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EQUILIBRIUM IN N-PLAYER COMPETITIVE SILENT GAMES OF TIMING

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ABSTRACT. In this paper, we shall discuss about the explicit derivation of the mixed strategy Nash equilibrium for some $n(\geq 3)$ player competitive silent games of timing. We extend some former results for 2-player games in the area to the n-player games. We deal with (a) shooting contest, (b) competitive prediction of a random variable and (c) war-games of attrition (or animals' display). We find that 3-player game in (b) is very difficult to derive the explicit solution.

0 Introduction. First we shall explain the two features in the games in this paper by taking the case of shooting contest.

(1) Each player in the game of timing has to decide his time to shoot under the condition that he is not informed of the shooting times of his rivals. That is, we deal with silent games of timing.

(2) Shooting by more than one player at the same time is quite unlikely event. So, payoffs in this event should not effect on the player's behavior. We deal with n-player games where players in the draw are not rewarded, *i.e.*, the sole player is the winner.

We deal with the shooting contest in Section 1, competitive prediction of a random variable in Section 2, and war-games of attrition in Sections 3 and 4. Three remarks are given in Section 5.

1 Silent Contest where Sole and the Earliest Hitter Wins. Consider an n-player silent contest with the same accuracy function. Each player has one silent bullet which may be fired at any time in $[0,1]$ aiming his target. He, starting at time $t=0$, walks toward his target at a distance 1 apart at a constant unit speed, with no opportunity to stop nor retreat. Let $A(t)$ (called accuracy function) be the probability for each player of hitting his target, if he fires at time $t \in [0,1]$. $A(t)$ is assumed to be differentiable with $A'(t)>0, A(0)=0$ and $A(1)=1$.

Payoff to each player is 1, if he hits his target at the earliest time among the other players who hit, and 0, if otherwise. Players who play draw (*i.e.*, hit by firing at the same time), get 0. Each player has to find the firing strategy under which his expected payoff is maximized.

The symmetric nature of the game suggests that we shall confine ourselves here to n-ply symmetric equilibria. Intuitively, the fact that players have no knowledge of each other's actions suggests randomization, and by symmetry, any mixed strategy that is optimal for one player is also optimal for the others, and the equilibrium payoffs of the game to the players are identical. Hence a reasonable guess of the equilibrium strategy (abbr. by EQS) for each player has the form of a p.d.f. $f(t), a \leq t \leq 1$, for some $a \in [0,1]$. Thus, the expected payoff to player 1, if he fires at time x, and all other players employ their EQS is given by

$$(1.1)\qquad M_1(x,\overbrace{f,\cdots,f}^{n-1}) = \begin{cases} A(x), & \text{if } 0 \le x < a, \\ A(x)\left[1-\int_a^x A(t)f(t)dt\right]^{n-1}, & \text{if } a \le x \le 1 \end{cases}$$

since, for $a \le x \le 1$, player 1 can get reward 1, only in the case where all players $2 \sim n$ do not fire, or fire with no-hitting before time x.

Let v be the common EQ payoff (EQ means equilibrium). Then the condition of EQ is

$$(1.2)\qquad M_1(x,f,\cdots,f)\left\{ \begin{matrix} \le \\ = \end{matrix} \right\} v, \quad \text{for} \left\{ \begin{matrix} 0 \le x < a \\ a \le x \le 1 \end{matrix} \right\}.$$

For $a \le x \le 1$, we obtain, by differentiating (1.1) and equating 0, the differential equation

$$(1.3)\qquad \frac{f'(x)}{f(x)} = -\frac{2n-1}{n-1}\left[\frac{A'(x)}{A(x)} - \frac{A''(x)}{A'(x)}\right].$$

Integration from a to x gives

$$(1.4)\qquad \frac{f(x)}{f(a)} = \frac{A'(x)}{A'(a)}\left(\frac{A(x)}{A(a)}\right)^{-\frac{2n-1}{n-1}},$$

that is,

$$(1.5)\qquad f(x) = c\,(A(x))^{-\frac{2n-1}{n-1}}A'(x).$$

The condition $\int_a^1 f(t)dt = 1$ gives

$$(1.6)\qquad c^{-1} = \int_a^1 (A(x))^{-\frac{2n-1}{n-1}}A'(x)dx = \left(\frac{n-1}{n}\right)\left[(A(a))^{-\frac{n}{n-1}} - 1\right].$$

