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Global Solution to the One-Dimensional Equations for a Self-Gravitating Viscous Radiative and Reactive Gas

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1 Introduction.

We consider the one-dimensional motion of a compressible, viscous and heat conductive gas driven by the self-gravitation in the free-boundary case. In addition to this situation, we take into account the energy producing process inside the medium, that is, the gas consists of a reacting mixture and the combustion process is current at the high temperature stage.

The motion mentioned above is described by the following four equations in the Euler coordinate system corresponding to the conservation laws of mass, momentum and energy, and an equation of reaction-diffusion type:

$$\begin{cases} \rho_t + v\rho_y &= -\rho v_y, \\ \rho(v_t + vv_y) &= (-p+\mu v_y)_y + \rho f, \\ \rho(e_t + ve_y) &= (\kappa\theta_y)_y + (-p+\mu v_y)v_y + \lambda\phi z, \\ \rho(z_t + vz_y) &= (d\rho z_y)_y - \phi z \end{cases} \tag{1.1}$$

in $\bigcup_{t>0}\big(\Omega(t)\times\{t\}\big)$, where $\Omega(t) := \big\{y\in\mathbb{R}\,|\,y_1(t)\le y\le y_2(t)\big\}$ and $y_i(\cdot)$ for $i=1,2$ are fluctuating boundary functions. Here the density $\rho=\rho(y,t)$, the velocity $v=v(y,t)$, the absolute temperature $\theta=\theta(y,t)$ and the mass fraction of the reactant $z=z(y,t)$ are the unknown functions, and positive constants μ, d and λ are the coefficients of viscosity, the species diffusion and the difference in heat between the reactant and the product.

The external force per unit mass $f=f(y,t)$ is given by $f=-U_y$, where U is the solution of the boundary value problem

$$\begin{cases} U_{yy} = G\rho & \text{in } \bigcup_{t>0}\big(\Omega(t)\times\{t\}\big), \\ U|_{y=y_1(t)} = U|_{y=y_2(t)} = 0 & \text{for } t>0. \end{cases} \tag{1.2}$$

Here G is the Newtonian gravitational constant. The rate function $\phi=\phi(\theta)$ is defined by the Arrhenius law

$$\phi(\theta) = \theta^\beta e^{-\frac{A}{\theta}}, \tag{1.3}$$

where A is the activation energy (a positive constant) and β is a non-negative number. At high tempereture regimes, pressure $p = p(\rho, \theta)$ and internal energy $e = e(\rho, \theta)$ are given by $p = p_G + p_R$ and

$$e = C_{\mathrm{v}}\theta + a\frac{\theta^4}{\rho}$$

with the specific heat at constant volume (positive constant) C_{v}, the Stefan-Boltzmann constant $a > 0$, respectively. Here $p_G = p_G(\rho, \theta)$ is the *gaseous* (elastic and thermal) pressure, which is described for example, by Benedict-Webb-Rubin equation of state [21]

$$\begin{aligned} p_G = R\rho\theta &+ \left(RB_0\theta - A_0 - C_0\theta^{-2}\right)\rho^2 \\ &+ \left(Rb_2\theta - b_1\right)\rho^3 + b_1\alpha\rho^6 + b_3\theta^{-2}\rho^3\left(1+\gamma\rho^2\right)\mathrm{e}^{-\gamma\rho^2} \end{aligned}$$

with the perfect gas constant R and positive constants $A_0, B_0, C_0, b_1, b_2, b_3, \alpha, \gamma$ dependent on the concrete media, and $p_R = p_R(\rho, \theta)$ is the *radiative* pressure given by Stefan law [14]

$$p_R = \frac{a}{3}\theta^4.$$

For technical reason, we neglect the terms except the first in the right hand side of p_G. We also assume the conductivity $\kappa = \kappa(\rho, \theta)$ has the following form (see for example, [1], [8]):

$$\kappa = \kappa_1 + \kappa_2\frac{\theta^q}{\rho},$$

where κ_1, κ_2 and q are positive constants.

We impose the dynamical and kinematic boundary conditions for $i = 1, 2$

$$\begin{cases} (-p + \mu v_x)|_{y=y_i(t)} = -p_e & \text{for } t > 0, \\ \dfrac{\mathrm{d}y_i(t)}{\mathrm{d}t} = v(y_i(t), t) & \text{for } t > 0, \end{cases}$$

where the positive constant p_e is the external pressure, and the thermal and chemical boundary conditions for $i = 1, 2$

$$\begin{cases} (\kappa\theta_y)|_{y=y_i(t)} = 0 & \text{for } t > 0, \\ (d\rho z_y)|_{y=y_i(t)} = 0 & \text{for } t > 0, \end{cases}$$

and the initial conditon

$$(\rho, v, \theta, z)|_{t=0} = \left(\rho_0(y), v_0(y), \theta_0(y), z_0(y)\right) \quad \text{for } y \in \overline{\Omega(0)}.$$

We introduce the Lagrangian transformation. For arbitrary fixed point $(y,t) \in \bigcup_{t\geq 0} \left(\overline{\Omega(t)} \times \{t\}\right)$, we consider the solution curve $Y_{y,t}(\tau)$ of the Cauchy problem

$$\begin{cases} \dfrac{\mathrm{d}Y_{y,t}(\tau)}{\mathrm{d}\tau} = v\left(Y_{y,t}(\tau),\tau\right) \quad \text{for } 0<\tau<t, \\ Y_{y,t}(t) = y. \end{cases}$$

The unique existence of such a solution curve is guaranteed from the fundamental existence theorem of an ordinary differential equation as long as v is suitably smooth. Let $Y_{y,t}(0)=\xi$. Then this is uniquely solvable in y,

$$y = Y_{\xi,0}(t) = \xi + \int_0^t v\left(Y_{\xi,0}(\tau),\tau\right)\mathrm{d}\tau.$$

