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Existence and Nonexistence of the Global Solutions for a Reaction-Diffusion System

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1. Introduction

We consider the Cauchy problem for the reaction-diffusion system:

$$(1.1)\qquad u_t - \Delta u = |x|^{\sigma_1} u^{p_1} v^{q_1}, \qquad x \in \boldsymbol{R}^N,\ t > 0,$$

$$(1.2)\qquad v_t - \Delta v = |x|^{\sigma_2} u^{p_2} v^{q_2}, \qquad x \in \boldsymbol{R}^N,\ t > 0,$$

$$u(x,0) = u_0(x) \geq 0, \not\equiv 0, \qquad x \in \boldsymbol{R}^N,$$

$$v(x,0) = v_0(x) \geq 0, \not\equiv 0, \qquad x \in \boldsymbol{R}^N,$$

where p_j, $q_j \geq 0$, $\sigma_j \geq 0$ $(j = 1,2)$, and $p_1, q_2 \neq 1$.

Our aim is to find conditions on the exponents σ_j, p_j, q_j $(j = 1,2)$ for the existence and the nonexistence of global solutions to the system (1.1)–(1.2).

At first we focus on the single equation : $u_t - \Delta u = u^p$. Let N be the space dimension. In [5], Fujita proved the existence of global solutions to the equation if $p > 1 + 2/N$ for exponential decaying small initial data. The author also proved the nonexistence of the global solutions if $p < 1 + 2/N$. In the critial case, $p = 1 + 2/N$, the nonexistence is proved in Hayakawa [6], Kobayashi, Sirao and Tanaka [7] and Weissler [11]. On the other hand, in the sublinear case, i.e. $0 < p < 1$, it is shown by Aguirre and Escobedo [1] that every solution for the equation exists globally in time.

There are various extensions of these results. For example, in [10] Pinsky showed the existence and nonexistence for the equation: $u_t - \Delta u = a(x)u^p$, where $p > 1$ and $a(x)$ behaves like $|x|^m$ for large $|x|$.

Next, we introduce the extended results to the system of the equations:

$$\begin{cases} u_t - \Delta u = F_1(u, v), \\ v_t - \Delta v = F_2(u, v). \end{cases}$$

Escobedo and Herrero [3] studied the system with the nonlinear teams $F_1 = v^p$ and $F_2 = u^q$ for nonnegative, continuous and bounded initial data, where p, $q > 0$. The situation is divided into three cases: (i) $pq > 1$ and $(\max\{p, q\}+1)/(pq-1) < N/2$, (ii) $pq > 1$ and $(\max\{p, q\}+1)/(pq-1) \geq N/2$, (iii) $pq < 1$. When $pq > 1$ and $(\max\{p, q\} + 1)/(pq-1) < N/2$, for small initial data there exist global solutions. For large data, there exist blowing up solutions. When $pq > 1$ and $(\max\{p, q\} + 1)/(pq - 1) \geq N/2$, there exist no global solutions. When $pq < 1$, every solution exists globally in time.

Now, we introduce two extenteded results of [3]. One is the result for the system with nonlinear teams $F_1 = |x|^{\sigma_1} v^p$ and $F_2 = |x|^{\sigma_2} u^q$ ($p, q > 1$, $0 \leq \sigma_j < N(p_j + q_j - 1)$, $j = 1, 2$). In [9], Mochizuki and Huang showed the existence and nonexistence result and the assymptotic behavior of the solution.

Another is for the system with $F_j = u^{p_j} v^{q_j}$, where $p_j, q_j \geq 0$, $0 < p_1 + q_1 \leq p_2 + q_2$ for each $j = 1, 2$. In [4], the situation is divided into two cases, $0 \leq p_1 \leq 1$ and $p_1 > 1$. In the former case, growth of the solutions by the interaction between two equation is stronger than self-growth of the solutions. In the latter case, self-growth of the solutions is stronger. These are understood from the following results: Put $\alpha = (q_1 - q_2 + 1)/\{p_2 q_1 - (1-p_1)(1-q_2)\}$, $\beta = (p_1 - p_2 + 1)/\{p_2 q_1 - (1-p_1)(1-q_2)\}$.

(i) Let $p_1 \leq 1$. If $0 \leq \max\{\alpha, \beta\} < N/2$, then global solution exists for small initial data. If $\max\{\alpha, \beta\} < 0$, then every solution exists globally in time.