The condition (1.2), for $a \le x \le 1$, requires that

$$A(x)\left[1-\int_a^x A(t)f(t)dt\right]^{n-1} \equiv v,$$

which becomes from (1.5), after simplified,

$$(1.7)\quad c(n-1)\left[(A(a))^{-\frac{1}{n-1}} - (A(x))^{-\frac{1}{n-1}}\right] = 1 - v^{\frac{1}{n-1}}(A(x))^{-\frac{1}{n-1}}, \quad \forall x \in [a,1].$$

Eliminating c, by using (1.6) and (1.7), we obtain

$$(1.8)\quad (A(a))^{-\frac{1}{n-1}} - (A(x))^{-\frac{1}{n-1}} = \frac{1}{n}\left[1-\left(\frac{v}{A(x)}\right)^{\frac{1}{n-1}}\right]\left[(A(a))^{-\frac{n}{n-1}} - 1\right], \quad \forall x \in (a,1).$$

Therefore we must have

$$(1.9) \qquad (A(a))^{-\frac{n}{n-1}} - n\,(A(a))^{-\frac{1}{n-1}} - 1 = 0 \quad \text{and} \quad v^{\frac{1}{n-1}}\left[(A(a))^{-\frac{n}{n-1}} - 1\right] = n.$$

The two equations above give $v^{-\frac{1}{n-1}} = (A(a))^{-\frac{1}{n-1}}$ and hence $v = A(a)$. Also the first one is, by multiplying $(A(a))^{\frac{n}{n-1}}$ on both sides, is identical to

$$(1.10) \qquad (A(a))^{\frac{n}{n-1}} + nA(a) - 1 = 0.$$

Now, in the final step, we have to confirm the condition $M_1(x, f, \cdots, f) \leq v,\ \forall x \in [0, a]$, in Eq.(1.2) is satisfied. This holds true since $A(x) \leq A(a) = v,\ \forall x \in [0, a)$.

All of the above arguments combined lead to following result.

Theorem 1 *Let α_n be a unique root in $[0,1]$ of the equation*

$$(1.11) \qquad \alpha^{\frac{n}{n-1}} + n\alpha - 1 = 0.$$

Then the common EQS *for each player is*

$$(1.12) \qquad f^*(x) = \frac{1}{n-1}(\alpha_n)^{\frac{1}{n-1}}(A(x))^{-\frac{2n-1}{n-1}}A'(x), \quad \textit{for } A^{-1}(\alpha_n) = a_n \leq x \leq 1.$$

The common EQV *of the game is equal to α_n.*

We find that (1) the common EQV of the game is independent of the accuracy function $A(t)$, and (2) the starting point of the support of the optimal p.d.f. depends on $A(t)$. Further more, we observe from (1.11), that the probability of draw (*i.e.*, all players get zero) is equal to $(\alpha_n)^{\frac{n}{n-1}}$.

Computed values of $\alpha_n, 2 \leq n \leq 50$ are given in Ref.[2 : Table 3]. For example,

$$\begin{aligned} \alpha_n &= \sqrt{2} - 1 \approx 0.4142, \quad \text{for } n = 2 \\ &\approx 0.2831, 0.2173, 0.1770, \quad \text{for } n = 3, 4, 5, \text{ resp..} \end{aligned}$$

Example 1. Let $A(x) = x^\gamma, \gamma > 0$. Then $a_n = A^{-1}(\alpha_n) = \alpha_n^{1/\gamma}$ and

$$f^*(x) = \frac{\gamma}{n-1}(\alpha_n)^{\frac{1}{n-1}}x^{-\left(\frac{n}{n-1}\gamma+1\right)}, \quad \text{for } \alpha_n^{1/\gamma} \leq x \leq 1.$$

For any fixed $n \geq 2$, a_n decreases as γ increases. Due to the competitive nature of the game, players' a_n is small(large), if his shooting skill is low(high).

Example 2. Let $A(x) = \dfrac{e^x - 1}{e - 1}$. Then $a_n = A^{-1}(\alpha_n) = \log\{1 + (e-1)\alpha_n\}$ and so, a_n decreases as n increases. For example, we have

$$\begin{aligned} a_n &= \log\left\{(\sqrt{2} - 1)(e + \sqrt{2})\right\} \approx 0.5375, \quad \text{for } n = 2 \\ &\approx 0.3964, 0.3173, 0.2655, \quad \text{for } n = 3, 4, 5, \text{resp..} \end{aligned}$$

Moreover

$$f^*(x) = \frac{1}{n-1}(\alpha_n)^{\frac{1}{n-1}}(e-1)^{-1}(e^x-1)^{-\frac{2n-1}{n-1}}e^x, \quad \text{for } a_n \le x \le 1.$$

2 N-player Competitive Prediction of a Ramdom Variable. N players compete in predicting the realized value u of a r.v. U which obeys $U_{[0,1]}$ (*i.e.*, uniform distribution in $[0,1]$). The sole player who has predicted the value not greater than u and nearest to u gets 1, and the other $n-1$ players get 0. Each player aims to maximize the expected reward he can get.

