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Growth and Habit Formation : the case of endogenous technical change *

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Abstract

This paper introduces external habit formation into one of the basic models of endogenous growth in which continuing expansion of product variety sustains long-term growth. We assume that households consume a range of final goods and they set a benchmark level of consumption for each good. The benchmark consumption is determined by external habit formation so that there are commodity-specific external effects. Each good is produced by a monopolistically competitive firm and the firm's optimal pricing decision exploits the fact that consumers' demand is subject to the external habit formation. Given those settings, we show that the introduction of consumption externalities may affect the balanced-growth characterization, transitional dynamics as well as policy impacts in fundamental manners.

JEL Classification code: E2, O3, O4
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1 Introduction

This paper based on Doi and Mino (2006). The purpose of this paper is to investigate the role of consumption external effects in the context of a variety-expansion model of growth. Before our discussion, let us show a simple survey about growth theory and habit formation at first.

Recent development of growth theory differs from the one-sector neoclassical model during the 1950s and 1960s, which considers capital accumulation as the engine of growth. It is sure for neoclassical models to be able to explain the growth process simply. However, such growth models could not explain long-run growth and technological progress on economic decisions. To overcome these points, recent growth theory has tried to determine the growth rate in economic activities, focusing on human capital (Lucas(1998)) and technological change (Romer (1990), Grossman-Helpman(1991), Aghion-Howitt(1992)) and so on. Since growth theorists have been still interested in the engine of growth, their effort has been devoted to exploring it. That is, they have paid attention to the external effects in production and knowledge creation activities. In other words, the role of consumption have ignored in much of growth literatures. However, since such externalities can affect consumers' behavior, they may change strategy of firm, for example, price setting. Thus, they may affect economic performance. This gives us motivation to show the role of consumption externalities in a growing economy.

As to consumption externality, Duesenberry (1949) showed two ideas; that is, individuals care about the average consumption around him (outward-looking), and it is difficult for a family to reduce its expenditures from a high level at a time in the past (inward-looking). These consumers' habit give rise to externalities in their utility. In the field of asset pricing and business cycles, these ideas have been introduced. In the former, these ideas are used in order to solve the equity premium puzzle, and in the latter, to consider how the mark-up ratio change in boom or recession. On the other hand, much of the theoretical literature on growth have not considerd these ideas, and have used the time separable and CRRA utility function.

The recent contributions by Alonso-Carrera et al. (2004 and 2005), Carroll et al. (1977 and 2000), Alvarez-Cuadrado et al. (2004) and Liu and Turnovsky (2005), however, have kindled a renewed interest in the role of consumption externalities in growing economies. Alonso-Carrera et al. (2004 and 2005), Alvarez-Cuadrado et al. (2004) and Liu and Turnovsky (2005) examine implications of consumption external effects in the standard neoclassical growth models. The main research concern of those authors is to explore the effects of interdependency among consumers on welfare and transitional dynamics of the economy. Carroll et al. (1997 and 2000) examine the roles of external as well as internal habit formation in an endogenous growth model with an Ak technology and analyze how the presence of consumption externalities affects savings and the pattern of growth.[1] Those studies have clearly demonstrated that external effects of consumption may have significant implications for growing economies in both qualitative and quantitative senses.

Departing from the existing studies on the effects of consumption externalities in growing economies, we investigate the role of consumption external effects in the context of a variety-expansion model of growth. The analytical framework of this paper is based on Grossman and Helpman (1991, Chapter 3). In our setting, there are a variety of consumption goods and each commodity is produced by a monopolistically competitive firm. The range of consumption goods variety is enhanced by R&D activities.[2] Those assumptions enable us to introduce two distinctive features of consumption external effects that have not been considered in the existing literature assuming a homogenous consumption good and perfect competition. First, we may assume that consumers set a benchmark consumption level for each good. The benchmark level of each consumption good is determined by external (outward-looking) habit formation so that there exist commodity-specific external effects. Second, since each commodity is produced by a monopolist, the firm may exploit the fact that consumer's demand for its own product is affected by the commodity-specific external effect. This means that the firm can internalize the consumption external effect when maximizing its profits. As a result, the firm's marginal cost involves the implicit 'internalization costs' of the consumption external effects, and hence the pricing decision of the firm is affected by the behavior of benchmark level of consumption set by the consumers. The basic idea of this kind of modelling has been proposed by Ravn et al. (2002 and 2006) who examine the effects of commodity-specific consumption externalities in a real business cycle model with monopolistic competition.[3] In this paper, we consider the implications of consumption external effects in an imperfectly competitive, growing economy.