(ii) Let $p_1 > 1$. If $p_1 + q_1 > 1 + 2/N$, then global solution exists for small initial data.

In (i), the condition for blowing up of the solutions consists of the exponents in both two equations. On the other hand, in (ii) the condition for consists of only the exponents in one equation.

We study (1.1)–(1.2) as an extention of these systems. Since our problem includes the sublinear case, p_j or $q_j < 1$, the contraction argument does not work to showing the global existence. In this paper, we show it by iteration argument in weighted L^∞ function space.

To show nonexistence theorems, the iteration argument of [4] is often used for reaction-diffusion systems. However, the method does not seem applicable for our problem because the nonlinear terms have the variable coefficients $|x|^{\sigma_j}$. In this paper, we improve the argument in [9] and apply it to our problem. The argument in [9] is to transform the system of PDEs into the ordinary differential inequalities. In our problem, multiplying the equation by negative power of unknown function makes the transformation possible.

REMARK 1.1. In [3], [4], [9], [6], [7], [10] and [11], the authors show that the solution blows up in critical case. This critical blow-up also occurs in our system (1.1)-(1.2).

2. MAIN RESULTS

For simplicity, let

$$\text{(2.1)} \qquad \begin{cases} \alpha = \dfrac{q_1(\sigma_2+2)+(1-q_2)(\sigma_1+2)}{2\{p_2q_1-(1-p_1)(1-q_2)\}}, \\ \beta = \dfrac{p_2(\sigma_1+2)+(1-p_1)(\sigma_2+2)}{2\{p_2q_1-(1-p_1)(1-q_2)\}}, \end{cases}$$

$$\text{(2.2)} \qquad \begin{cases} \delta_1 = \dfrac{q_1\sigma_2+(1-q_2)\sigma_1}{p_2q_1-(1-p_1)(1-q_2)}, \\ \delta_2 = \dfrac{p_2\sigma_1+(1-p_1)\sigma_2}{p_2q_1-(1-p_1)(1-q_2)}. \end{cases}$$

For $a \in \boldsymbol{R}$, we define the function spaces:

$$I^a = \{w \in C(\boldsymbol{R}^N); w(x) \geq 0, \ \limsup_{|x|\to\infty} |x|^a w(x) < \infty\},$$

and

$$L_a^\infty = \{w \text{ is measurable function on } \boldsymbol{R}^N; \\ w(x) \geq 0, \ \|w\|_{\infty,a} \equiv \| \langle x\rangle^a w(x)\|_\infty < \infty\},$$

where $\langle x\rangle = (1+|x|^2)^{1/2}$. We also define

$$E_T = \{(u,v);\ [0,T] \to L^\infty_{\delta_1} \times L^\infty_{\delta_2},\ \|(u,v)\|_{E_T} < \infty\},$$

where

$$\|(u,v)\|_{E_T} = \sup_{t\in[0,T]} (\|u(t)\|_{\infty,\delta_1} + \|v(t)\|_{\infty,\delta_2}).$$

Now, we state our main results. We assume that the initial data $(u_0, v_0) \in I^{\delta_1} \times I^{\delta_2}$.

THEOREM 2.1. *Let* $p_1 < 1$, $q_2 < 1$.
(i) *If* $\max(\alpha,\beta) \geq N/2$, *then no nontrivial global solutions of (1.1)–(1.2) exist.*
(ii) *If* $0 < \max(\alpha,\beta) < N/2$, *then there exist global solutions of (1.1)–(1.2) for small initial data, and there exist no global solutions for large initial data.*
(iii) *If* $\max(\alpha,\beta) < 0$, *then every solution of (1.1)–(1.2) exists globally in time.*

THEOREM 2.2. *Let* $p_1 > 1$, $q_2 < 1$.
(i) *If* $\alpha \geq N/2$ *or* $p_1 + q_1 \leq 1 + (2+\sigma_1)/N$, *then no nontrivial global solutions of (1.1)–(1.2) exist.*
(ii) *If* $\alpha < N/2$ *and* $p_1 + q_1 > 1 + (2+\sigma_1)/N$, *then there exist global solutions of (1.1)–(1.2) for small initial data, and there exist no global solutions for large initial data.*

THEOREM 2.3. *Let* $p_1 > 1$, $q_2 > 1$.
(i) *If* $p_1 + q_1 \leq 1 + (2+\sigma_1)/N$ *or* $p_2 + q_2 \leq 1 + (2+\sigma_2)/N$, *then no nontrivial global solutions of (1.1)–(1.2) exist.*
(ii) *If* $p_1+q_1 > 1+(2+\sigma_1)/N$ *and* $p_2+q_2 > 1+(2+\sigma_2)/N$, *then there exist global solutions of (1.1)–(1.2) for small initial data, and there exist no global solutions for large initial data.*

We can also rewrite the theorems into the way in Escobedo-Levine [4].