A reasonable guess of the EQS for each player has the form of a p.d.f. $g(x), 0 \le x \le a$, for some $a \in (0,1)$. Let $G(x) = I(x < a)\int_0^x g(t)dt + I(x \ge a)$. Then the expected reward to player 1, if he predicts x, and the other players employ their EQS is

$$M_1(x, \overbrace{g, \cdots, g}^{n-1}) = \bar{x}, \quad \text{for } a < x < 1. \tag{2.1}$$

For $0 < x < a$, it is

$$\begin{aligned} M_1(x, g, \cdots, g) &= (G(x))^{n-1}\bar{x} \\ &+ \sum_{k=1}^{n}\binom{n-1}{k} k\,(G(x))^{n-1-k}\int_x^a g(t)\left(\bar{G}(t)\right)^{k-1}(t-x)dt. \end{aligned} \tag{2.2}$$

since k players $(1 \le k \le n-1)$ may predict greater than x and the other $n-1-k$ players predict less than x. Note that p.d.f. of $\max(X_1, \cdots, X_k)$ is $k\left(\bar{G}(t)\right)^{k-1}g(t)$.

Since we have, by integration by parts,

$$\int_x^a (t-x)\left(\bar{G}(t)\right)^{k-1}g(t)dt = \frac{1}{k}\int_x^a \left(\bar{G}(t)\right)^k dt,$$

we can rewrite (2.2) as

$$M_1(x, \overbrace{g, \cdots, g}^{n-1}) = (G(x))^{n-1}\bar{x} + \sum_{k=1}^{n}\binom{n-1}{k}(G(x))^{n-1-k}\int_x^a \left(\bar{G}(t)\right)^k dt, \tag{2.3}$$

for $0 < x < a$. Let v be the common EQV of the game. The condition of EQ is

$$M_1(x, g, \cdots, g)\left\{\begin{matrix} \equiv \\ \le \end{matrix}\right\} v, \quad \text{for } \left\{\begin{matrix} 0 \le x < a \\ a < x \le 1 \end{matrix}\right\}. \tag{2.4}$$

From (2.3)-(2.4), we compute $\frac{\partial}{\partial x}M_1(x, g, \cdots, g) = 0$, for $0 \le x < a$. After dividing both sides by $(G(x))^{n-1}$, and simplifying, we get finally

$$\begin{aligned} 1 + \sum_{k=1}^{n-1}\binom{n-1}{k}\left(\frac{\bar{G}(x)}{G(x)}\right)^k = \frac{g(x)}{G(x)}\Bigg[&(n-1)\bar{x} \\ &+ \sum_{k=1}^{n-1}\binom{n-1}{k}(n-1-k)\int_x^a \left(\frac{\bar{G}(t)}{G(x)}\right)^k dt\Bigg]. \end{aligned} \tag{2.5}$$

This equation becomes very quickly unmanageable. We show the solutions for $n = 2$ and 3 only.

For $n = 2$, (2.5) becomes $\bar{x}g(x) = 1, \ 0 \le x < a$, and therefore, we obtain

$$g^*(x) = \frac{1}{\bar{x}} \text{ and } G^*(x) = -\log \bar{x}, \text{ for } 0 \le x \le a = 1 - e^{-1} (\approx 0.63212).$$

The other requirement in (2.4) is satisfied, since by (2.1),

$$M_1(x, g^*) = \bar{x} \le \bar{a} = e^{-1}, \quad \text{for } a \le x \le 1$$

and

$$M_1(a, g^*) = \bar{a}G^*(a) = -\bar{a}\log\bar{a} = e^{-1}.$$

So, the common EQV of the game is $e^{-1} (\approx 0.36788)$.
The above result is alrealy known, for example, by Ref.[1 ; p.120, and 3].
For the case $n = 3$, we obtain the result below.