Given the analytical framework described above, we explore the balanced-growth equilibrium and transitional dynamics. We find that the presence of consumption externalities may yield significant effects on the balanced-growth-path (BGP) characterization as well as on equilibrium dynamics of the model economy. First, if the BGP establish saddle stability, the behaviors of key variables such as the rate of technical

[1] Harbaugh (1996) also discusses the relation between growth and saving in the presence of consumption externalities by using a two-period model with uncertainty.

[2] See Gancia and Zilbotti (2005) for a detailed survey on this class of models.

[3] Since Ravan et al. (2002 and 2006) explore real business cycles in the context of a stochastic dynamic general equilibrium framework, their discussion riles on a model calibration. In contrast, we use a simpler deterministic, continuous-time model of growth, which enables us to study the behavior of the model economy analytically.

change may be different depending on whether external effects are negative or positive. Second, the policy implications obtained in our model can be quite different from those established in the original Grossman and Helpman model. For example, in our framework a policy that stimulates R&D investment does not necessarily promote long-term growth. In addition, due to the presence of consumption external effects, the level of R&D spending determined in the competitive equilibrium may not be lower than its optimal level that attains the efficient resource allocation.

The reminder of the paper is organized as follows. Section 2 constructs the analytical framework. Section 3 derives a complete dynamic system. Section 4 examines the balanced-growth equilibrium and investigates equilibrium dynamics out of the steady state. Section 5 considers the effects of R&D subsidy and the socially optimal level R&D spending. A brief conclusion is given in Section 6.

2 The Model

2.1 Households

There is a continuum of identical households whose number is normalized to one. The representative household consumes a variety of consumption goods, ranging from index 0 to n. We assume that the consumer's felicity depends not only on her own consumption of each good but also on the benchmark level of consumption that is determined by outward-looking habit formation. The instantaneous sub-utility of the household is given by

$$C = \left(\int_0^n \left(c_i s_i^{-\theta} \right)^{\frac{\alpha-1}{\alpha}} di \right)^{\frac{\alpha}{\alpha-1}}, \quad \theta < 1, \quad \alpha > 1, \tag{1}$$

where c_i is consumption of good $i \in [0,n]$, α denotes the elasticity of substitution between consumption goods and C is the composite consumption. Here, s_i is the household's benchmark level of consumption of good i, which represents a commodity-specific external habit formation. It is accumulated by the following dynamic equation,

$$\dot{s}_i = \beta \left(\bar{c}_i - s_i \right), \quad \beta > 0. \tag{2}$$

where $\bar{c}_i(\tau)$ denotes the average consumption of good i in the economy at large. To understand meanings of this utility, the instantaneous utility of consumption of good i can be written as

$$c_i s_i^{-\theta} = c_i^{1-\theta} \left(\frac{c_i}{s_i} \right)^{\theta}; \quad \theta \neq 0, \quad \theta < 1.$$

This shows that the felicity obtained by consuming the i-th good depends on the relative consumption, c_i/s_i, as well as on the absolute level of consumption, c_i. If

θ is positive, a rise in s_i negatively affects the felicity of consumer. Namely, each consumer's preference exhibits jealousy as to consumption of others. In contrast, if θ is negative, then the felicity of consumer increases with the benchmark consumption. In this case consumers' preferences show admiration for consumption of other members in the society.[4] It is also to be noted that if $\beta = +\infty$, then $s_i = \bar{c}_i$ so that the external effects are only intratemporal. In addition, if $\theta = 0$, then each consumer's preference becomes the standard one in which her felicity depends on the absolute levels of private consumption alone.

