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On Lie algebras of vector fields of manifolds with singularities

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§1. Introduction

In this talk we shall consider Pursell-Shanks type theorem for some manifolds with singularities.

Let $\mathcal{X}(M)$ be the Lie algebra of smooth vector fields on a connected smooth manifold M with compact support. Then Pursell and Shanks proved the following.

Theorem 1.1 (Pursell-Shanks [PS])
Let M and N be connected smooth manifolds. If $\mathcal{X}(M)$ and $\mathcal{X}(N)$ are isomorphic as a Lie algebra, then M and N are diffeomorphic.

There are many analogous results on the Lie algebra of smooth vector fields which preserve a geometric structures (c.f. [AM], [BA], [FU], [GP], [GR], [OM], [KO]). We extended Theorem 1.1 to the case of smooth orbifold.

Theorem 1.2 (K. Abe [AB2])
Let M and N be connected smooth orbifold. If $\mathcal{X}(M)$ and $\mathcal{X}(N)$ are isomorphic as a Lie algebra, then M and N are diffeomorphic.

Note that a smooth orbifold is locally diffeomorphisc to the orbit space V/Γ of a representation space V of a finite group Γ. In this paper we consider when Γ is a discrete subgroup of $SL(2,\mathbf{Z})$.

§2. Statement of the result

Let $\mathcal{H}$ denote the upper half complex plane. Let $SL(2,\mathbf{R})$ be the group of real matrix with determinant 1. Then $SL(2,\mathbf{R})$ acts on $\mathcal{H}$ by the Möbius as the following.

For $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$, $z \in \mathcal{H}$,

$$g \cdot z = \frac{az+b}{cz+d}.$$

Then $SL(2,\mathbf{R})$ acts transitively on $\mathcal{H}$ and the isotropy subgroup at $i = \sqrt{-1}$ is

$$SL(2,\mathbf{R})_i = SO(2).$$

The kernel of the action is $\mathbf{Z}_2 = \{\pm 1\}$ and $PSL(2,\mathbf{R}) = SL(2,\mathbf{R})/\{\pm 1\}$ acts effectively on $\mathcal{H}$ and

$$\mathcal{H} \cong SL(2,\mathbf{R})/SO(2).$$

The action can be extended to the Riemannian sphere $\bar{\mathbf{C}} = \mathbf{C} \cup \{\infty\}$.
For $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$, $z \in \bar{\mathbf{C}}$,

$$g \cdot z = \begin{cases} \frac{az+b}{cz+d} & (z \neq -\frac{d}{c}, \infty) \\ \infty & (z = -\frac{d}{c},\ z = d = 0) \\ \frac{a}{c} & (z = \infty) \end{cases} .$$

Set

$$R_1 = \left\{ \pm \begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix} \mid a > 0 \right\}$$

and

$$R_2 = \left\{ \pm \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \ \pm \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \right\}.$$

Then each $g \in SL(2,\mathbf{R})$ is conjugate to one of the elements of $SO(2) \cup R_1 \cup R_2$, and $g \neq \pm 1$ is called *elliptic*, *hyperbolic* and *parabolic* if g is conjugate to an element in $SO(2)$, R_1 and R_2, respectively.

Let Γ denote a discrete subgroup of $SL(2,\mathbf{R})$. $z \in \mathcal{H}$ is called *elliptic point* if there exits an elliptic element $g \in \Gamma$ such that $g \cdot z = z$. $x \in \mathbf{R} \cup \{\infty\}$ is called *cusp point* if there exists a parabolic element $g \in \Gamma$ such that $g \cdot z = z$.