Proposition 2 *The 3-player "prediction game", here, the common* EQS $g^*(x)$, *if it exists, satisfies a simple but non-separable differential equation*

$$g'(x) = 2\,(g(x))^2\,(1 - \bar{x}g(x)), \quad 0 < x < a, \textit{with } g(a) = \frac{1}{2\bar{a}}. \tag{2.6}$$

The common EQV *is* $\bar{a}$, *where* a *is a unique root in* $(0,1)$ *of the equation*

$$\frac{1}{2}\int_0^a \left(\bar{G}(t)\right)^2 dt = \bar{a}. \tag{2.7}$$

Proof. Eq.(2.2), for $n = 3$, becomes

$$M_1(x, g, g) = (G(x))^2\,\bar{x} + 2G(x)\int_x^a (t - x)g(t)dt + 2\int_x^a (t - x)\bar{G}(t)g(t)dt \tag{2.8}$$

which gives

$$M_1(0, g, g) = \frac{1}{2}\int_0^a \left(\bar{G}(t)\right)^2 dt \quad \text{and} \quad M_1(a, g, g) = \bar{a}. \tag{2.9}$$

Eq.(2.5), for $n = 3$, becomes after simplification,

$$G(x) - x + \int_x^a tg(t)dt = \frac{1}{2g(x)}. \tag{2.10}$$

Differentiating both sides, we get a simple but non-separable differential equation (2.6).
An approach to (2.6) is to consider a transformation

$$s(x) = \left[G(x) - x + \int_x^a tg(t)dt\right] \Big/ \bar{x}. \tag{2.11}$$

This function is positive and increasing for $0 < x < a$, by (2.9)~(2.11), with values $\int_0^a tg(t)dt$, at $x = 0$, and 1 at $x = a$.
Differentiating both sides and using (2.10), we obtain a simple and separable equation

$$\frac{ss'}{s^2 - s + \frac{1}{2}} = \frac{1}{\bar{x}}, \quad 0 < x < a, \quad \text{with } s(a) = 1. \tag{2.12}$$

Integral both sides from x to a, and we arrive at

$$(2.13)\qquad \left(s^2 - s + \frac{1}{2}\right)^{\frac{1}{2}} e^{\tan^{-1}(2s-1)} = \left(\frac{1}{\sqrt{2}} e^{\pi/4}\right) \bar{a}/\bar{x},$$

which is too complicate to obtain $s(x)$ explicity.

If $s(x)$ and hence $g(x) = s'(x) + \bar{s}(x)/\bar{x}$ are determined, then a is determined by (2.9) as a unique root of the equation (2.7). Then, from (2.1), the common EQV of the game is $\bar{a}$. □

We remark that Eq.(2.6) has a particular solution $g(x) = k/\bar{x}$, where k satisfies $k\bar{k} = \frac{1}{2}$, *i.e.*, $k = \frac{1}{2}\left(1 \pm \sqrt{-1}\right)$.
This fact, however, seems useless to derive the explicit solution.

See Remark 2 in Section 5.

3 Games of War of Attrition. We discuss, in this section, about an n-player game close to the animals' competition game first considered by Maynard Smith (see Ref.[5]). It consists of displays by animals in which victory goes to the sole animal which displays longest. The sole winner gets the prize $V(> 0)$ and pays cost equal to the length of the time until the second longest displayer leaves the contest. Animals who play draw are considered as losers. The losers get 0 and pay cost equal to the length of their displaying time.

Let $x_i \in [0, \infty), i = 1, \cdots, n$, be the pure strategy of player i, to mean that he stops his display at time x_i. Payoffs for the players are given as

$$(3.1)\qquad M_i(x_1, x_2, \cdots, x_n) = \begin{cases} V - x_j, & \text{if } x_i > x_j \geq \max\limits_{k(\neq i,j)} x_k, \\ -x_i, & \text{if } x_i < \max\limits_{k(\neq i)} x_k. \end{cases}$$

The situation of the game here is a silent game. Players do not know at what time the rivals leave the contest. So, in the real world of animals, the situation may be unsuitable except the case for $n = 2$ or 3. Rather, it is suitable for the Sealed-Bid-Auction with the rule under which the winner pays the second winner's bid. Also, see Remark 3 in Section 5.

The expected reward for player 1, if he bids $x \in [0, \infty)$, and all if his rivals employ their common mixed strategy $h(t)$, is given by

$$(3.2)\quad M_1(x, h, \cdots, h) = \int_0^x (V - t) d\,(H(t))^{n-1} - x\left\{1 - (H(x))^{n-1}\right\}, 0 < x < \infty,$$

where $H(x) = \displaystyle\int_0^x h(t)dt$.