Given (1), the households maximizes a discounted sum of subutilities

$$U = \int_0^\infty e^{-\rho t} \log C dt, \quad \rho > 0,$$

subject to the flow budget constraint:

$$\dot{a} = ra + wN - \int_0^n c_i p_i di, \tag{3}$$

where a denotes the asset holding of the household, r is the real interest rate, w is the real wage rate and p_i denotes the price of consumption good i. We assume that in each moment the representative household supplies N units of labor inelastically. Notice that the habit formation is external to an individual household, so that when deciding her optimal plan, the household takes the future sequence of benchmark consumption, $\{s(t)\}_{t=0}^{\infty}$, as given.

Denoting $\hat{c}_i = c_i s_i^{-\theta}$ and $\hat{p}_i = p_i s_i^{\theta}$, we first consider the following cost minimization problem:

$$\begin{aligned} &\min \int_0^n \hat{c}_i \hat{p}_i di \\ &s.t. \quad C = \left(\int_0^n \hat{c}_i^{\frac{\alpha-1}{\alpha}} di \right)^{\frac{\alpha}{\alpha-1}}. \end{aligned}$$

Solving this problem gives the demand equation of good i is thus given by

$$c_i = s_i^{\theta(1-\alpha)} \left(\frac{\hat{P}}{p_i} \right)^{\alpha} C. \tag{4}$$

where $\hat{P} \equiv \left(\int_0^n \hat{p}_i^{1-\alpha} di \right)^{\frac{1}{1-\alpha}}$, that is, this denotes a price index of the subutility (aggregate consumption). This equation states that given prices and the composite consumption, C, the demand for good i decreases with s_i if $\theta > 0$. When $\theta < 0$, a higher s_i increases c_i. Similarly to Grossman and Helpman(1991), we rewrite intertemporal

[4]See Dupor and Lin (2003) for a useful taxonomy as to formulating consumption externalities.

maximization problem using expenditure function, $E \equiv \int_0^n c_i p_i di$, and solve the problem of expenditure minimization. The optimization conditions for this new problem give the Euler equation,

$$\frac{\dot{E}}{E} = r - \rho, \tag{5}$$

together with the transversality condition, $\lim_{t\to\infty} (a/E)\, e^{-\rho t} = 0$. Following Grossman and Helpman (1991), setting prices as a numeraire gives us a constant nominal spending, E. Thus, by setting $E = 1$ for all $t \geq 0$, from (5) the real interest rate, r, equals the time discount rate in every moment:

$$r = \rho. \tag{6}$$

2.2 Producers

Each consumption good is produced by a monopolistically competitive firm. The profits of the firm producing consumption good i are given by

$$\pi_i = p_i c_i - wbc_i, \quad b > 0.$$

The firm produces by using labor alone and the production function of good i is assumed to be $c_i = (1/b)\, l_i$, where l_i is labor devoted to production of the i-th good. Following Ravn et al.(2006), we assume that the firm exploits the fact that consumers' demand behavior is affected by the benchmark consumption level, s_i, and that s_i changes according to (2). This means that the firm maximizes a discounted sum of its profits over an infinite-time horizon subject to (2). The optimization behavior of the firm is thus formulated as follows:

$$\begin{aligned} \max \quad & \int_0^\infty \exp\left(-\int_0^t r(\xi)\, d\xi\right) \pi_i(t)\, dt \\ s.t. \quad & \pi_i = s_i^{\theta(1-\alpha)} \hat{p}^\alpha C \left[p_i^{1-\alpha} - wbp_i^{-\alpha}\right] \\ & \dot{s}_i = \beta \left[s_i^{\theta(1-\alpha)} \hat{p}^\alpha C p_i^{-\alpha} - s_i\right], \end{aligned}$$

where $s_i(0)$ is given. In this problem, the firm's control and state variables are p_i and s_i, respectively.