It is well known that the kinematic boundary condition implies that for each $t\geq 0$ this mapping $(y,t)\mapsto(\xi,t)$ is one-to-one from $\overline{\Omega(t)}\times\{t\}$ onto $\overline{\Omega(0)}\times\{t\}$. We put $y_1(0)=0$ and $y_2(0)=L$. Futhermore, we introduce the mass transformation

$$\xi \mapsto x = \int_0^\xi \rho_0(s)\,\mathrm{d}s.$$

Then problem (1.2) is reduced to

$$\begin{cases} (\tilde{\rho}\tilde{U}_x)_x = G \quad \text{in } (0,M)\times(0,\infty), \\ \tilde{U}|_{x=0,\,M} = 0 \quad \text{for } t>0, \end{cases}$$

where $M=\int_0^L \rho_0(\xi)\,\mathrm{d}\xi$ and the tilde "~" means transformed functions. Through the relations $\tilde{f}=-\tilde{\rho}\tilde{U}_x$ we can get the explicit formula

$$\tilde{f}(x,t) = -G\left(x - \frac{\int_0^M \eta\tilde{\rho}(\eta,t)^{-1}\,\mathrm{d}\eta}{\int_0^M \tilde{\rho}(\eta,t)^{-1}\,\mathrm{d}\eta}\right). \tag{1.4}$$

Consequently, by putting $v(x,t):=1/\tilde{\rho}(x,t)$, $u(x,t):=\tilde{v}(x,t)$ and normalizing $M=1$ our problem becomes

$$\begin{cases} v_t = u_x, \\ u_t = \left(-p+\dfrac{\mu}{v}u_x\right)_x - G\left(x-\dfrac{\int_0^1 \eta v(\eta,t)\,\mathrm{d}\eta}{\int_0^1 v(\eta,t)\,\mathrm{d}\eta}\right), \\ e_t = \left(\dfrac{\kappa}{v}\theta_x\right)_x + \left(-p+\dfrac{\mu}{v}u_x\right)u_x + \lambda\phi z, \\ z_t = \left(\dfrac{d}{v^2}z_x\right)_x - \phi z \end{cases} \tag{1.5}$$

in $(0,1)\times(0,\infty)$ with the boundary conditions

$$\left(-p+\frac{\mu}{v}u_x,\ \frac{\kappa}{v}\theta_x,\ \frac{d}{v^2}z_x\right)\Big|_{x=0,\,1}=(-p_e,\ 0,\ 0)\quad \text{for } t>0, \tag{1.6}$$

and the initial condition

$$(v,u,\theta,z)|_{t=0}=\big(v_0(x),u_0(x),\theta_0(x),z_0(x)\big)\quad \text{for } x\in[0,1]. \tag{1.7}$$

One-dimensional problems have been studied under various conditions. For the viscous polytropic ideal gas a pioneering work of global in time existence with large initial data was due to Kazhikhov and Shelukhin [11] under Dirichlet boundary condition with respect to the velocity. In the free-boundary case, Nagasawa [15] discussed the global existence problem and the asymptotic behavior for the polytropic ideal gas with the external pressure depending on time. Dafermos and Hsiao [3], Kawohl [10] and Jiang [9] considered problems for some general fluids. Also Chen [2] studied a model equations for a reacting mixture. All works mentioned above were not taken into account the influence of an external force.

Ducomet [4] - [7] treated a one-dimensional self-gravitating gaseous model as some large-scale structure of the universe, called "pancakes" in the astrophysical literature (see [12], [17]). Following the spirit of [17], he adopted as the self-gravitational term

$$\tilde{f}(x,t)=-G\left(x-\frac{1}{2}M\right)$$

not the exact form (1.4), and also assumed that the initial data and the solution are symmetric.

Now, by integration of $(1.5)^2$ with respect to x over $[0,1]$ we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1 u\,\mathrm{d}x=-G\left(\frac{1}{2}-\frac{\int_0^1\eta v(\eta,t)\,\mathrm{d}\eta}{\int_0^1 v(\eta,t)\,\mathrm{d}\eta}\right). \tag{1.8}$$

Denoting $u-\int_0^1 u\,\mathrm{d}x$ by u again, we obtain the final form:

$$\left\{\begin{array}{rcl} v_t &=& u_x,\\ u_t &=& \left(-p+\dfrac{\mu}{v}u_x\right)_x-G\left(x-\dfrac{1}{2}\right),\\ e_t &=& \left(\dfrac{\kappa}{v}\theta_x\right)_x+\left(-p+\dfrac{\mu}{v}u_x\right)u_x+\lambda\phi z,\\ z_t &=& \left(\dfrac{d}{v^2}z_x\right)_x-\phi z \end{array}\right. \tag{1.9}$$

in $(0,1)\times(0,\infty)$ with the same initial-boundary conditions (1.6) and (1.7). For this system it is natural that initial function u_0 (which corresponds to $u_0-\int_0^1 u_0\,\mathrm{d}x$ for the original system (1.5)) satisfies

$$\int_0^1 u_0 \, \mathrm{d}x = 0. \tag{1.10}$$

In this paper we construct the unique global classical solution of system (1.9), (1.6), (1.7) with the equations of state

$$p = R\frac{\theta}{v} + \frac{a}{3}\theta^4, \qquad e = C_v\theta + av\theta^4 \tag{1.11}$$

and the conductivity

$$\kappa = \kappa_1 + \kappa_2 v\theta^q, \tag{1.12}$$

without the symmetric assumption to the initial data and the solution. From (1.8) it is easily seen that this solution leads to the one for the original problem (1.5)-(1.7) describing the exact one-dimensional self-gravitating fluid model, not the approximated one : "pancakes" which has been considered by Ducomet. The difficulty of our problem is mainly caused by radiative components of equations of state and θ-dependency of the conductivity. We can solve the problem only for the case of some $q \geq 4$, which is physically valid [22]. Similar result has been obtained in [7], but the proof in it is not clear for the authors.

Let $\Omega := (0,1)$, m be a nonnegative integer and $0 < \sigma < 1$. T is a positive constant and $Q_T := \Omega \times (0,T)$. We denote

$$|u|^{(0)} := \sup_{(x,t)\in Q_T} |u(x,t)|$$

and use the familiar notations $C^{m+\sigma}(\Omega)$, $C^{\sigma,\sigma/2}_{x,t}(Q_T)$, $C^{2+\sigma,1+\sigma/2}_{x,t}(Q_T)$ for the Hölder spaces (see for example, [13]).