Let

$$(3.3)\qquad K(x) = 1 - (H(x))^{n-1}.$$

Then

$$(3.4)\qquad M_1(x, h, \cdots, h) = -\int_0^x (V - t) K'(t) dt - xK(x).$$

the condition $\frac{\partial M_1(x,h,\cdots,h)}{\partial x} = 0$ gives $\frac{K'(x)}{K(x)} = -\frac{1}{V}$, and therfore $K(x) = e^{-x/V}$. Thus we find that

$$(3.5)\qquad H(x) = (1 - K(x))^{\frac{1}{n-1}} = \left(1 - e^{-x/V}\right)^{\frac{1}{n-1}}.$$

Substituting $K(x)$ into (3.4), we have

$$M_1(x,h,\cdots,h) = \frac{1}{V}\int_0^x (V-t)e^{-t/V}dt - xe^{-x/V}.$$

The first term in the r.h.s. is equal to

$$V\int_0^{x/V}(1-u)e^{-u}du = V\left[ue^{-u}\right]_0^{x/V} = xe^{-x/V},$$

and therefore $M_1(x,h,\cdots,h) \equiv 0,\ \forall x \in [0,\infty)$.

Theorem 3 *For the n-player silent game, considered in this section, the common* EQS *is given by*

$$H^*(x) = \int_0^x h^*(t)dt = \left(1-e^{-x/V}\right)^{\frac{1}{n-1}}, \quad 0 \le x < \infty$$

and the common EQV *is* 0.

The result for $n=2$ is the known result, for example Ref.[1 ; pp.119~120]. The EQV is zero for all $n \geq 2$, and any $V > 0$.

4 Games of War of Attrition — Continued. Another version of war-of-attrition game is the case where the contest ends at time 1. The prize given to the sole winner, if he stops at time $x \in [0,1]$ is $V(x)$, which is positive and decreasing for $0 \le x \le 1$, with $0 \le V(1) \le 1$ and $V'(x) < 0,\ \forall x \in (0,1)$. The pure strategies and payoffs are the same as in (3.1), expect that the pure strategy space is changed from $[0,\infty)$ to $[0,1]$.

Suppose that players' common EQS has the form of $H(x) = I(0 \le x < a)\int_0^x h(t)dt + I(a \le x \le 1)$. The expected reward for player 1, if he bids $x \in [0,1]$ and all his rivals employ their common EQS is

$$\text{(4.1)} \quad M_1(x,\overbrace{h,\cdots,h}^{n-1}) = \begin{cases} \displaystyle\int_0^x (V(x)-t)d\,(H(t))^{n-1} - x\left\{1-(H(x))^{n-1}\right\}, & \text{if } 0 \le x < a, \\ \displaystyle\int_0^a (V(x)-t)\,d\,(H(t))^{n-1}, & \text{if } a < x \le 1. \end{cases}$$

where t is the "second winner"s stopping time. Let

$$\text{(4.2)} \qquad Q(x) = V(x)\,(H(x))^{n-1}, \quad \text{for } 0 < x < a.$$

Then (4.1) becomes, for $0 < x < a$,

$$\text{(4.3)} \quad \begin{aligned} M_1(x,h,\cdots,h) &= Q(x) - \int_0^x t d\,(H(t))^{n-1} - x\left\{1-\frac{Q(x)}{V(x)}\right\} \\ &= Q(x) + \int_0^x \frac{Q(t)}{V(t)}dt - x. \end{aligned}$$

Then $\frac{\partial M_1}{\partial x} = 0$ gives a linear differential equation

(4.4)
$$Q'(x) + \frac{Q(x)}{V(x)} = 1, \quad \text{with } Q(0) = 0$$

and the solution is

(4.5)
$$Q(x) = e^{-\int (V(x))^{-1}dx} \left[\int e^{\int (V(x))^{-1}dx} dx + c \right]$$

where c is an arbitration constant.

For one of the easiest example, let $V(x) = \bar{x}, 0 \leq x \leq 1$. Then we have

$$Q(x) = \bar{x} \left[\int_0^x dt/\bar{t} + c \right] = \bar{x}(-\log \bar{x} + c).$$

By the boundary condition $Q(0) = 0$ we have $c = 0$, and so,

(4.6)
$$Q(x) = -\bar{x} \log \bar{x},$$

which gives from (4.2)

(4.7)
$$H(x) = (-\log \bar{x})^{\frac{1}{n-1}}, \quad 0 \leq x \leq a,$$

an increasing function with $H(0) = 0$ and $H(a) = (-\log \bar{a})^{\frac{1}{n-1}}$.
The condition $H(a) = 1$ gives $a = 1 - e^{-1} \approx 0.63212$.