Proposition 2.1 (1) *If z is a elliptic point, then Γ_z is a cyclic group which is conjugate to a cyclic subgroup of $SO(2)$.*

(2) *If x is a cusp point, then Γ_x is isomorphic to $\mathbf{Z}$ which is conjugate to a subgroup of the group*

$$\Gamma_\infty = \left\{ \pm \begin{pmatrix} 1 & nk \\ 0 & 1 \end{pmatrix} \mid n \in \mathbf{Z} \right\} \quad (\exists k \in \mathbf{Z}).$$

Let E_Γ denote the set of all elliptic points in $\mathcal{H}$ and C_Γ be the set of cusp points of Γ. Set $\mathcal{H}^* = \mathcal{H} \cup C_\Gamma$,

We shall give the following topology on $\mathcal{H}^*$.
(1) We give the canonical topology on $\mathcal{H}$.
(2) Let $x \in C_\Gamma$.
(2.1) If $x \neq \infty$, then we take all the family of the form
$\{x\} \cup \{$ the interior of a circle in $\mathcal{H}$ tangent to the real axis at x $\}$
as a fundamental system of open neighborhoods of x.
(2.2) If $x = \infty$, then

$$\{\infty\} \cup \cup_{c>0}\{z \in \mathcal{H} | \ \Im z > c\}$$

as a fundamental system of open neighborhood of the point ∞. Then Γ acts on $\mathcal{H}^*$ as a topological transformation group. Set

$$\mathcal{R}_\Gamma = \mathcal{H}^*/\Gamma = \mathcal{H}/\Gamma \cup C_\Gamma/\Gamma$$

Then $\mathcal{R}_\Gamma$ is a Hausdorff space.

Lemma 2.2 *For each $x \in C_\Gamma$, there exists an open neighborhood $\tilde{U}_x$ of x in $\mathcal{H}^*$ such that*

$$\Gamma_x = \{\gamma \in \Gamma | \ \gamma \cdot \tilde{U}_x \cap \tilde{U}_x \neq \emptyset\}.$$

Take $x \in C_\Gamma$. Let $\iota_x : \tilde{U}_x/\Gamma_x \hookrightarrow \mathcal{R}_\Gamma$ be a map defined by $\iota_x(\Gamma_x \cdot z) = \Gamma \cdot z$ for $z \in \tilde{U}_x$. Put $p = \Gamma \cdot x$. Then $U_p = \iota_x(\tilde{U}_x/\Gamma_x)$ is an open neighborhood of p in $\mathcal{R}_\Gamma$.

For $x \in C_\Gamma$, there exist $g \in SL(2, \mathbf{R})$ and integer k such that $g \cdot x = \infty$ and

$$g\Gamma_x g^{-1} = \left\{ \pm \begin{pmatrix} 1 & nk \\ 0 & 1 \end{pmatrix} | \ n \in \mathbf{Z} \right\}.$$

Proposition 2.3 *Let $\varphi_p : \tilde{U}_x/\Gamma_x \to \mathbf{C}$ be a map given by*

$$\varphi_p(\Gamma_x z) = \begin{cases} \exp(\frac{2\pi\sqrt{-1}}{k}(g \cdot z)) & (z \in \tilde{U}_x \setminus \{x\}), \\ 0 & (z = x). \end{cases}$$

Then φ_p is homeomorphic to an open subset W_p of $\mathbf{C}$.

By Proposition 2.3, the map
$\psi_p = \varphi_p \circ \iota_x^{-1} : \ U_p \to \tilde{U}_x/\Gamma_x \to W_p$
is regarded as a local coordinate of $\mathcal{R}_\Gamma$ around p.
$\bar{C}_\Gamma = C_\Gamma/\Gamma$

Definition 2.4 *$f : \mathcal{R}_\Gamma \to \mathbf{R}$ is defined to be smooth if*
(1) $f \circ \pi_\Gamma$ is smooth, where $\pi_\Gamma : \mathcal{H} \to \mathcal{H}/\Gamma$ is the natural projection,
(2) for each $p \in \bar{C}_\Gamma$, $f \circ \psi_p^{-1}$ is smooth.

