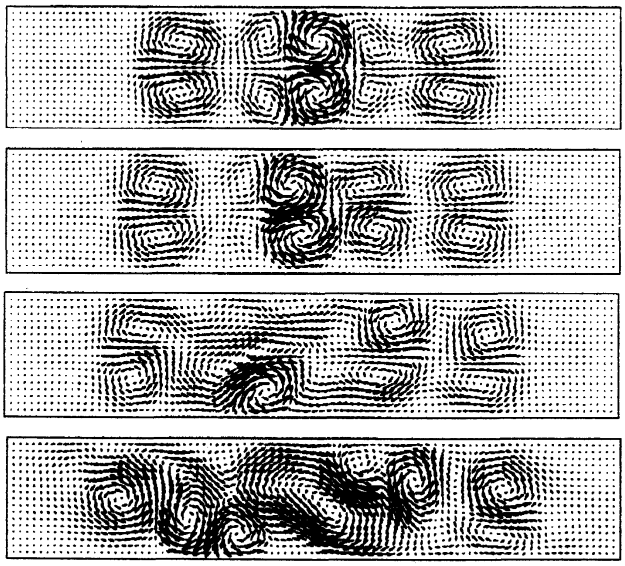

FIGURE 7. Development of asymmetric streaming pattern for $M = 0.001$ and $L/W = 5$.

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