Our main result is

Theorem 1 (Global Solution) *Let $\alpha \in (0,1)$, $4 \leq q \leq 16$ and $0 \leq \beta \leq 13/2$. Assume that*

$$(v_0, u_0, \theta_0, z_0) \in C^{1+\alpha}(\Omega) \times \left(C^{2+\alpha}(\Omega)\right)^3 \tag{1.13}$$

satisfies the compatibility conditions, (1.10) *and*

$$v_0(x),\ \theta_0(x) > 0, \quad 0 \leq z_0(x) \leq 1 \quad \textit{for } x \in \overline{\Omega}. \tag{1.14}$$

Then there exists a unique solution (v,u,θ,z) of the initial-boundary value problem (1.9), (1.6), (1.7) *with* (1.3), (1.11), (1.12) *such that for any $T > 0$*

$$(v,\ v_x, v_t) \in \left(C^{\alpha,\alpha/2}_{x,t}(Q_T)\right)^3, \quad (u,\theta,z) \in \left(C^{2+\alpha,1+\alpha/2}_{x,t}(Q_T)\right)^3, \tag{1.15}$$

$$v(x,t),\ \theta(x,t) > 0, \quad 0 \leq z(x,t) \leq 1 \quad \textit{for } (x,t) \in \overline{Q_T}. \tag{1.16}$$

Proof of Theorem 1 is based on the local existence theorem and a priori estimates. The fundamental theorem about the existence and the uniqueness of the local in time solution in three-dimensional case was firstly established by Tani [19], [20] under sufficiently general initial-boundary conditions. For a radiative fluid, Secchi [16] obtained the corresponding result.

We can easily obtain suitable unique local solution to our problem in the same manner as these works (see for example, [18]). Therefore, to prove Theorem 1 it is sufficient to establish the following a priori boundedness.

Proposition 1 (A priori estimates) *Let T be an arbitrary positive constant, $4 \leq q \leq 16$ and $0 \leq \beta \leq 13/2$. Assume that the initial data satisfy the hypotheses of Theorem 1 and problem (1.9), (1.6), (1.7) with (1.3), (1.11), (1.12) has a solution (v, u, θ, z) such that*

$$(v,\ v_x, v_t) \in \left(C_{x,t}^{\alpha,\alpha/2}(Q_T)\right)^3, \quad (u, \theta, z) \in \left(C_{x,t}^{2+\alpha,1+\alpha/2}(Q_T)\right)^3. \tag{1.17}$$

Then there exists a positive constant M depending on the initial data and T such that

$$|v,\ v_x, v_t|_{\alpha,\alpha/2},\ |u, \theta, z|_{2+\alpha,1+\alpha/2} \leq M, \tag{1.18}$$

$$v(x,t),\ \theta(x,t) \geq 1/M, \quad 0 \leq z(x,t) \leq 1 \quad \text{for } (x,t) \in \overline{Q_T}. \tag{1.19}$$

2 Proof of Proposition 1.

In proving Proposition 1, we need several lemmas concerning the estimates of the solution and its derivatives. Our methods are mainly based on the techniques in Kawohl [10] and Jiang [9]. We use C as positive constants, and $\|\cdot\|$ denotes usual L^2 norm.

Lemma 1 *For any $t \in [0,T]$*

$$\begin{aligned}\int_0^1 \left(\frac{1}{2}u^2 + e + \lambda z + f(x)v\right) \mathrm{d}x \\ = \int_0^1 \left(\frac{1}{2}{u_0}^2 + e_0 + \lambda z_0 + f(x)v_0\right) \mathrm{d}x := E_0,\end{aligned} \tag{2.1}$$

$$U(t) + \int_0^t V(\tau)\, \mathrm{d}\tau \leq C, \tag{2.2}$$

$$\int_0^1 \frac{1}{2}z^2\, \mathrm{d}x + \int_0^t \int_0^1 \left(\frac{d}{v^2}{z_x}^2 + \phi z^2\right) \mathrm{d}x\, \mathrm{d}\tau = \int_0^1 \frac{1}{2}{z_0}^2\, \mathrm{d}x. \tag{2.3}$$

Here $e_0 := C_v\theta_0 + av_0{\theta_0}^4$, $f(x) := p_e + \frac{1}{2}Gx(1-x)$ and

$$\begin{cases} U(t) &:= \displaystyle\int_0^1 \Big[C_v(\theta - 1 - \log\theta) + R(v - 1 - \log v)\Big]\, \mathrm{d}x, \\ V(t) &:= \displaystyle\int_0^1 \left(\frac{\mu {u_x}^2}{v\theta} + \frac{\kappa{\theta_x}^2}{v\theta^2} + \lambda\frac{\phi}{\theta}z\right) \mathrm{d}x. \end{cases}$$

Proof. It is easy to see from (1.9) and (1.6)

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1 \left(\frac{1}{2}u^2 + f(x)v\right)\mathrm{d}x + \int_0^1 \frac{\mu}{v}{u_x}^2\,\mathrm{d}x = \int_0^1 pu_x\,\mathrm{d}x \tag{2.4}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1 (e+\lambda z)\,\mathrm{d}x = \int_0^1 \left(-p + \frac{\mu}{v}u_x\right)u_x\,\mathrm{d}x.$$

Adding these and integrating over $[0,t]$, we obtain (2.1).

Rewriting $(1.9)^3$ as

$$e_\theta\theta_t + \theta p_\theta u_x = \frac{\mu}{v}{u_x}^2 + \left(\frac{\kappa}{v}\theta_x\right)_x + \lambda\phi z \tag{2.5}$$

and multiplying this by θ^{-1}, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(C_{\mathrm{v}}\log\theta + R\log v + \frac{4}{3}av\theta^3\right) = \frac{\mu{u_x}^2}{v\theta} + \frac{1}{\theta}\left(\kappa\frac{\theta_x}{v}\right)_x + \lambda\frac{\phi}{\theta}z.$$

Integrating this over $(0,1)\times(0,t)$ yields

$$U(t) + \int_0^t V(\tau)\,\mathrm{d}\tau \le C\left(1 + \int_0^1 v\theta^3\,\mathrm{d}x\right).$$

From Hölder's inequality

$$\int_0^1 v\theta^r\,\mathrm{d}x \le \left(\int_0^1 v\theta^4\,\mathrm{d}x\right)^{r/4}\left(\int_0^1 v\,\mathrm{d}x\right)^{(4-r)/4} \quad \text{for } 0\le r\le 4 \tag{2.6}$$

and (2.1), (2.2) follows.

