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Solution Method of Multi-Objective Decision Problem for Eco-Conscious Management by Particle Swarm Optimization

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1. Introduction

Supply chain management (SCM) [1, 2] is the technique of carrying out management of the supply chain which treated from manufacture to the customer and includes global operation. SCM aims at the profit improvement, a cost cut and shortening the product time for delivery in the whole supply chain. The management activities supported by modern mass production and mass consumption [3-5] are increasing the environment load [6] in many actual problems. In future management activities, it is necessary to take notice of the influence on environment etc., replying to various needs from a consumer. A life cycle assessment is in the technique of evaluating the environmental influence of the life cycle of a product. A Life Cycle Assessment (LCA) [7, 8] is one of the evaluation methods for environmental preservation and evasion of resources drain. LCA gives not only the indicator of the recycling design for realizing sustainable development, but also the estimation of effective mechanism of manufacture. In LCA, it is important to reduce the environmental load when the products are manufactured at machine tool and the products are transported in logistics. In order to solve an optimization problem, we use the Particle Swarm Optimization which is one of the meta-heuristics. Particle Swarm Optimization (PSO) [9] is a method solving a continuous nonlinear optimization problem efficiently which is developed by Kennedy and used as the simulation of the simplified social model. It is known as a result of much old numerical simulations that it is possible to calculate the semi optimal solution which is equivalent to the global optimal solution of multimodal function of a continuous variable or it high accuracy. In this paper, we propose the target order quantity considering environmental issue for SCM by PSO. First, we formulate a model whose aim is to minimize cost in the multistage SCM, taking environmental issue into consideration. Next the optimum target order quantity is calculated using PSO.

2. Model Formulation

2.1 Basic Model of Single Stage in Supply Chain

It is thought that consumer causes demand for one retailer. Here, demand at stage i in period t is shown by normal distribution $D_t^i \in N(d, \sigma^2)$ with average d and variance σ^2. Demand forecast $\tilde{d}_t^i$ during lead time at stage i is formulated by

$$\tilde{d}_t^i = L^i \hat{d}_t^i \tag{1}$$

Where, L^i is lead times for stage i and $\hat{d}_t^i$ is demand forecast at stage i in period t using moving average method with period p.

$$\hat{d}_t^i = \frac{\sum_{j=1}^{p} D_{t-j}}{p} \tag{2}$$

Standard deviation of demand forecast during lead time at stage i is given by $\tilde{\sigma}_t^i = \sqrt{L^i}\sigma$. Under there environment, demand variable level at stage i in period t is expressed by

$$y_t^i = \tilde{d}_t^i + a^i \tilde{\sigma}_t^i \tag{3}$$

where, a^i is control parameter about customer satisfaction at stage i. And, order quantity O_t^i at stage i in period t is given by

$$O_t^i = D_t^i + y_{t+1}^i - y_t^i \tag{4}$$

2.2 Formulation of Multistage Supply Chain

We formulate fundamental model which is discussed in this paper. In problem definition and formulation, we consider these quantities such as D_t^i, y_t^i, O_t^i, S_t^i, where S_t^i denotes inventory at stage i in period t. Index i expresses retailer by 1, wholesaler by 2, distributor by 3, and factory by 4. We consider price of product, p_D^i, order and logistics cost, p_O^i, restocking fee in order, p_r^i, restocking fee in demand, p_b^i, holding cost, p_S^i, and stock out cost, p_u^i, per unit. In the same way we consider environment load, cost for production, e_D^i, logistics, e_O^i, restocking in order, e_r^i, restocking in demand, e_b^i, and holding inventory, e_S^i, per unit.