To derive the optimization conditions, let us set up the following Hamiltonian function:

$$H_i = s_i^{\theta(1-\alpha)} \hat{p}^\alpha C \left[p_i^{1-\alpha} - wbp_i^{-\alpha}\right] + \lambda_i \beta \left[s_i^{\theta(1-\alpha)} \hat{p}^\alpha C p_i^{-\alpha} - s_i\right],$$

where λ_i is the shadow value of the benchmark consumption level, s_i. Maximizing the Hamiltonian function with respect p_i, we obtain the optimal pricing formula in such a way

$$p_i = \frac{\alpha}{\alpha - 1} (bw - \beta \lambda_i). \tag{7}$$

Equation (7) means that the price of good i equals the marginal cost of labor input, bw, plus the shadow cost of habit formation, $-\beta\lambda_i$, multiplied by a coefficient, $\alpha/\left(\alpha-1\right)$. In the conventional expression (7) may be written as

$$p_i = \frac{\alpha}{\alpha-1}\left(1-\frac{\beta\lambda_i}{bw}\right)bw.$$

Since the explicit cost is labor cost, bw, alone, the markup ratio is represented by $\frac{\alpha}{\alpha-1}\left(1-\frac{\beta\lambda_i}{bw}\right)$.So that it changes with the relative costs, $\left(-\beta\lambda_i\right)/bw$. Therefore, in our setting with consumption externality, the markup ratio is endogenously determined[5]. This endogenous markup ratio is the sources that make the analytical results diverge from those obtained in the original Grossman and Helpman model.

The shadow value λ_i changes according to

$$\begin{aligned}\dot{\lambda}_i &= r\lambda_i - \frac{\partial H_i}{\partial s_i} \\ &= \left(r+\beta\right)\lambda_i - \theta\left(\frac{1}{\alpha}-1\right)s_i^{\theta(1-\alpha)-1}\hat{p}^{\alpha}Cp_i^{1-\alpha}. \end{aligned} \tag{8}$$

where we use $p_i - bw + \beta\lambda = p_i/\alpha$. From the solution of (8), we obtain the value of $\lambda\left(t\right)$. As to the sign of $\lambda\left(t\right)$ and the characterization of price, we summarize the relationship between them as the next table.

	admiration($\theta < 0$)	G-H	jealousy($\theta > 0$)
sign λ	$+$	0	$-$
$\partial p_i/\partial s_i$	$-$	0	$+$

If the firm i sells an additional unit of product, then an increase in consumption of good i raises the benchmark consumption, s_i. When $\theta > 0$, such an increase in s_i will lower the future consumption demand for good i. Therefore, an increment in production of good i yields two types of additional costs: the marginal cost of labor employment, bw, and the marginal penalty cost, $-\beta\lambda_i$, that counts the expected reduction of future consumption demand for good i due to the marginal increase in s_i. Therefore, the optimal price in this model is higher than original Grossman and Helpman, in order to accumulate s_i more slowly. In contrast, when $\theta < 0$, the effect of a rise in s_i works reversely.

Next, we consider R&D sector. We assume that R&D sector behave in the same way as Grossman and Helpman(1991). The knowledge production function is

$$\dot{n} = \delta L_R n, \quad \delta > 0, \tag{9}$$

[5]In the standard formulation without consumption externalities, the markup formula is given by $p_i = \alpha wb/\left(\alpha-1\right)$, which has a constant markup rate, $\alpha/\left(\alpha-1\right)$.

where L_R denote labor input for R&D activities. Denoting the patent price by v, we see that the zero-excess-profit condition for the R&D sector, i.e. $v\dot{n} - wL_R = 0$, gives

$$w = \delta n v. \tag{10}$$

The arbitrage condition is

$$\frac{\dot{v}}{v} = r - \frac{\pi_i}{v}. \tag{11}$$

Finally, to close this model, the full employment condition of labor market is

$$L_R + L_f = N, \tag{12}$$

where L_f is the total labor used for consumption goods production, $L_f = \int_0^n l_i di$.