Equality (2.3) is easily obtained by integrating $(1.9)^4$ over $(0,1)\times(0,t)$ and using (1.6). □

Next lemma concerning the pointwise estimate of z is obtained in the same manner as Chen [2]. We omit the proof.

Lemma 2 *For any* $(x,t)\in\overline{Q_T}$

$$0\le z(x,t)\le 1. \tag{2.7}$$

Kazhikhov and Shelukhin firstly derived the useful representation formula for v. In the present case, we can obtain the following similar form (see [11]).

Lemma 3 *The identity*

$$v(x,t) = \frac{1}{B(x,t)Y(x,t)D(x,t)} \times\left(v_0 + \int_0^t \frac{R}{\mu}\theta(x,\tau)B(x,\tau)Y(x,\tau)D(x,\tau)\,\mathrm{d}\tau\right) \tag{2.8}$$

holds, where

$$B(x,t) := \exp\left[\frac{1}{\mu}\int_0^x \Big(u_0(\xi) - u(\xi,t)\Big)\,\mathrm{d}\xi\right], \quad Y(x,t) := \exp\left(\frac{1}{\mu}f(x)t\right),$$
$$D(x,t) := \exp\left(-\frac{a}{3\mu}\int_0^t \theta(x,\tau)^4\,\mathrm{d}\tau\right).$$

Since $\left(\mu\frac{u_x}{v}\right)_x = \mu(\log v)_{xt}$ follows from $(1.9)^1$, integrating $(1.9)^2$ over $(0,x)\times(0,t)$ yields

$$\frac{1}{\mu}\int_0^x (u-u_0)\,\mathrm{d}\xi = \log v - \log v_0 - \frac{1}{\mu}\int_0^t p\,\mathrm{d}\tau + \frac{1}{\mu}f(x)t.$$

From this one can easily find the lower bound of v:

$$\min_{(x,t)\in\overline{Q_T}} v(x,t) \ge \min_{x\in\overline{\Omega}} v_0(x)\exp\left\{-\frac{1}{\mu}\left[4E_0^{1/2} + \left(p_e + \frac{G}{8}\right)t\right]\right\}. \tag{2.9}$$

This together with (2.6) leads to

$$\int_0^1 \theta^r\,\mathrm{d}x \le C \quad \text{for } 0 \le r \le 4. \tag{2.10}$$

Lemma 4 *For any $t\in[0,T]$ and $q \ge 0$*

$$\int_0^t \max_{0\le x\le 1}\theta(x,\tau)^r\,\mathrm{d}\tau \le C, \quad 0\le r\le q+4. \tag{2.11}$$

Proof. For each $t\in[0,T]$, there exists $x^*(t)\in[0,1]$ such that

$$\theta(x^*(t),t) = \int_0^1 \theta\,\mathrm{d}x,$$

and therefore, for any $r\ge 0$ and $(x,t)\in Q_T$ we have

$$\begin{aligned}
\theta(x,t)^{r/2} &= \left(\int_0^1\theta\,\mathrm{d}x\right)^{r/2} + \frac{r}{2}\int_{x^*(t)}^x \theta(\xi,t)^{r/2-1}\theta_\xi(\xi,t)\,\mathrm{d}\xi \\
&\le C\left(1 + \int_0^1 \frac{\kappa^{1/2}|\theta_x|}{v^{1/2}\theta}\cdot\frac{v^{1/2}\theta^{r/2}}{\kappa^{1/2}}\,\mathrm{d}x\right) \\
&\le C\left[1 + \left(\int_0^1 \frac{v\theta^r}{1+v\theta^q}\,\mathrm{d}x\right)^{1/2} V(t)^{1/2}\right].
\end{aligned} \tag{2.12}$$

Since $\theta^r \le C(1+\theta^{q+4})$ holds for $0\le r\le q+4$, we have from (2.10)

$$\int_0^1 \frac{v\theta^r}{1+v\theta^q}\,\mathrm{d}x \le C\int_0^1 (v+\theta^4)\,\mathrm{d}x \le C,$$

which yields from (2.12) and (2.2)

$$\int_0^t \max_{0\le x\le 1} \theta(x,\tau)^r \,\mathrm{d}\tau \le C\int_0^t \Big(1+V(\tau)\Big)\,\mathrm{d}\tau \le C. \quad \square$$

Lemma 5 *For any* $(x,t)\in\overline{Q_T}$

$$C^{-1}\le v(x,t)\le C. \tag{2.13}$$

Proof. The lower bound of v is already obtained in (2.9). For the upper bound of v we use representaiton (2.8).

From (2.2) and Jensen's inequality we have

$$\int_0^1 \theta\,\mathrm{d}x - \log\int_0^1 \theta\,\mathrm{d}x - 1 \le \frac{C}{C_\mathrm{v}}. \tag{2.14}$$

Then, by applying the mean value theorem there exists a point $x^{**}(x,t)\in[0,1]$ for each fixed $t\in[0,T]$ such that

$$\theta(x^{**}(t),t) - \log\theta(x^{**}(t),t) - 1 \le \frac{C}{C_\mathrm{v}}. \tag{2.15}$$

Here we define α_0 and β_0 $(0<\alpha_0<\beta_0)$ as two positive roots of the equation

$$y-\log y - 1 = \frac{C}{C_\mathrm{v}},$$

which are independent of t. We get from (2.14) and (2.15)

$$\alpha_0\le\int_0^1\theta\,\mathrm{d}x\le\beta_0,\qquad \alpha_0\le\theta(x^{**}(t),t)\le\beta_0.$$

From (2.1) it is easily seen that for any $(x,t)\in\overline{Q_T}$

$$C^{-1}\le B(x,t)\le C.$$

On the other hand, since

$$\theta(x,t)^2 = \theta(x^{**}(t),t)^2 + 2\int_{x^{**}(t)}^x \theta(\xi,t)\theta_\xi(\xi,t)\,\mathrm{d}\xi,$$

we have

$${\alpha_0}^2 - CV(t)^{1/2}\le\theta(x,t)^2\le{\beta_0}^2+CV(t)^{1/2},$$

hence

$$\frac{1}{2}{\alpha_0}^4 - CV(t)\le\theta(x,t)^4\le 2{\beta_0}^4 + CV(t). \tag{2.16}$$

