Evolution model described by iteration dynamical systems of discrete Laplacians on the plane lattice

(離散ラプラス作用素の反復力学系による進化モデル)

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Key words: dynamical system, discrete Laplacian, designs, evolution

Recently several authors have interests in the discretization of differential operators, for example, the Dirac operator and the Cauchy-Riemann operator ([4]). Also we know that the iteration dynamical system of a quadratic polynomial can describe many fluctuations ([2]). In this study we introduce a concept of a dynamical system defined by an iteration of a discrete Laplacian on the plane lattice and give several computer simulations. Then we consider the evolution of extinct animals. The several kinds of discrete Laplacians and their iteration dynamical systems can not be found in the literatures. Hence we may say that the introduction is quite new and original.

1. Iteration dynamical system of discrete Laplacian

We choose the lattice L on the real plane and consider {0,1}-valued functions on L. We calculate sums and products in mod 2 calculation rule. A set of cells which attach the referenced cell is called a neighborhood Up of p. We list several examples:
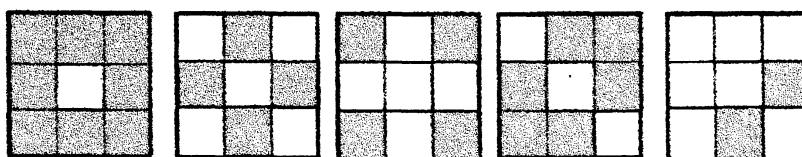
Moor   Neuman   Diag Neuman   Hexagonal   Sierpinski

(1) **(Discrete Laplacian)**Choosing a Up , we define the discrete Laplacian:

$$\Delta_{Up} f(p) = \sum_{q \in Up-p} (f(q) - f(p)).$$

(2)**(Iteration dyamical system of discrete Laplacian)**Choosing an initial function $f_0 \in F$, we define the dynamical system defined by the iteration of the Laplacian:

$$\{f_n\}, \quad f_n = \Delta_U f_{n-1} \ (n \geq 1).$$

(3)**(Source)** We call p a source (or seed) of the dynamical system when $f_n(p)=1$ for any $n \in N$. We regard the sources as boundary conditions.

**.2. Several examples of computer simulations([1])**

We give several computer simulations by suitable choices of sources and neighborhoods. We may expect to realize examples of evolutions and organizations by these simulators. We notice that we choose the lattice of a suitable size M with the periodicity condition:
(1) **Designs:** We can produce designs of carpets, laces and embroideries systematically choosing suitable neighborhoods. We have a software Designer KENTAURUS(2005).
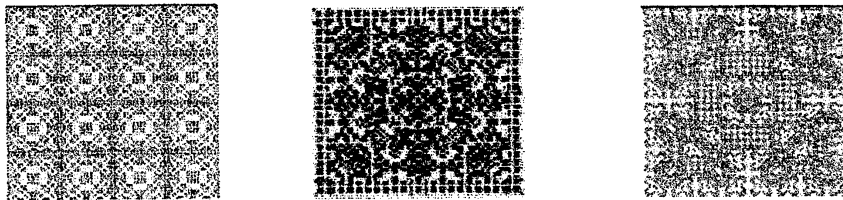

(2) **Crystals of water:** Hexagonal neigh. with a source can realize crystals of waters quite well. We may make a theory of the crystallization by this dynamical system..
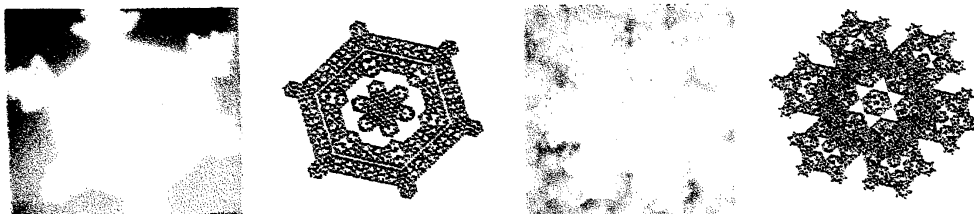