3 Dynamic System

3.1 Symmetric Equilibrium

In order to make our model analytically tractable, we focus on the symmetric equilibrium in which the following conditions are fulfilled:

$$c_i = c, \quad p_i = p, \quad s_i = s, \ \lambda_i = \lambda, \tag{13}$$

for all $i \in [0, n]$. In the symmetric equilibrium each consumer sets the same amount of benchmark consumption for every good, regardless of its timing of introduction into the market. If we assume that $s_i = s$, it also holds that $\lambda_i = \lambda$ for all $i \in [0, n]$. Hence, the prices are the same for all goods, $p_i = \alpha b w / [bw - \beta\lambda] = p$. Additionally, due to the normalization of the number of households, in equilibrium the instantaneous level of average consumption satisfies that $\bar{c}_i = c$ for all $i \in [0, n]$.

3.2 A Complete Dynamic System

At the symmetric equilibrium, after some manipulation, we have derived a complete dynamic system consisting of (14), (15) and (16) that describe the dynamic motions of, the aggregate level of benchmark consumption $x\,(= ns)$, the aggregate value of knowledge ($vn = w/\delta$) and the shadow value of the benchmark consumption, λ.

Now denote $sn \equiv x$. Then (2), (9) and $\frac{\dot{x}}{x} = \frac{\dot{s}}{s} + \frac{\dot{n}}{n}$ give

$$\frac{\dot{x}}{x} = \beta \left[\frac{wb(\alpha - 1)}{x\alpha (bw - \beta\lambda)} - 1 \right] + \delta \left[N - \frac{b(\alpha - 1)}{\alpha (bw - \beta\lambda)} \right]. \tag{14}$$

In view of (10) and (11), we obtain the following:

$$\frac{\dot{w}}{w} = \frac{\dot{n}}{n} + \frac{\dot{v}}{v} = \rho + \delta N - \frac{\delta}{w}. \tag{15}$$

Finally, from (8) the implicit price of the benchmark consumption changes according to

$$\dot{\lambda} = (\rho + \beta)\lambda - \theta\left(\frac{1}{\alpha} - 1\right)\frac{1}{x}. \tag{16}$$

4 Balanced-Growth and Equilibrium Dynamics

4.1 Existence of the Balanced-Growth Equilibrium

In the balanced-growth equilibrium, x, w and λ stay constant over time. Hence, it holds that

$$\begin{aligned}\frac{\dot{s}}{s} &= \frac{\dot{c}}{c} = \frac{\dot{v}}{v} = -\frac{\dot{n}}{n} \\ &= -\delta\left(N - L_f^*\right) \\ &= -\delta\left[N - \frac{b(\alpha-1)}{\alpha(bw^* - \beta\lambda^*)}\right] < 0,\end{aligned} \tag{17}$$

where L_f^*, w^* and λ^* denotes the steady-state values of L_f, w and λ, respectively. Because of normalization, in the balanced-growth equilibrium where n grows at a constant rate, s, c and v contract at the rate of $-\dot{n}/n$[6].

Condition $\dot{w} = 0$ gives the steady-state level of the real wage rate:

$$w^* = \frac{\delta}{\delta N + \rho}. \tag{18}$$

The steady-state values of x and λ are obtained by setting $\dot{\lambda} = 0$ and $\dot{x} = 0$ in (14) and (16), respectively. Substituting (18) into these conditions, we find that the steady-state values of x and λ are respectively given by

$$x^* = \frac{(\alpha-1)\beta\left[\delta b(\rho+\beta) + \theta(\delta N - \beta)(\delta N + \rho)\right]}{(\rho+\beta)\delta b\left[(\alpha-1)(\delta N + \rho) - (\delta N - \beta)\alpha\right]}, \tag{19}$$

$$\lambda^* = \frac{\theta\delta b\left[\alpha(\delta N - \beta) - (\alpha-1)(\delta N + \rho)\right]}{\alpha\beta\left[\delta b(\rho+\beta) + \theta(\delta N - \beta)(\delta N + \rho)\right]}. \tag{20}$$

Since $L_f^* = b(\alpha-1)/\alpha(bw^* - \beta\lambda^*)$, from (18) and (20) we obtain

$$L_f^* = \frac{(\alpha-1)(\delta N + \rho)\left[\delta b(\rho+\beta) + \theta(\delta N - \beta)(\delta N + \rho)\right]}{\delta\left[\alpha\delta b(\rho+\beta) + \theta(\alpha-1)(\delta N + \rho)^2\right]}. \tag{21}$$

In order to define a feasible steady state, we show the conditions which the parameter values should satisfy.