Definition 2.4 (2) does not depend on the choice of x with $\Gamma \cdot x = p$.

Let $C^\infty(\mathcal{R}_\Gamma)$ denote the set of all real valued smooth functions on $\mathcal{R}_\Gamma$.

Definition 2.5 *For discrete subgroups Γ, Γ' of $SL(2, \mathbf{R})$, $h : \mathcal{R}_\Gamma \to \mathcal{R}_{\Gamma'}$ is said smooth if for each real valued smooth function $f : \mathcal{R}_{\Gamma'} \to \mathbf{R}$ $f \circ h$ is smooth. h is said diffeomorphic if h and h^{-1} are smooth.*

Definition 2.6 *A derivation X of $C^\infty(\mathcal{R}_\Gamma)$ is called a smooth vector field on $\mathcal{R}_\Gamma$ if X vanishes on C_Γ. Let $\mathcal{L}(\mathcal{R}_\Gamma)$ denote the set of all smooth vector field on $\mathcal{R}_\Gamma$ and let $\mathcal{X}(\mathcal{R}_\Gamma)$ be the subalgebra of $\mathcal{L}(\mathcal{R}_\Gamma)$ which consists of vector fields with compact suport.*

Then we have the following.

Theorem 2.7 *Let Γ and Γ' be discrete subgroups of $SL(2, \mathbf{R})$. Then $\mathcal{R}_\Gamma$ and $\mathcal{R}_{\Gamma'}$ are diffeomorphic if and only if $\mathcal{X}(\mathcal{R}_\Gamma)$ and $\mathcal{X}(\mathcal{R}_{\Gamma'})$ are isomorphic as a Lie algebra.*

§3. Maximal ideals of $\mathcal{X}(\mathcal{R}_\Gamma)$

In order to pvove Theorem 2.7 we investigate the maximal ideals of $\mathcal{X}(\mathcal{R}_\Gamma)$. Let Γ be a discrete subgroup of $SL(2, \mathbf{R})$. Let $\bar{E}_\Gamma = E_\Gamma/\Gamma$ and $\bar{C}_\Gamma$ denote the set of elliptic singularities and cusp singularities in $\mathcal{R}_\Gamma$, respectively. Set $\mathcal{S}_\Gamma = \bar{E}_\Gamma \cup \bar{C}_\Gamma$ which is the set of singularities in $\mathcal{R}_\Gamma$. We abbreviate $\mathcal{R}_\Gamma, \mathcal{S}_\Gamma$ and $\bar{E}_\Gamma$ to $\mathcal{R}, \mathcal{S}$ and $\bar{E}$, respectively. Let $\mathcal{R}_1 = \mathcal{R} \setminus \mathcal{S}$ be the set of regular points of $\mathcal{R}$. For each $p \in \mathcal{R}_1$, set

$$\mathcal{X}_p(\mathcal{R}) = \{X \in \mathcal{X}(\mathcal{R}) |\ X(p) = 0\}.$$

Proposition 3.1 *For each $p \in \mathcal{R}_1$, there exists a unique maximal ideal $\mathcal{M}_p$ of $\mathcal{X}(\mathcal{R})$ which is contained in $\mathcal{X}_p(\mathcal{R})$. Moreover $\mathcal{M}_p$ is an infinite codimensional subalgebra in $\mathcal{X}(\mathcal{R})$.*

Next we shall find the maximal ideals of $\mathcal{X}(\mathcal{R})$ which correspond to the singularities in $\mathcal{R}$. Here we recall the results by Bierstone and Schwarz. Let G be a finite group and V be a representation space of G. Let $\pi : V \to V/G$ be the natural projection. $\mathcal{X}_G(V)$ denotes the Lie algebra of G-invariant smooth vector fields on V with compact support.

Theorem 3.2 (Bierstone [BI] and Schwarz [SC])
The induced map $\pi_* : \mathcal{X}_G(V) \to \mathcal{X}(V/G)$ is a Lie algebra isomorphism.