COROLLARY 2.4. *Assume that*

$$\frac{p_1+q_1-1}{\sigma_1+2} \leq \frac{p_2+q_2-1}{\sigma_2+2}, \tag{2.3}$$

and let $p_1 < 1$, $q_2 \neq 1$.
(i) *If* $\max(\alpha,\beta) \geq N/2$, *then there exist no global solutions for large initial data.*
(ii) *If* $0 < \max(\alpha,\beta) < N/2$, *then there exist global solutions for small initial data, and there exist no global solutions for large initial data.*
(iii) *If* $\max(\alpha,\beta) < 0$, *every solutions exists globally in time.*

COROLLARY 2.5. *Assume* (2.3), *and let* $p_1 > 1$, $q_2 \neq 1$.
(i) *If* $p_1 + q_1 \leq 1 + (2+\sigma_1)/N$, *then no nontrivial global solutions exist.*
(ii) *If* $p_1 + q_1 > 1 + (2+\sigma_1)/N$, *then no global solutions exist for large data.*

3. PROOF OF THEOREMS 2.1-2.3 : GLOBAL EXISTENCE

First, we show the local existence of classical solutions of (1.1)–(1.2).

THEOREM 3.1. *Let* δ_1 *and* δ_2 *be defined in* (2.2). *Assume that* $(u_0, v_0) \in I^{\delta_1} \times I^{\delta_2}$. *Then there exist classical solutions* $(u(t), v(t)) \in E_T$ *for the system* (1.1)–(1.2) *for some* $T > 0$.

PROOF. See Theorem 3.1 in [2]. □

Next, we introduce a comparison theorem and the existence of super-solutions.

COMPARISON PRINCIPLE

PROPOSITION 3.2. *Let* $f(u,v)$ *and* $g(u,v)$ *be strictly monotone increasing in* u *and* v *for* u, $v \geq 0$. *Assume that* $\bar{u}$, $\bar{v}$, $\underline{u}$, $\underline{v}$ *are non-negative and satisfy,*

$$\begin{cases} \bar{u}_t - \Delta\bar{u} \geq |x|^{\sigma_1} f(\bar{u},\bar{v}), \\ \bar{v}_t - \Delta\bar{v} \geq |x|^{\sigma_2} g(\bar{u},\bar{v}), \\ \underline{u}_t - \Delta\underline{u} \leq |x|^{\sigma_1} f(\underline{u},\underline{v}), \\ \underline{v}_t - \Delta\underline{v} \leq |x|^{\sigma_2} g(\underline{u},\underline{v}), \end{cases} \qquad \text{in } \boldsymbol{R}^N \times (0,T),$$

$$\begin{cases} \bar{u}(x,0) - \underline{u}(x,0) \geq 0, \not\equiv 0, \\ \bar{v}(x,0) - \underline{v}(x,0) \geq 0, \not\equiv 0. \end{cases} \qquad x \in \boldsymbol{R}^N.$$

Then we have $\bar{u}(x,t) \geq \underline{u}(x,t)$ *and* $\bar{v}(x,t) \geq \underline{v}(x,t)$ *on* $\boldsymbol{R}^N \times (0,T)$.