(2.12) with $r = 2$ implies

$$C - CV(t) \leq \theta(x,t) \leq C\Big(1 + V(t)\Big). \tag{2.17}$$

Let us decompose $v(x,t) = v_1(x,t) + v_2(x,t)$, where

$$v_1(x,t) = \frac{v_0(x)}{B(x,t)Y(x,t)D(x,t)},$$

$$v_2(x,t) = \frac{R}{\mu}\int_0^t \frac{B(x,\tau)Y(x,\tau)D(x,\tau)}{B(x,t)Y(x,t)D(x,t)}\theta(x,\tau)\,\mathrm{d}\tau.$$

Using (2.16), we immediately obtain

$$C\,\mathrm{e}^{-\left(f(x)-\frac{1}{6}a\alpha_0{}^4\right)\frac{t}{\mu}} \leq v_1(x,t) \leq C\,\mathrm{e}^{-\left(f(x)-\frac{2}{3}a\beta_0{}^4\right)\frac{t}{\mu}}. \tag{2.18}$$

Also (2.16) and (2.17) yields

$$v_2(x,t) \leq C\int_0^t \mathrm{e}^{-\frac{1}{\mu}\left(f(x)-\frac{2}{3}a\beta_0{}^4\right)(t-\tau)}\Big(1+V(\tau)\Big)\,\mathrm{d}\tau \tag{2.19}$$

$$\leq C\,\mathrm{e}^{CT}\int_0^t \Big(1+V(\tau)\Big)\,\mathrm{d}\tau. \tag{2.20}$$

Consequently, from (2.18) and (2.20) the upper bound of v follows. $\square$

Lemma 6 *If $q \geq 2$, for any $t \in [0,T]$*

$$\|v_x\|^2 + \int_0^t\int_0^1 \theta {v_x}^2\,\mathrm{d}x\,\mathrm{d}\tau \leq C. \tag{2.21}$$

Proof. $(1.9)^2$ implies

$$\left(u - \frac{\mu}{v}v_x\right)_t = -p_x - G\left(x - \frac{1}{2}\right).$$

Multiplying this by $u - \frac{\mu}{v}v_x$ and integrating over [0,1], we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1 \frac{1}{2}\left(u - \frac{\mu}{v}v_x\right)^2\mathrm{d}x + \int_0^1 \frac{\mu R}{v^3}\theta {v_x}^2\,\mathrm{d}x$$
$$= \int_0^1 \frac{R}{v^2}u\theta v_x\,\mathrm{d}x - \int_0^1 \left[\left(\frac{R}{v} + \frac{4}{3}a\theta^3\right)\theta_x + G\left(x - \frac{1}{2}\right)\right]\left(u - \frac{\mu}{v}v_x\right)\mathrm{d}x. \tag{2.22}$$

Firstly, we have for any $\varepsilon > 0$

$$\left|\int_0^1 \frac{R}{v^2}u\theta v_x\,\mathrm{d}x\right| \leq \varepsilon\int_0^1 \theta {v_x}^2\,\mathrm{d}x + C_\varepsilon \max_{0\leq x\leq 1}\theta\cdot\int_0^1 u^2\,\mathrm{d}x,$$

where C_ε is a positive number depending on ε. The second term of the right hand side of (2.22) is estimated as follows.

$$\begin{aligned}
&\left|\int_0^1 \left[\left(\frac{R}{v}+\frac{4}{3}a\theta^3\right)\theta_x + G\left(x-\frac{1}{2}\right)\right]\left(u-\frac{\mu}{v}v_x\right)\mathrm{d}x\right| \\
&\quad \le C\left[\int_0^1 \frac{\kappa{\theta_x}^2}{v\theta^2}\,\mathrm{d}x + \int_0^1 \frac{\theta^2(1+\theta^3)^2}{\kappa}\left(u-\frac{\mu}{v}v_x\right)^2\mathrm{d}x\right] \\
&\quad \le C\left[V(t) + \max_{0\le x\le 1}\left(1+\theta^2+\frac{\theta^8}{1+\theta^q}\right)\cdot\int_0^1\left(u-\frac{\mu}{v}v_x\right)^2\mathrm{d}x\right].
\end{aligned}$$

Since

$$\int_0^t \max_{0\le x\le 1}\frac{\theta^8}{1+\theta^q}\,\mathrm{d}\tau \le C$$

holds with $q\ge 2$, by taking ε small, Gronwall's inequality implies (2.21). □

Lemma 7 *For any $t\in[0,T]$*

$$\int_0^t \|u_x\|^2\,\mathrm{d}\tau \le C. \tag{2.23}$$

Proof. Integrate (2.4) with respect to t and using Lemma 5 and Cauchy-Schwarz inequality, we have

$$\begin{aligned}
\int_0^t \|u_x\|^2\,\mathrm{d}\tau &\le C\left(1+\int_0^t\int_0^1(\theta+\theta^4)^2\,\mathrm{d}x\,\mathrm{d}\tau\right) \\
&\le C\left[1+\int_0^t\left(\int_0^1\theta^2\,\mathrm{d}x + \max_{0\le x\le 1}\theta^4\cdot\int_0^1\theta^4\,\mathrm{d}x\right)\mathrm{d}\tau\right].
\end{aligned}$$

The right hand side is bounded by (2.10) and (2.11). □

Lemma 8 *If $q\ge 4$, for any $t\in[0,T]$*

$$\int_0^t \|u_x\|^3_{L^3(\Omega)}\,\mathrm{d}\tau \le C. \tag{2.24}$$

Proof. We use a method due to Dafermos and Hsiao [3]. Putting $w=\int_0^x u\,\mathrm{d}\xi$ and using (1.10), we get a new system:

$$\begin{cases}
w_t = \dfrac{\mu}{v}w_{xx} - p + f(x) & \text{in } Q_T, \\
w|_{t=0} = w_0(x) := \int_0^x u_0(\xi)\,\mathrm{d}\xi & \text{for } x\in[0,1], \\
w|_{x=0,1} = 0 & \text{for } t\in[0,T].
\end{cases}$$

General theory of linear parabolic equations (see for example, [13]) gives

$$\int_0^t \|w_{xx}\|^3_{L^3(\Omega)}\,\mathrm{d}t \le C\Big(\int_0^t \|-p+f(x)\|^3_{L^3(\Omega)}\,\mathrm{d}t + \|w_0\|_{W_3^{4/3}(\Omega)}\Big).$$