We formulate problem deciding the order variable $x_t^i\left(x_{ts-L_e^i+1}^i, x_{ts+1}^i, \cdots, x_{ts+tp}^i\right)$. ts is plan start time. And tp is plan period. x_t^i satisfies maximizing profit of total supply chain as follows;

$$\min \sum_{i=1}^{m} \sum_{t=ts-L_e^i+1}^{ts-tp} \left(c_t^i - p_D^i D_t^i\right), \tag{5}$$

$$\min \sum_{i=1}^{m} \sum_{t=ts-L_e^i+1}^{ts-tp} g_t^i, \tag{6}$$

$$s.t. \quad c_t^i = p_s^i S_t^i + p_u^i u_t^i + p_O^i O_t^i + p_r^i r_t^i + p_b^i b_t^i, \tag{7}$$

$$s.t. \quad g_t^i = e_S^i S_t^i + e_O^i O_t^i + e_r^i r_t^i + e_b^i b_t^i + e_D^i D_t^i, \tag{8}$$

$$z_{t+1}^i = S_t^i - y_t^i + O_{t-L_O^i+1}^i - r_t^i, \tag{9}$$

$$y_t^i = D_t^i - b_t^i, \tag{10}$$

$$S_t^i = \min\left(f_+\left(z_t^i\right), \hat{S}^i\right), \tag{11}$$

$$u_t^i = f_-\left(z_t^i\right), \tag{12}$$

$$O_t^i = f_+\left(x_t^i\right), \tag{13}$$

$$r_t^i = \min\left(S_t^i, f_-\left(x_t^i\right)\right), \tag{14}$$

$$D_t^i = O_t^{i-1}, \tag{15}$$

$$b_t^i = r_{t-L_r^{i-1}}^{i-1}, \tag{16}$$

$$L_e^i \geq L_O^i \geq 0, \tag{17}$$

$$\hat{S}^i \geq 0, \tag{18}$$

$$L_r^i \geq 0, \tag{19}$$

c_t^i denotes total cost of multistage, g_t^i denotes total environmental load of multistage and from eq. (9) to eq. (16) are derived by model assumptions. r_t^i denotes restocking in order, b_t^i denotes restocking in demand, u_t^i denotes quantity out of stock and z_t^i denotes inventory variable. $\hat{S}^i$ denotes limit inventory in stage i. And inventory quantity S_t^i must not be beyond limit inventory $\hat{S}^i$ in each stage. We consider lead time of order, L_O^i, restocking, L_r^i, and plan, L_e^i. L_O^i is a lead time which until it orders form the next stage $i+1$ and returns to the present stage i, and L_r^i is a lead time which until it returns the goods to the next stage $i+1$ from the present stage i. L_e^i is lead time of period to build a plan.

3. Solution Method

We use Multi Objective Particle Swarm Optimization (MOPSO) [10, 11] which extended PSO as a multiple-purpose optimization technique, in order to decide order quantity. PSO consists of very brief algorithm. However, it is the technique of the ability to solve a continued type nonlinear optimization problem efficiently. It is observed as the optimization technique for the single purpose function in recent years. The multiple-purpose optimization technique MOPSO which improved the algorithm of PSO so that it could deal with a multiple-purpose optimization problem is proposed. MOPSO can ask for the multiple-purpose optimal solution set, i.e., the Pareto solution set, efficiently.

In MOPSO, the searching point $\mathbf{x} \in R^m$ which is distributed in the shape of a group and moves in the search space of m dimension generates the move vector $\mathbf{v} \in R^m$ using the position information $\mathbf{gbest} \in R^m$ on the Pareto solution shared with group's position information $\mathbf{pbest} \in R^m$ of the best solution which self has in groups, and it searches for a solution. And it is the technique of considering a set of **gbest** which finally remained as the Pareto optimum meeting set. The search method of MOPSO is shown below.