(I) For each $p \in \bar{E}$, take $x_p \in E$ with $\Gamma \cdot x_p = p$. Let V_{x_p} be the linear slice at x_p. Then V_{x_p} is a Γ_{x_p}-mdule. Let

$$(\pi_{x_p})_* : \mathcal{X}_{\Gamma_{x_p}}(V_{x_p}) \to \mathcal{X}(V_{x_p}/\Gamma_{x_p}) \hookrightarrow \mathcal{X}(\mathcal{R})$$

be the natural Lie algebra homomorphism. By Theorem 3.2, for each $X \in \mathcal{X}(\mathcal{R})$ there exists $Y_{x_p} \in \mathcal{X}_{\Gamma_{x_p}}(V_{x_p})$ such that $(\pi_{x_p})_*(Y_{x_p}) \doteq X$ on a neighborhood of p in $\mathcal{R}$. Let $\mathfrak{gl}_{\Gamma_{x_p}}(V_{x_p})$ be the set of Γ_{x_p}-invariant linear endmorphisms. Let

$$J_p : \mathcal{X}(\mathcal{R}) \to \mathfrak{gl}_{\Gamma_x}(V_{x_p})$$

be the homomorphism defined by $J_p(X) = j^1_{x_p}(Y_{x_p})$, where $j^1_{x_p}(Y_{x_p})$ is the 1-jet of Y_{x_p} at x_p.

(II) For $p \in \bar{C}$ there is a chart $\psi_p : U_p \to W_p \subset \mathbf{C} = \mathbf{R}^2$ around the open neighborhood U_p of p in $\mathcal{R}$. Let

$$J_p : \mathcal{X}(\mathcal{R}) \to \mathfrak{gl}(2, \mathbf{R})$$

be the Lie algebra homomorphism defined by $J_p(X) = j^1_p(X|_{U_p})$.

Combining (I) and (II) we set

$$J(\mathcal{R}) = \bigoplus_{p \in \bar{E}} \mathfrak{gl}_{\Gamma_x}(V_{x_p}) \oplus \bigoplus_{p \in \bar{C}} \mathfrak{gl}(2, \mathbf{R}).$$

Let $J : \mathcal{X}(\mathcal{R}) \to J(\mathcal{R})$ be a Lie algebra homomorphism defined by

$$J(X) = \bigoplus_{p \in \bar{E}} J_p(X) \oplus \bigoplus_{p \in \bar{C}} J_p(X).$$

Lemma 3.3 *J is an onto Lie algebra homomorphism.*

Proposition 3.4 *If $\mathfrak{M}$ is a maximal ideal of $\mathcal{X}(\mathcal{R})$, then we have the following.*
(1) *If $\mathfrak{M}$ is contained in $\mathcal{X}_p(\mathbf{R})$ for some $p \in \mathcal{R}_1$, then $\mathfrak{M} = \mathcal{M}_p$, and $\mathfrak{M}$ is an infinite codimensional subalgebra of $\mathcal{X}(\mathcal{R})$.*
(2) *If $\mathfrak{M} \not\subset \mathcal{X}_p(\mathbf{R})$ for any $p \in \mathcal{R}_1$, then there exists a maximal ideal $\mathfrak{L}$ of $J(\mathcal{R})$ such that $\mathfrak{M} = J^{-1}(\mathfrak{L})$, and $\mathfrak{M}$ is a finite codimensional subalgebra of $\mathcal{X}(\mathcal{R})$.*

§4. Stone topology of the maximal ideals

Let $\mathcal{R}^*$ be the set of all maximal ideals of $\mathcal{X}(\mathcal{R})$.