Therefore, we have

$$\begin{aligned}\int_0^t \|u_x\|^3_{L^3(\Omega)}\,\mathrm{d}\tau &\le C\left(1+\int_0^t \|p\|^3_{L^3(\Omega)}\,\mathrm{d}\tau\right)\\ &\le C\left[1+\int_0^t\left(\int_0^1 \theta^3\,\mathrm{d}x + \max_{0\le x\le 1}\theta^8\cdot\int_0^1\theta^4\,\mathrm{d}x\right)\mathrm{d}\tau\right].\end{aligned}$$

If $q \ge 4$, the right hand side is bounded. □

As [10] and [9], we introduce the quantities

$$X := \int_0^t\int_0^1 \left(1+\theta^{q+3}\right)\theta_t{}^2\,\mathrm{d}x\,\mathrm{d}\tau, \quad Y := \max_{0\le t\le T}\int_0^1 \left(1+\theta^{q+4}\right)\theta_x{}^2\,\mathrm{d}x$$

$$Z := \max_{0\le t\le T}\int_0^1 u_{xx}{}^2\,\mathrm{d}x.$$

Firstly, by Cauchy-Schwarz inequality we have

$$|\theta|^{(0)} \le C\left(1+Y^{\frac{1}{q+10}}\right). \tag{2.25}$$

Secondly, it is easily seen that by interpolation

$$\max_{0\le t\le T}\|u_x\|^2 \le C\left(1+Z^{1/2}\right) \tag{2.26}$$

holds and for any $t\in[0,T]$

$$\begin{aligned}\max_{0\le x\le 1} u_x{}^2 &\le \|u_x\|^2 + 2\|u_x\|\|u_{xx}\|\\ &\le C\left(1+Z^{3/4}\right),\end{aligned}$$

which gives

$$|u_x|^{(0)} \le C\left(1+Z^{3/8}\right). \tag{2.27}$$

In the same manner as in [10] and [9] we introduce the function

$$K(v,\theta) := \int_0^\theta \frac{\kappa(v,\xi)}{v}\,\mathrm{d}\xi.$$

Multiplying (2.5) by K_t and integrating it over $(0,1)\times(0,t)$, we have

$$\begin{aligned}&\int_0^t\int_0^1\left(e_\theta\theta_t+\theta p_\theta u_x-\frac{\mu}{v}u_x{}^2\right)K_t\,\mathrm{d}x\,\mathrm{d}\tau + \int_0^t\int_0^1\frac{\kappa}{v}\theta_x K_{xt}\,\mathrm{d}x\,\mathrm{d}\tau\\ &=\int_0^t\int_0^1 \lambda\phi z K_t\,\mathrm{d}x\,\mathrm{d}\tau.\end{aligned} \tag{2.28}$$

Here

$$\begin{cases} K_t = \dfrac{\kappa}{v}\theta_t + K_v u_x, \\ K_{xt} = \left(\dfrac{\kappa}{v}\theta_x\right)_t + K_{vv} v_x u_x + K_v u_{xx} + \left(\dfrac{\kappa}{v}\right)_v v_x \theta_t, \\ |K_v|,\ |K_{vv}| \le C\theta. \end{cases}$$

Estimating each term in (2.28), we get the following.

Lemma 9 *If* $4 \le q \le 16$ *and* $0 \le \beta \le 13/2$, *we have*

$$X + Y \le C\left(1 + Z^{7/8}\right). \tag{2.29}$$

Sketch of proof. At first, note that

$$\begin{aligned} \int_0^t \int_0^1 e_\theta \theta_t \left(\frac{\kappa}{v}\theta_t\right) \mathrm{d}x\, \mathrm{d}\tau &\ge C \int_0^t \int_0^1 \left(1+\theta^3\right)\left(1+\theta^q\right) {\theta_t}^2 \,\mathrm{d}x\, \mathrm{d}\tau \\ &\ge CX, \end{aligned}$$

$$\begin{aligned} \int_0^t \int_0^1 \frac{\kappa}{v}\theta_x \left(\frac{\kappa}{v}\theta_x\right)_t \mathrm{d}x\, \mathrm{d}\tau &= \frac{1}{2}\int_0^1 \left(\frac{\kappa}{v}\theta_x\right)^2 \mathrm{d}x - \frac{1}{2}\int_0^1 \left(\frac{\kappa}{v}\theta_x\right)^2 (x,0)\,\mathrm{d}x \\ &\ge CY - C. \end{aligned}$$

Other terms are also estimated from (2.25)-(2.27), Lemmas 4, 6, 7 and 8, for example,

$$\begin{aligned} &\left| \int_0^t \int_0^1 \left(\theta p_\theta u_x - \frac{\mu}{v}{u_x}^2\right) \frac{\kappa}{v}\theta_t \,\mathrm{d}x\, \mathrm{d}\tau \right| \\ &\quad \le \varepsilon X + C_\varepsilon |{u_x}^2|^{(0)} \int_0^t \max_{0\le x\le 1} (1+\theta)^{q+1} \int_0^1 (1+\theta)^4 \,\mathrm{d}x\, \mathrm{d}\tau \\ &\qquad\qquad + C_\varepsilon \left|1+\theta^{q-3}\right|^{(0)} |u_x|^{(0)} \int_0^t \|u_x\|_3{}^3 \,\mathrm{d}\tau \\ &\quad \le \varepsilon X + C_\varepsilon \left(1 + Z^{3/4} + Y^{\frac{q-3}{q+10}} Z^{3/8}\right) \\ &\quad \le \varepsilon (X+Y) + C_\varepsilon \left(1 + Z^{3/4}\right), \end{aligned}$$

$$\begin{aligned} \left| \int_0^t \int_0^1 \frac{\kappa}{v}\theta_x K_v u_{xx} \,\mathrm{d}x\, \mathrm{d}\tau \right| &\le C|\kappa^{1/2}\theta^2|^{(0)} Z^{1/2} \int_0^t V(\tau)^{1/2} \,\mathrm{d}\tau \\ &\le C\left(1 + Y^{\frac{q+4}{2(q+10)}} Z^{1/2}\right) \\ &\le \varepsilon Y + C_\varepsilon \left(1 + Z^{7/8}\right). \end{aligned}$$

Combining these estimates and taking ε small, we obtain (2.29). $\square$

Lemma 10 *If* $4 \le q \le 16$ *and* $0 \le \beta \le 13/2$, *we have for any* $t \in [0,T]$

$$\|u_x\|^2 + \|\theta_x\|^2 + \|u_{xx}\|^2 + \|u_t\|^2 + \int_0^t \left(\|\theta_t\|^2 + \|u_{xt}\|^2\right) \mathrm{d}\tau \le C, \tag{2.30}$$

and

$$|u_x|^{(0)} + |u|^{(0)} + |\theta|^{(0)} \le C. \tag{2.31}$$

Proof. The following calculations are formal because the regularity of the solution is not sufficient. But as usual we can easily derive the rigorous results by using the arguments of mollifiers and passing to the limit.