[6]In this model, welfare expansion requires that $-\frac{1}{\alpha-1} < \theta$. When $\theta > 0$, this condition is always satisfied. In what follows, we assume that this condition holds for the case of $\theta < 0$ as well.

In the case of negative consumption externalities $(\theta > 0)$, the economy has a unique, feasible balanced-growth path, if the parameter values satisfy

$$-\frac{\delta b\,(\beta+\rho)}{\theta\,(\delta N+\rho)} < \delta N - \beta < \frac{(\alpha-1)\,(\delta N+\rho)}{\alpha}.$$

In the case of positive consumption externalities $(\theta < 0)$, the presence of a unique and feasible balanced-growth path is ensured if

$$\delta N - \beta < \min\left\{\tfrac{(\alpha-1)(\delta N+\rho)}{\alpha},\ -\tfrac{\delta b(\rho+\beta)}{\theta(\delta N+\rho)^2}\right\}$$
$$\text{and } \alpha\delta b\,(\rho+\beta) + \theta\,(\alpha-1)\,(\delta N+\rho)^2 > 0.$$

4.2 The Long-Run Growth Rate

By use of (18) and (20), we may express the balanced-growth rate as a function of given parameters:

$$g = \delta N - \frac{(\alpha-1)\,(\delta N+\rho)\,[\delta b\,(\rho+\beta) + \theta\,(\delta N-\beta)\,(\delta N+\rho)]}{\alpha\delta b\,(\rho+\beta) + \theta\,(\alpha-1)\,(\delta N+\rho)^2}. \tag{22}$$

If there is no consumption external effect, i.e. $\theta = 0$, the balanced-growth rate determined by (22) is reduced to

$$\hat{g} = \delta\left(N - \hat{L}_f\right) = \delta\left(N - \frac{\alpha-1}{\alpha w^*}\right) = \frac{\delta N}{\alpha} - \rho. \tag{23}$$

where $\hat{L}_f$ is the total labor for a final goods sector in the economy without consumption externalities. As (23) shows, the balanced-growth rate in the standard model with product-variety expansion increases with the labor supply, N, and the efficiency of R&D, δ, while it decrease with the elasticity of substitution among consumption goods, α, and the time discount rate, ρ. In contrast to these simple results in the standard model, (22) shows that the effects of parameter changes on the long-term growth rate are rather complex in the presence of external habit formation.

First, compare the balanced-growth rate given by (22) with that determined by (23), we can summarize the results as following table,

	admiration$(\theta < 0)$	G-H	jealousy$(\theta > 0)$
sign λ	$+$	0	$-$
$L_f - \hat{L}_f$	$+$	0	$-$
$g - \hat{g}$	$-$	0	$+$

Let us consider economic interpretations of these results. In the case of $\theta > 0$, a higher growth of consumption demand will enhance s, which in turn depresses the future consumption demand and thus future profits of firms. Since each firm correctly anticipates such an effect of social habit formation on the consumers' decision, it has an incentive to set a higher price in order to slow down the growth of habit accumulation. Consequently, in the symmetric equilibrium the aggregate consumption demand will decline and thus L_f decreases. This means that labor will shift from the production activities to R&D sector, which accelerates the long-term growth. In the case of $\theta < 0$, the exposition given above is completely reversed. We have thus shown:

Proposition 1 *Other things being equal, the economy with negative consumption externalities attains a higher balanced-growth rate than the economy without externalities. In contrast, if there are positive consumption externalities, the balanced-growth rate is lower than that sustained by the economy without externalities.*

Second, consider the effect of a change in the level of labor supply, N and the time preference, ρ. The results are

	admiration($\theta < 0$)	G-H	jealousy($\theta > 0$)
$\partial g/\partial N$	$-$	$+$	ambiguous
$\partial g/\partial \rho$	ambiguous	$-$	$-$