(2) **Growth of cities:** We can try to make simulations of the growth of cites and ecological systems. Here we give an example of the growth of city Numberg(the

right side is the city in 1882 and the left side is the computer simulation)
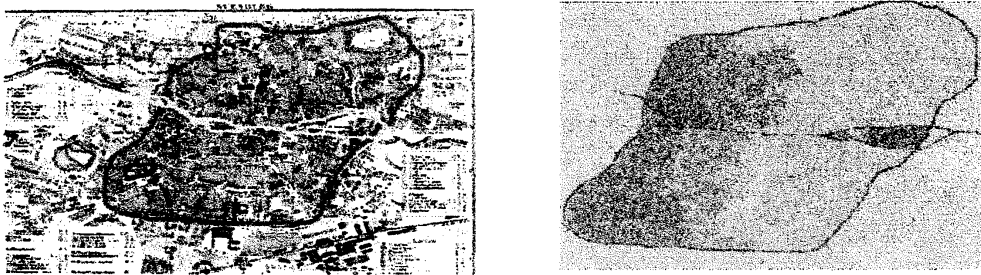

3. Evolutions of extinct animals:

In this section we make computer simulations of the time changes of the numbers of families by use of our dynamical systems. At first we notice that the logistic curve can be realized by our simulations. Hence we see that the Cambrian explosion of species can be realized by our simulations([3]).
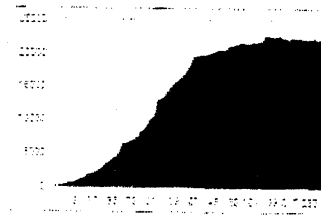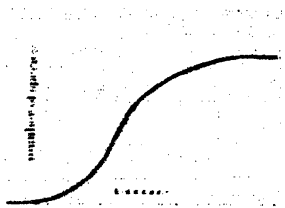

There are many references on the time change of the numbers of extinct animals and the background extinctions and mass extinctions have been discussed([6],[7]). One of the most important results are given by Sepkovski. He has collected many samples of the time changes of extinct animals and made the theoretical background([7]). Also he has applied the factor analysis on them and obtained three classes of extinct animals :Cambrian fauna, Paleozoic fauna and Modern fauna. Here we give several computer simulations corresponding fauna
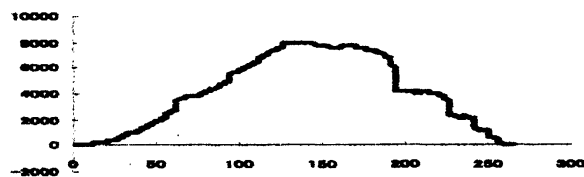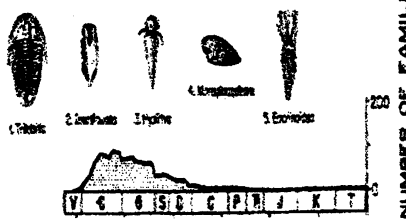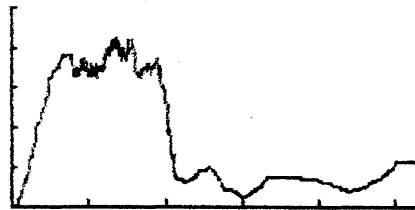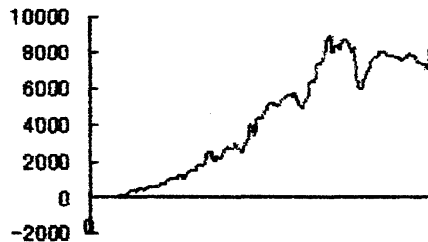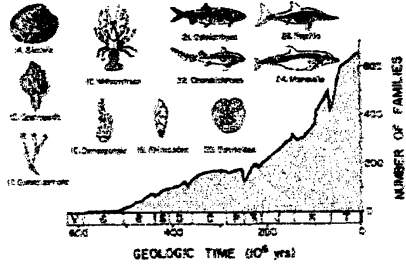
PALEOZOIC FAUNA

N-NE-E-S-W-NW(N=3)/N-W(\=1,V-1)I



MODERN FAUNA

GEOLOGIC TIME (10⁶ yrs)
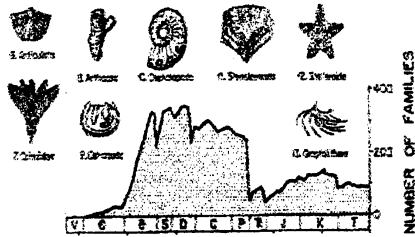
4. The mutation

In this section we propose a new idea on the description of mutation in terms of the change of neighborhoods. At first we notice that our simulations tell us the following facts:

(1) The time change of Cambrian fauna can be described by use of 2-neighborhoods or 4-neighborhoods without high symmetries.

(2) The time change of Paleozoic fauna can be described by use of 4,6,8-neighborhoods with high symmetries.

(3)The time change of Modern fauna can be described by use of 4,6-neighborhoods without high symmetries.