Step (A) Initialize

First, the searching point number N_I, the number of times of repetition N_K, and saving point number maximum $N_R^{\max}$ are determined. And initial setting of $\mathbf{x}(i)$, $\mathbf{pbest}(i)$, $\mathbf{gbest}(r)$, $\mathbf{v}(i)$ is performed. However, i expresses a searching point number and r expresses a saving point number. $\mathbf{x}(i)(1 \le i \le N_I)$ is determined at random within a limit value, and sets with $\mathbf{v}(i) = 0(1 \le i \le N_I)$, $\mathbf{pbest}(i) = x(i)(1 \le r \le N_I)$, $\mathbf{gbest}(r) = x(i)(r = i, 1 \le r \le N_I)$. $\mathbf{gbest}(r)(N_I + 1 \le r \le N_R^{\max})$ does not have an initial value, and it is $N_R = N_I$ in initial setting when the saving point number is set to N_R.

Step (B) Generate hypercube

In the case of dealing with n purpose optimization problem $(n > 1)$, Since a searching point $\mathbf{x}(i)$ has n purpose functions, the position in the n-dimensional purpose functional space is decided by those values. And each searching point can be evaluated. The position information $\mathbf{pbest}(i)$ on the best solution which these searching points itself has, and the position information $\mathbf{gbest}(r)$ on the Pareto solution shared in groups have n objective functions similarly, and exist in n-dimensional objective function space. A Hypercube (n-dimensional cube) is generated so that only arbitrary numbers may divide the n-dimensional purpose functional space where all $\mathbf{gbest}(r)$ exists.

Step (C) Selection of $\mathbf{gbest}(h)$

The procedure which chooses $\mathbf{gbest}(h)$ which is needed when generating a move vector $\mathbf{v}(i)$ at Step 1-4 is as follows. In objective-function space, the number of $\mathbf{gbest}(r)$ which belongs to each Hypercube is set to c paying attention to all the Hypercube containing at least one $\mathbf{gbest}(r)$. $rand()$ is set to the uniform random numbers from 0 to 1. $rand()/c$ specifies one Hypercube which becomes the maximum and sets to as Hypercube h. $\mathbf{gbest}(r)$ is chosen at random from Hypercube h, and selected $\mathbf{gbest}(r)$ is set to $\mathbf{gbest}(h)$. Thus, it draws near to the domain where the density of $\mathbf{gbest}(r)$ is low by choosing $\mathbf{gbest}(h)$. And it is effective in the ability to perform wide range search.

Step (D) New searching point generation

In the $k+1$th search, the ith searching point $\mathbf{x}_{k+1}(i)$ moves to the new position shown by eq. (20) according to the move vector $\mathbf{v}_{k+1}(i)$ described by formula eq. (20) in search space.

$$\mathbf{v}_{k+1}(i) = w \cdot \mathbf{v}_k(i) + rand_1() \cdot (\mathbf{pbest}(i) - \mathbf{x}_k(i)) + rand_2() \cdot (\mathbf{gbest}(h) - \mathbf{x}_k(i)) \tag{20}$$

$$\mathbf{x}_{k+1}(i) = \mathbf{x}_k(i) + \mathbf{v}_{k+1}(i) \tag{21}$$

In eq. (20), w expresses an inertia weight and $rand_1()$, $rand_2()$ expresses the uniform random numbers from 0 to 1. The 1st clause of the right-hand side is a vector showing the inertia to the direction to which it moved last time. The 2nd clause of the right-hand side is a vector which draws a searching point near to the position of the best solution which self has. The 3rd clause of the right-hand side is a vector which draws a searching point near to the position of $\mathbf{gbest}(h)$. In addition, various search becomes realizable by random number $rand_1()$, $rand_2()$.

Step (E) Searching point evaluation

An objective-function value is calculated from the position of a searching point $\mathbf{x}_{k+1}(i)$.

Step (F) Updating and preservation of $\mathbf{pbest}(i)$, $\mathbf{gbest}(r)$

This step consists of the following step A-F.

Step (F-1)

When $\mathbf{x}_{k+1}(i)$ is superior to $\mathbf{pbest}(i)$ to a certain objective-function value, it updates $\mathbf{pbest}(i)$ to $\mathbf{x}_{k+1}(i)$.