When N increases in the economy without consumption externalities, depresses w^*, which has a negative impact on growth. At the same time, a rise in N stimulates technical progress, because it allows the R&D sector to employ a larger amount of labor. In the standard model, the latter effect always dominates. However, in this model with consumption externalities, in addition to these two effect, we can find an increase in N may change the value of λ^* (see (20)). When $\theta > 0$, the effect of a raise in N on the magnitude of λ^* is ambiguous. If $\theta < 0$, then an increase in N raises λ^*. If this increase in L_f^* is large enough to hold $dL_f^*/dN > 1$, then a larger labor supply lowers the balanced-growth rate: we may have an anti-scale effect even though there are knowledge spillovers in the R&D sector.

Similarly, in the absence of consumption externalities, a higher ρ decreases w^*, which increases L_f^*. Hence, the balanced growth rate will decline. If $\theta > 0$, (20) states that an increase in ρ lowers the absolute value of λ^*. Therefore, we obtain $dL_f^*/d\rho > 0$, so that the balanced growth rate decreases. However, if $\theta < 0$, it is seen that we cannot determine the sign of $dL_f^*/d\rho$ without imposing further constraints on the magnitudes of the parameter values.

4.3 Equilibrium Dynamics

Note that the dynamic behavior of w given by (18) is independent of other variables and it is completely unstable, thus $w = w^*$ always holds. Consequently, we may focus on the dynamics of x and λ under the fixed level of $w = w^*$. Keeping in mind that the predetermined variable in our system is $x\,(= sn)$ alone, we see that there is a unique stable converging path around the balanced-growth equilibrium if the dynamic system consisting of (14) and (16) exhibits a saddle-point property[7]. However, in the case $\theta < 0$ and $\beta < \delta N$, we can say the balanced-growth path may exhibits local indeterminacy.

5 Discussion

5.1 R&D Subsidy

In the original Grossman and Helpman model, any policy that promotes R&D activities has a clear implication. Such an unambiguous policy implication may not hold in our model. To see this, consider a simple R&D subsidy scheme in which a portion of labor costs of the R&D firms is subsidized at a rate of $\phi \in (0,1)$. We assume that the government finances the R&D subsidies by levying a lump-sum tax on the households' income. Then profits of the R&D firms is $v\dot{n} - (1-\phi)\, wL_R$, so that the zero-excess-profit condition for the R&D firms is given by, $(1-\phi)\, w = \delta n v$. In this setting, the steady-state conditions are given by:

$$\beta\left(\frac{wL_f}{x} - 1\right) + \delta\left(N - L_f\right) = 0 \Longleftrightarrow \dot{x} = 0, \tag{24}$$

$$(\rho+\beta)\,\lambda - \theta\left(\frac{1}{\alpha} - 1\right)\frac{1}{x} = 0 \Longleftrightarrow \dot{\lambda} = 0, \tag{25}$$

$$\rho + \delta N - \frac{\delta}{(1-\phi)\,w} + \frac{\phi\delta L_f}{1-\phi} = 0 \Longleftrightarrow \dot{w} = 0. \tag{26}$$

Keeping in mind that $L_f = b\,(\alpha - 1)\,/\alpha\,(bw - \beta\lambda)$, and in order to examine the growth effect of a change in ϕ in the presence of external habit formation, let us write L_f as

$$L_f = \frac{1}{\phi}\left(\frac{1}{w} - \frac{\delta N + \rho}{\delta}\right) + \frac{\delta N + \rho}{\delta}. \tag{27}$$

Then the steady-state levels of w and L_f are determined at the intersection of the two graphs[8]. As a result, we know that a higher R&D subsidy does not always raise the growth rate, unlike the standard model without consumption externalities.

The intuition behind this is the following. Remember again that the steady-state level of employment in the final good sector is written as

$$L_f^* = \frac{b}{p^*} = \frac{1}{\frac{\alpha}{\alpha-1}\left(1 - \frac{\beta\lambda^*}{bw^*}\right) w^*}.$$

[7]Linearizing (14) and (16) around the steady state where $\dot{x} = \dot{\lambda} = 0$, we can confirm the stability of this model.