Differentiating $(1.9)^2$ with respect to t, multiplying it by u_t and integrating it with respect to x, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1 \frac{1}{2}{u_t}^2\,\mathrm{d}x + \int_0^1 \frac{\mu}{v}{u_{xt}}^2\,\mathrm{d}x = \int_0^1 \left(p_t u_{xt} + \frac{\mu}{v^2}{u_x}^2 u_{xt}\right)\mathrm{d}x,$$

hence

$$\begin{aligned}
&\|u_t\|^2 + \int_0^t \|u_{xt}\|^2\,\mathrm{d}\tau \\
&\le C\left[1 + \int_0^t \left(\|p_t\|^2 + \|u_x\|^4_{L^4(\Omega)}\right)\mathrm{d}\tau\right] \\
&\le C\left[\int_0^t\int_0^1 \left(1+\theta^6\right){\theta_t}^2\,\mathrm{d}x\,\mathrm{d}\tau \right. \\
&\qquad \left. + \left|{u_x}^2\right|^{(0)}\int_0^t\int_0^1 \theta^2\,\mathrm{d}x\,\mathrm{d}\tau + |u_x|^{(0)}\int_0^t \|u_x\|^3_{L^3(\Omega)}\,\mathrm{d}\tau\right] \\
&\le C\left(1 + X + Z^{3/4} + Z^{3/8}\right) \\
&\le C\left(1 + Z^{7/8}\right)
\end{aligned} \tag{2.32}$$

by Lemma 9. From $(1.9)^2$ again together with the estimate above, it follows for any $t \in [0,T]$

$$\begin{aligned}
\|u_{xx}\|^2 &\le C\left[1 + \int_0^1 \left({u_t}^2 + {p_x}^2 + {u_x}^2{v_x}^2\right)\mathrm{d}x\right] \\
&\le C\left[1 + \|u_t\|^2 + \int_0^1 \left(1+\theta^6\right){\theta_x}^2\,\mathrm{d}x + \left(|\theta^2|^{(0)} + \left|{u_x}^2\right|^{(0)}\right)\|v_x\|^2\right] \\
&\le C\left(1 + Y^{\frac{2}{q+10}} + Y + Z^{3/4} + Z^{7/8}\right) \\
&\le C\left(1 + Z^{7/8}\right).
\end{aligned}$$

This means

$$Z \le C\left(1 + Z^{7/8}\right).$$

Therefore, we conclude that Z is bounded. Then one can see that X, Y, $|\theta|^{(0)}$, $\|u_x\|$, $|u_x|^{(0)}$, $\|u_t\|$ and $\int_0^t \|u_{xt}\|^2 \,\mathrm{d}\tau$ are also bounded from Lemma 9, (2.25)-(2.27), and (2.32).

The boundedness of u is easily derived from

$$|u|^{(0)} \leq \max_{0\leq t\leq T} \left(\|u\|_{L^1(\Omega)} + \|u_x\|\right). \quad \square$$

Lemma 11 *For any* $(x,t) \in \overline{Q_T}$

$$\theta(x,t) \geq C. \tag{2.33}$$

Proof. By putting $\Theta := \dfrac{1}{\theta}$, (2.5) becomes

$$e_\theta \Theta_t = \left(\frac{\kappa}{v}\Theta_x\right)_x + \frac{vp_\theta{}^2}{4\mu} - \left[\frac{2\kappa\Theta_x{}^2}{v\Theta} + \frac{\mu\Theta^2}{v}\left(u_x - \frac{vp_\theta}{2\mu\Theta}\right)^2 + \lambda\Theta^2\phi z\right],$$

from which by (2.31) there exists a positive constant C_1 such that

$$\Theta_t < \frac{1}{e_\theta}\left(\frac{\kappa}{v}\Theta_x\right)_x + C_1.$$

Standard comparison arguments imply

$$\min_{(x,t)\in\overline{Q_T}} \left(C_1 t + \max_{0\leq x\leq 1} \frac{1}{\theta_0(x)} - \Theta(x,t)\right) \geq 0. \quad \square$$

Lemma 12 *For any* $t \in [0,T]$

$$\|z_x\|^2 + \|z_{xx}\|^2 + \|z_t\|^2 + \int_0^t \|z_{xt}\|^2 \,\mathrm{d}\tau \leq C. \tag{2.34}$$

Proof. Multiplying $(1.9)^4$ by z_{xx} and integrating it over $[0,1]$, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1 \frac{1}{2}z_x{}^2 \,\mathrm{d}x + \int_0^1 \frac{d}{v^2}z_{xx}{}^2 \,\mathrm{d}x = \int_0^1 \left(\frac{2d}{v^2}v_x z_x z_{xx} + \phi z z_{xx}\right) \mathrm{d}x.$$

In a standard manner, we easily obtain the boundedness of $\|z_x\|$ and $\int_0^t \|z_{xx}\|^2 \,\mathrm{d}\tau$, which implies $\int_0^t \|z_t\|^2 \,\mathrm{d}\tau \leq C$ by $(1.9)^4$ again.