By this we may expect that we can describe the mutation in terms of the change of neighborhoods:It can be described by the change of neighborhoods from simple(or complicate) ones to complicate(resp. simple) ones. We give a simulation of trilobita.

Mutation happens
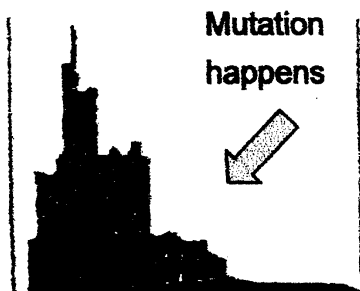
change of Neigh.
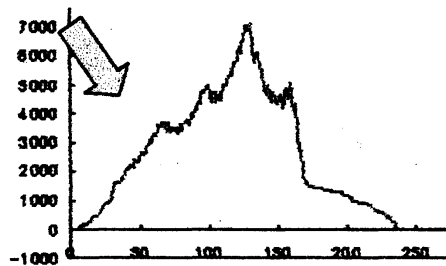
5. Mass extinction

In the evolution of extinct animals, we can find several times of the mass extinction. The big mass extinctions happen in the Cecile, Ordobis and Permian periods. There are many references on the causes of mass extinctions and we have not definite conclusions on the causes ([6], for example). Here we want to show that our simulations can describe the three mass extinctions automatically.

Crinoid

Erchinoderms
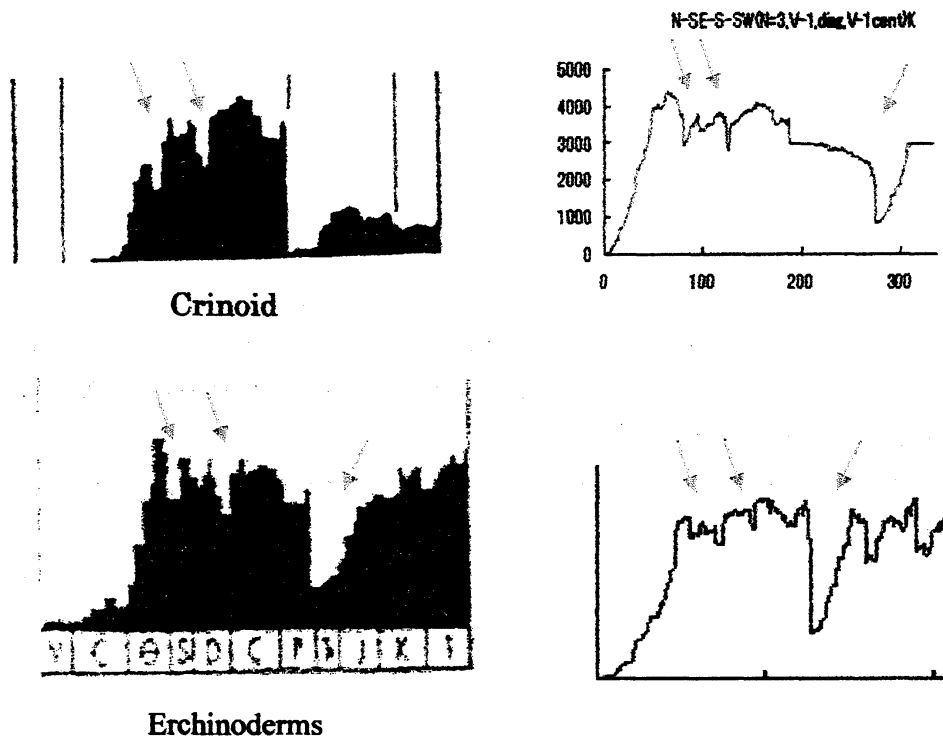
6 Conclusions and discussions

From above discussions, we can conclude our simulations in the following manner:

(I)Our discrete Lapalcians can describe the increase property of the number of families quite well.
(i)They can realize the logistic curves quite well when the numbers of sources are big.
(ii)They can describe the three peaks in the time change for Paleozoic fauna.
(iii)They can describe the two peaks in the time change for Cambrian fauna.
(iii)They can describe the two pauses in the time cange for Modern fauna.

(II)Our discrete Lapalcians can describe the decreases and extinctions quite well:
(i)They can describe the concave properties of decreasings quite well
(ii)The causes of extinction can be discussed in the following two cases:

(1)The cause of extinction is included in the process of the natural growth. Hence the extinction arises automatically and independently from the change of environments.
(2)The cause of extinction comes from the change of environments. It can be simulated by the change of sources. If the changes happen after the changes the numbers become   stable, then the changes are mild. Otherwise there happen big fluctuations in the changes and they become mild finally.

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