Definition 4.1 *The Stone topology on $\mathcal{R}^*$ is defined by the closure operator $C\ell$ as following.*

(1) $C\ell(\phi) = \phi$,

(2) *For a subset B of $\mathcal{R}^*$ with $B \neq \emptyset$,*

$$C\ell(B) = \left\{ \mathfrak{M} \in \mathcal{R}^* \,\middle|\, \mathfrak{M} \supset \bigcap_{\mathfrak{M}' \in B} \mathfrak{M}' \right\}.$$

Let $\mathcal{O}(\mathcal{S})$ denote the family of all subsets of $\mathcal{S}$. We define a map

$$\tau_{\mathcal{R}} : \mathcal{R}^* \to \mathcal{R}_1 \cup \mathcal{O}(\mathcal{S})$$

by the following way.

(1) For $p \in \mathcal{R}_1$, $\tau_{\mathcal{R}}(\mathcal{M}_p) = p$.

(2) If $\mathfrak{M} \in \mathcal{R}^*$ such that $\mathfrak{M} \not\subset \mathcal{X}_p(\mathcal{R})$ for any $p \in \mathcal{R}_1$, then

$$\tau_{\mathcal{R}}(\mathfrak{M}) = \{p \in \mathcal{S} |\ J(\mathcal{M}) \not\supset J_p(\mathcal{X}(\mathcal{R}))\}.$$

Set $\mathcal{R}_1^* = \{\mathcal{M}_p \in \mathcal{R}^* |\ p \in \mathcal{R}_1\}$.

Proposition 4.2
The map $\tau_{\mathcal{R}} : \mathcal{R}_1^* \to \mathcal{R}_1$ is homeomorphic.

Definition 4.3 (End)
Let $\mathfrak{K}(\mathcal{R}_1) = \{K_i | i \in I\}$ denote the family of compact subset in $\mathcal{R}_1$. For each $K \in \mathfrak{K}(\mathcal{R}_1)$, let $\mathfrak{C}_K$: be the set of connected component of $\mathcal{R}_1 \setminus K$.

$$\textstyle\prod_{K_i \in \mathfrak{K}(\mathcal{R}_1)} C_{K_i} \in \prod_{K_i \in \mathfrak{K}(\mathcal{R}_1)} \mathfrak{C}_{K_i}$$

is said to be an end of $\mathcal{R}_1$ if $C_{K_i} \subset C_{K_j}$ for any pair $i, j \in I$ with $K_j \subset K_i$.
$\mathcal{E}(\mathcal{R}_1)$: the set of all ends of $\mathcal{R}_1$

For each $p \in \mathcal{S}$ there exists a unique end $\mathcal{E}_p = \prod_{K_i \in \mathfrak{K}(\mathcal{R}_1)} C_{K_i}$ in $\mathcal{R}_1$ such that $\bigcap_{K_i \in \mathfrak{K}(\mathcal{R}_1)} cl(C_{K_i}) = \{p\}$, where $cl(C_{K_i})$ is the closure of C_{K_i} in $\mathcal{R}$. Set

$$\mathcal{E}_0(\mathcal{R}_1) = \{\mathcal{E}_p |\ p \in \mathcal{S}\}, \quad \bar{\mathcal{R}}_1 = \mathcal{R}_1 \cup \mathcal{E}(\mathcal{R}_1).$$

Then $\bar{\mathcal{R}}_1$ has the natural topology such that

$$\{C_{K_j} \cup \prod_{K_i \in \mathfrak{K}(\mathcal{R}_1)} C_{K_i} |\ K_j \in \mathfrak{K}(\mathcal{R}_1)\}$$

is the fundamental system of neighborhood of a point $\prod_{K_i \in \mathfrak{K}(\mathcal{R}_1)} C_{K_i} \in \mathcal{E}(\mathcal{R}_1)$.