Next, we differentiate $(1.9)^4$ with respect to t, multiply it by z_t and integrate over $[0,1]$. Then we have

$$\|z_t\|^2 + \int_0^t \|z_{xt}\|^2 \,\mathrm{d}\tau \leq C\left[1 + \int_0^t \left(\|\theta_t\|^2 + \|z_x\|^2 + \|z_t\|^2\right) \mathrm{d}\tau\right],$$

whose right hand side is bounded. From $(1.9)^4$ again we obtain

$$\begin{aligned}\|z_{xx}\|^2 &\leq C\int_0^1 \left(1 + z_t{}^2 + z_x{}^2 v_x{}^2 + \phi^2\right) \mathrm{d}x \\ &\leq \varepsilon\|z_{xx}\|^2 + C_\varepsilon,\end{aligned}$$

which completes (2.34). $\square$

Lemma 13 *For any $t \in [0,T]$*

$$\|\theta_{xx}\|^2 + \|\theta_t\|^2 + \int_0^t \|\theta_{xt}\|^2 \,\mathrm{d}\tau \le C. \tag{2.35}$$

Proof. Differentiating (2.5) with respect to t, multiplying it by $e_\theta \theta_t$ and integrating over $[0,1]$, we have

$$\|\theta_t\|^2 + \int_0^t \|\theta_{xt}\|^2 \,\mathrm{d}\tau \le C\left[1 + \int_0^t \int_0^1 \left({v_x}^2 + {\theta_x}^2\right) {\theta_t}^2 \,\mathrm{d}x\,\mathrm{d}\tau\right].$$

Then, by virtue of the interpolation inequality

$$\max_{0\le x\le 1} \theta_t(x,t)^2 \le \varepsilon \|\theta_{xt}\|^2 + C_\varepsilon \|\theta_t\|^2,$$

we conclude that $\|\theta_t\|$ and $\int_0^t \|\theta_{xt}\|^2 \,\mathrm{d}\tau$ are bounded. Also from (2.5), one can obtain

$$\begin{aligned}
\|\theta_{xx}\|^2 &\le C \int_0^1 \frac{1}{\kappa^2} \left(1 + \theta^2 + {u_x}^2 + {u_x}^4 + {v_x}^2{\theta_x}^2 + {\theta_x}^4\right) \mathrm{d}x \\
&\le C\left[1 + \max_{0\le x\le 1} {\theta_x}^2 \cdot \int_0^1 \left({v_x}^2 + {\theta_x}^2\right) \mathrm{d}x\right] \\
&\le \varepsilon \|\theta_{xx}\|^2 + C_\varepsilon. \quad \square
\end{aligned}$$

Let us begin the Hölder estimate. From (2.30), (2.34), (2.35) and interpolations we obtain the bounds of $|\theta_x|^{(0)}$ and $|z_x|^{(0)}$. Therefore, u, θ, z are Lipschitz continuous in x. Applying Cauchy-Schwarz and interpolation inequalities, we have

$$\begin{aligned}
\left|u(x,t) - u(x,t')\right| &\le \left(\int_{t'}^t {u_t}^2 \,\mathrm{d}\tau\right)^{1/2} |t-t'|^{1/2} \\
&\le \left[\int_{t'}^t \left(\|u_t\|^2 + 2\|u_t\|\|u_{xt}\|\right) \mathrm{d}\tau\right]^{1/2} |t-t'|^{1/2}, \\
\left|u_x(x,t) - u_x(x',t)\right| &\le \left(\int_{x'}^x {u_{xx}}^2 \,\mathrm{d}\xi\right)^{1/2} |x-x'|^{1/2},
\end{aligned}$$

from which together with (2.30) u is Hölder continuous in t with exponent $1/2$ and u_x is also Hölder continuous in x with exponent $1/2$. Thus by a standard interpolation (see for example, [13], chapter II, Lemma 3.1) one can get $u_x \in C_{x,t}^{1/3,1/6}(Q_T)$. We can also get same Hölder estimates for θ, z. Recalling that $v_t = u_x$ and $v|_{t=0} = v_0 \in C^{1+\alpha}(\Omega)$, we deduce $v \in C_{x,t}^{1/3,1/6}(Q_T)$. Since it follows from (2.8) that

$$\begin{aligned}
v_x(x,t) = {} & \frac{1}{B(x,t)Y(x,t)D(x,t)} \Big\{ {v_0}'(x) - A(x,t)v_0(x) \\
& + \int_0^t \frac{R}{\mu}\Big[\theta_x(x,\tau) + \theta(x,\tau)\big(A(x,\tau) - A(x,t)\big)\Big] B(x,\tau)Y(x,\tau)D(x,\tau)\,\mathrm{d}\tau \Big\}
\end{aligned} \tag{2.36}$$

with

$$A(x,t) := \frac{1}{\mu}\left[u_0(x) - u(x,t) - \frac{4}{3}a\int_0^t \theta(x,\tau)^3\theta_x(x,\tau)\,\mathrm{d}\tau - G\left(x-\frac{1}{2}\right)t\right],$$

we can easily check $v_x \in C_{x,t}^{\sigma,\sigma/2}(Q_T)$ with $\sigma := \min\{\alpha, 1/3\}$.

Next we consider $(1.9)^2$, $(1.9)^3$ and $(1.9)^4$ as the linear parabolic equations:

$$\begin{cases} u_t - \frac{\mu}{v}u_{xx} + \left(\frac{\mu}{v^2}v_x\right)u_x = -\frac{R}{v}\theta_x + \frac{R}{v^2}\theta v_x - \frac{4}{3}a\theta^3\theta_x - G\left(x-\frac{1}{2}\right), \\ \theta_t - \frac{1}{e_\theta}\frac{\kappa}{v}\theta_{xx} - \frac{1}{e_\theta}\left[\left(\frac{\kappa}{v}\right)_\theta\theta_x + \left(\frac{\kappa}{v}\right)_v v_x\right]\theta_x + \left(\frac{p_\theta}{e_\theta}u_x\right)\theta = \frac{1}{e_\theta}\left(\frac{\mu}{v}{u_x}^2 + \lambda\phi z\right), \\ z_t - \frac{d}{v^2}z_{xx} + \left(\frac{2d}{v^3}v_x\right)z_x = -\phi \end{cases}$$

whose coefficients and right hand sides are Hölder continuous in x with exponent σ and in t with exponent $\sigma/2$. By the classical Schauder estimates (see for example, [13] or [3]) we obtain $|u,\theta,z|_{2+\sigma,1+\sigma/2} \le C$. This also implies that v, u_x, θ_x, z_x are Lipschitz continuous in x and Hölder in t with exponent $1/2$. Going back to (2.36), we obtain $v_x \in C_{x,t}^{\alpha,\alpha/2}(Q_T)$.

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