For $H(x)$, given by (4.6), payoff (4.1)-(4.3) for player 1 becomes, for $0 < x < a$,

$$M_1(x, h, \cdots, h) = -\bar{x} \log \bar{x} + \int_0^x (-\log \bar{t}) dt - x = 0,$$

since the second term in the r.h.s. is equal to $\bar{x} \log \bar{x} + x$, by using $\int (1 + \log \bar{t}) dt = -\bar{t} \log \bar{t}$.

For $a < x \leq 1$, $M_1(x, h, \cdots, h)$ is a decreasing function of x, by (4.1).
Hence, if $H^*(x)$ is chosen as defined by (4.7), then

$$M_1(h, h^*, \cdots, h^*) \leq M_1(h^*, h^*, \cdots, h^*) = 0, \quad \forall \text{p.d.f. } h(x).$$

Thus we arrive at

Theorem 4 *For the n-player silent game, we discuss in this section, with $V(x) = \bar{x}$, the common* EQS *is*

$$H^*(x) = I(0 \leq x < a)(-\log \bar{x})^{\frac{1}{n-1}} + I(a < x \leq 1),$$

and the common EQV *is* 0, *where* $a = 1 - e^{-1} (\approx 0.63212)$.

The prize and cost are in balance in the EQ.

The solution is the same as one for 2-player competitive prediction for the uniform distributed r.v.(see Section 2).

More generally we have

Theorem 4′ *For* $V(x) = \frac{1}{k}\bar{x}, (0 < k \leq 1)$, *the common* EQS *is*

$$H^*(x) = \left[(k/\bar{k})\left\{(\bar{x})^{k-1} - 1\right\}\right]^{\frac{1}{n-1}}, \quad 0 \le x < a,$$

where a is a unique root in $(0,1)$ of the equation

$$-\bar{k}\log\bar{a} = -\log k.$$

The common EQV *is* 0.

Note that $\lim_{k\to 1-0} \frac{(\bar{x})^{k-1} - 1}{\bar{k}} = -\log\bar{x}$ and hence $\lim_{k\to 1-0} H^*(x) = (-\log\bar{x})^{\frac{1}{n-1}}$.
Also see Remark 3 in Section 5.

5 Three Remarks.

Remark 2 Consider the 3-player "guess game", instead of 3-player "prediction game" of the Section 2. Because of the symmetric nature of the game, a reasonable guess of the EQS has the form of

$$G(x) = I(\bar{a} < x < a)\int_{\bar{a}}^{x} g(t)dt + I(a \le x \le 1),$$

for some $a \in \left[\frac{1}{2}, 1\right]$. The role player who has guessed the value nearest to the realized value $u \sim U_{[0,1]}$ gets 1, and the other two players get 0. Each player aims to maximize the expected reward he can get. The expected reward for player 1, if he guesses x, and his rivals employ their EQS is, instead of (2.8),

$$(5.1) \qquad M_1(x,g,g) = 2\int_{\bar{a}}^{x}\left(1 - \frac{x+t}{2}\right)g(t)G(t)dt$$
$$+ \; 2\iint_{\bar{a}<t_1<x<t_2<a} \frac{t_2 - t_1}{2} g(t_1)g(t_2)dt_1dt_2 + 2\int_{x}^{a}\frac{x+t}{2}g(t)\bar{G}(t)dt \quad \text{for } \bar{a} < x < a.$$

The solution to this game is surprizingly simple. A. Shaked (Ref.[4]) showed that the common EQS is

$$(5.2) \qquad g^*(t) = 2, \quad \text{for } \frac{1}{4} < t < \frac{3}{4}.$$

by using the fact that it should be symmetric ($i.e.$, $g(x) = g(\bar{x}), \forall x \in [0,1]$).

The three terms in the r.h.s. of (5.1) at $x = \frac{1}{4}$ are equal to 0, 0, and $\frac{1}{3}$, resp., in this order, and the common EQV is $\frac{1}{3}$.

It is interesting to prove that there is no mixed-strategy EQ in 2-player guess game.

Remark 1, Remark 3 and REFERENCES are omitted.

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