Put $\bar{\mathcal{R}}_0 = \mathcal{R}_1 \cup \mathcal{E}_0(\mathcal{R}_1)$. Let $\kappa_{\mathcal{R}} : \mathcal{R} \to \bar{\mathcal{R}}_0$ be the natural map defined by

$$\kappa_{\mathcal{R}}(p) = \begin{cases} p & \text{for } p \in \mathcal{R}_1 \\ \mathcal{E}_p & \text{for } p \in \mathcal{S}. \end{cases}$$

Lemma 4.4 *The map $\kappa_{\mathcal{R}} : \mathcal{R} \to \bar{\mathcal{R}}_0$ is a homeomorphism.*

§5. Outline of the proof of Theorem 2.7

Let Γ, Γ' be discrete subgroups. Assume that there exists a Lie algebra isomorphism $\Phi : \mathcal{X}(\mathcal{R}_\Gamma) \to \mathcal{X}(\mathcal{R}_{\Gamma'})$. We abbreviate $\mathcal{R}_{\Gamma'}, \mathcal{S}_{\Gamma'}, \bar{E}_{\Gamma'}, \ldots$ to $\mathcal{R}', \mathcal{S}', \bar{E}', \ldots$, respectively. By Propsosition 4.2 we have.

Proposition 5.1

(1) *$\Phi_* : \mathcal{R}^* \to \mathcal{R}'^*$ is homeomorphic.*

(2) *The compsosition $\sigma_1 = \tau_{\mathcal{R}'} \circ \Phi_* \circ \tau_{\mathcal{R}}^{-1} : \mathcal{R}_1 \to \mathcal{R}'_1$ is homeomorphic.*

By Proposition 5.1 we have.

Corollary 5.2 *There exists a homeomorphism $\bar{\sigma} : \bar{\mathcal{R}} \to \bar{\mathcal{R}}'$ which is an extension of σ_1 such that the following diagram is commutative :*

$$\begin{array}{ccc} \mathcal{R}^* & \xrightarrow{\Phi_*} & \mathcal{R}^* \\ {\scriptstyle \tau_{\mathcal{R}}}\downarrow & & \downarrow{\scriptstyle \tau_{\mathcal{R}'}} \\ \bar{\mathcal{R}} & \xrightarrow{\bar{\sigma}} & \bar{\mathcal{R}}' \end{array}$$

Lemma 5.3 *For $p \in \mathcal{S}$ let U be a neighborhood of p in $\mathcal{R}$ such that $cl(U) \cap \mathcal{S} = \{p\}$. Then we have*

$$C\ell(\tau_{\mathcal{R}}^{-1}(U)) = \tau_{\mathcal{R}}^{-1}(cl(U))$$

From Corollary 5.2, Lemma 5.3 and Lemma 4.4, we have the following.

Proposition 5.4 *We can extend the homeomorphism* $\sigma_1 : \mathcal{R}_1 \to \mathcal{R}'_1$ *to the homeomorphism* $\sigma : \mathcal{R} \to \mathcal{R}'$ *such that the following diagram is commutative :*

$$\begin{array}{ccc} \mathcal{R} & \xrightarrow{\sigma} & \mathcal{R}' \\ {\scriptstyle \kappa_{\mathcal{R}}}\downarrow & & \downarrow{\scriptstyle \kappa_{\mathcal{R}'}} \\ \bar{\mathcal{R}}_0 & \xrightarrow[\bar{\sigma}]{} & \bar{\mathcal{R}}'_0 \end{array}$$

Lemma 5.5 *Let* $p \in \mathcal{R}_1$ *and* $X \in \mathcal{X}(\mathcal{R})$. *Then* $X_p \neq 0$ *if and only if*

$$[X,\ \mathcal{X}(\mathcal{R})] + \mathcal{M}_p = \mathcal{X}(\mathcal{R}).$$

Corollary 5.6 $\sigma_1 : \mathcal{R}_1 \to \mathcal{R}'_1$ *is diffeomorphic.*

By the method Koriyama [KO] and Abe [AB1] we can prove that $\sigma : \mathcal{R} \to \mathcal{R}'$ is diffeomorphic.

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