[8]One is derived from (24) and (25), the other is (27).

In the presence of consumption externalities, an increase in ϕ yields the indirect as well as direct effects on the equilibrium price level p^*. First, a rise in subsidy to the R&D sector increases the labor demand of the R&D firms, and hence, other things being equal, the real wage rate tends to rise, which increases the equilibrium price p^*. At the same time, a higher p^* reduces consumption demand so that the external habit formation will be slow down. This lowers the implicit 'internalization costs' for the firm, i.e. the absolute value of λ^*, which depresses the mark-up rate, $\frac{\alpha}{\alpha-1}\left(1-\frac{\beta\lambda^*}{bw^*}\right)$.[9] If this reduction in the mark-up rate dominates the initial increase in the real wage rate, p^* may fall down so that L_f^* increases. As a consequence, a higher ϕ lowers the real wage and raises L_f^*, which depresses the balanced-growth rate. This conclusion tends to hold if the initial level of w^* is less than $\delta/\left(\delta N+\rho\right)$. In contrast, if the initial w^* exceeds $\delta/\left(\delta N+\rho\right)$, then a decrease in the mark-up rate cannot cancel the direct effect of a rise in w^*, and therefore in the new steady state p increases to lower L_f^*.

5.2 The Optimal Level of R&D

As emphasized above, since in our model externalities present both in production and consumption sides, we may not establish a straightforward result in the standard R&D based growth model. To confirm this, let us derive the social optimal allocation in the product-variety expansion model of growth without any distortion. This is examined by solving the following planning problem. If we focus on the symmetric equilibrium, the objective function for the planner is given by

$$U=\int_0^\infty e^{-\rho t}\log C dt=\int_0^\infty e^{-\rho t}\left[\frac{1}{\alpha-1}\log n-\theta\log s+\log L_f-\log b\right]dt.$$

In the above, we use $C=n^{\frac{\alpha}{\alpha-1}}s^{-\theta}c$ and $nc=L_f/b$. We assume that the planner maximizes U by controlling labor allocation to production, L_f, subject to

$$\dot{n}=\delta n\left(N-L_f\right), \tag{28}$$

$$\dot{s}=\beta\left(\frac{L_f}{bn}-s\right), \tag{29}$$

and the initial values of n and s.

After some manipulation, we find that the above set of equations can be summarized as a single equation such that

$$\frac{\delta}{(\rho-\delta)(\alpha-1)}L_f=\frac{\rho b}{\beta(\rho-\delta)}+\frac{\theta\left[\beta-\delta\left(N-L_f\right)\right]}{\delta(N-L_f)-(\rho+\beta)}. \tag{30}$$

[9] Remember that we are concerned with the case of negative externalities ($\theta>0$) so that λ has a negative value.

A positive solution of this equation gives the steady-state level of L_f in the socially optimum balanced-growth path in which every distortion is internalized. Suppose that (30) has a unique solution in between 0 and N. We can confirm that the steady-state value of L_f determined by (30) would be larger than L_f^* given by (20). Thus the competitive level of R&D is not necessarily smaller than the optimal level of R&D that realizes the social optimum. This finding as well as one shown in the previous subsection, indicate that we need careful consideration as to the policy recommendation in the R&D-based growth model if the consumers' preferences involve external habit formation.

6 Conclusion

This paper has introduced commodity-specific external effects into one of the standard models of endogenous growth in which continuing growth is sustained by expansion of product variety. We have shown that the presence of consumption externalities may significantly affect both the balanced-growth equilibrium and transitional dynamics of the economy. In addition, the scale effect, the effect of R&D subsidy and the characterization of efficient growth in our setting would be fundamentally different form those obtained in the standard model without consumption externalities. Obviously, unlike production externalities, the presence of consumption externalities cannot be the main engine of growth. Our study have, however, demonstrated that they may yield significant implications for growing economies in both positive and normative senses.

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