Step (F-2)

Although $\mathbf{x}_{k+1}(i)$ is superior to $\mathbf{pbest}(i)$ to a certain purpose function value, when inferior to $\mathbf{pbest}(i)$ to other purpose function values, it decides at random whether to update $\mathbf{pbest}(i)$ to $\mathbf{x}_{k+1}(i)$.

Step (F-3)

When $\mathbf{gbest}(r)$ inferior to $\mathbf{x}_{k+1}(i)$ exists to all the objective-function values, one is updated to $\mathbf{x}_{k+1}(i)$.

Except it, it is deleted by next processing **Step (F-6)**.

Step (F-4)

When $\mathbf{x}_{k+1}(i)$ is excellent in at least one purpose function value to all $\mathbf{gbest}(r)$ (that is, it is the Pareto solution), if it is $N_R < N_R^{max}$, $\mathbf{x}_{k+1}(i)$ is saved as new $\mathbf{gbest}(r)(r = N_R + 1)$.

Moreover, since the one saving point number increases at this time, the saving point number is set to $N_R + 1$.

Step (F-5)

If it is $N_R \geq N_R^{max}$ on condition of **Step (F-4)**, the number of $\mathbf{gbest}(r)$ saves only $x_{k+1}(i)$ belonging to the Hypercube which is below a certain value as new $\mathbf{gbest}(r)(r = N_R + 1)$.

Moreover, at this time, since the one saving point number increases, the saving point number is set to $N_R + 1$.

Step (F-6)

$\mathbf{gbest}(r)$ which became a non-PARETO solution at the time of preservation and updating is deleted. The saving point number at this time is newly set to N_R.

Step (G) Search end

The procedure of **Step (B-F)** is repeated until it reaches the specified number N_K of repeated calculation. Search will be ended if the number of calculation reaches N_K. And a set of $\mathbf{gbest}(r)$ which finally remained is considered as the Pareto optimum solution set.

4. Numerical Example

In this section, we use MOPSO to solve the proposed model. Figure 1 and 2 shows the result of Cost and CO_2. In other words period 16-30 are prediction periods. As for these graphs, the solid lines shows stage 1, the dashed lines shows stage 2 and the dotted lines shows stage 3. In MOPSO, We use a population of 45 particles, a repository size of 100 particles and from 100 to 10000 search times. Cost and CO_2 have little change on a stage 1. The cost of the sum total on every stage is generally the same. However, it turns out that it takes for going to a stage 3 by CO_2, and is increasing.

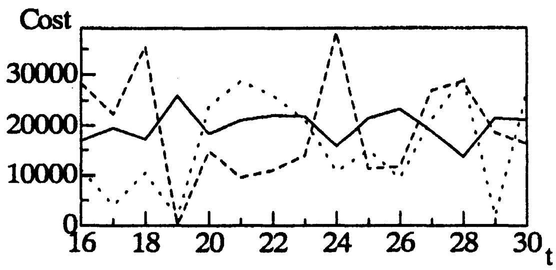

Figure 1. Result of Cost

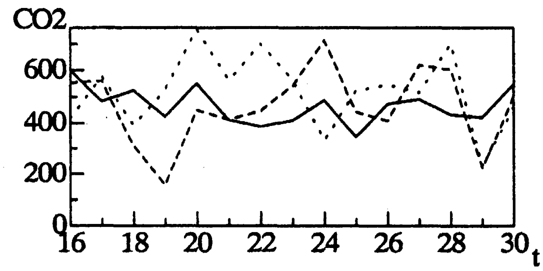

Figure 2. Result of CO_2

5. Conclusions

In this paper, we proposed the optimum target order quantity considering environmental issuer for supply chain management by MOPSO. Proposed model provides new optimal strategy which focus on environmental conscious in logistics and inventory of supply chain management from supplier to maker.

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