PROOF. See Proposition 4.1 in [2]. □

EXISTENCE OF SUPER-SOLUTIONS

PROPOSITION 3.3. *(i) Let* $p_1 > 1$, $q_2 > 1$ *or* $p_2q_1-(1-p_1)(1-q_2) > 0$, *and let* $p_1 + q_1 > 1$, $p_2 + q_2 > 1$. *Assume that one of the following conditions is satisfied:*
(A) $p_1, q_2 > 1$, $p_1 + q_1 > 1 + (2+\sigma_1)/N$, $p_2 + q_2 > 1 + (2+\sigma_2)/N$.
(B) $p_1 > 1 > q_2$, $p_1 + q_1 > 1 + (2+\sigma_1)/N$, $\alpha < N/2$.
(C) $p_1, q_2 < 1$, $p_2q_1 - (1-p_1)(1-q_2) > 0$, $\alpha, \beta < N/2$.
Then there exist C_1 ,C_2, α_1, $\beta_1 > 0$, $t_0 > 1$ *such that*

$$\bar{u}(x,t) = C_1(t+t_0)^{\alpha_1 - \frac{N}{2}} \exp\left(-\frac{|x|^2}{4(t+t_0)}\right), \tag{3.1}$$

$$\bar{v}(x,t) = C_2(t+t_0)^{\beta_1 - \frac{N}{2}} \exp\left(-\frac{|x|^2}{4(t+t_0)}\right) \tag{3.2}$$

are super-solutions of (1.1)–(1.2).
(ii) Let $p_1 > 1$, $q_2 > 1$ *or* $p_2q_1-(1-p_1)(1-q_2) > 0$. *And let* $p_1+q_1 > 1$, $p_2 + q_2 \leq 1$. *Assume that one of the following conditions is satisfied:*
(D) $p_1 > 1 > q_2$, $p_1 + q_1 > 1 + (2+\sigma_1)/N$, $\alpha < N/2$,
(E) $p_1, q_2 \leq 1$, $p_2q_1 - (1-p_1)(1-q_2) > 0$, $\alpha, \beta < N/2$.
Then there exist C_1 ,C_2, α_1, $\beta_1 > 0$, $t_0 > 1$, $a > 0$ *such that*

$$\bar{u}(x,t) = C_1(t+t_0)^{\alpha_1 - \frac{N}{2}} \exp\left(-\frac{|x|^2}{4(t+t_0)}\right), \tag{3.3}$$

$$\bar{v}(x,t) = C_2(t+t_0)^{\beta_1 - \frac{Na}{2}} \exp\left(-\frac{a|x|^2}{4(t+t_0)}\right), \tag{3.4}$$

are super-solutions of (1.1)–(1.2).
(iii) Let $p_1 < 1$, $q_2 < 1$ *and* $p_2q_1 - (1-p_1)(1-q_2) < 0$. *Then there exist* C_1 ,C_2, k, $a > 0$ *such that*

$$\bar{u}(x,t) = C_1\langle x\rangle^{-2\delta_1} \exp(kt), \tag{3.5}$$

$$\bar{v}(x,t) = C_2\langle x\rangle^{-2\delta_2} \exp(akt), \tag{3.6}$$

are super-solutions of (1.1)–(1.2).

Proof of Proposition 3.3 (i) Put

$$\bar{u}(x,t) = C_1(t+t_0)^{\alpha_1-\frac{N}{2}} \exp\left(-\frac{|x|^2}{4(t+t_0)}\right), \tag{3.7}$$

$$\bar{v}(x,t) = C_2(t+t_0)^{\beta_1-\frac{N}{2}} \exp\left(-\frac{|x|^2}{4(t+t_0)}\right), \tag{3.8}$$

where C_1 ,C_2, α_1, $\beta_1 > 0$, $t_0 > 1$. We can see that $(\bar{u}, \bar{v})$ are global super-solutions for small $C_1, C_2 > 0$ and large $t_0 > 1$, provided that

(3.9)

$$\begin{cases} \alpha_1 - N/2 - 1 > p_1(\alpha_1 - N/2) + q_1(\beta_1 - N/2) - \sigma_1/2, & \text{and} \\ \beta_1 - N/2 - 1 > p_2(\alpha_1 - N/2) + q_2(\beta_1 - N/2) - \sigma_2/2, \end{cases}$$

which (3.9) is equivalent to

$$(p_1 - 1)\alpha_1 + q_1\beta_1 < (p_1 + q_1 - 1)N/2 - (\sigma_1 + 2)/2, \quad \text{and} \tag{3.10}$$

$$p_2\alpha_1 + (q_2 - 1)\beta_1 < (p_2 + q_2 - 1)N/2 - (\sigma_2 + 2)/2. \tag{3.11}$$

Now, we shall show the existence of α_1, $\beta_1 > 0$ on the (α_1, β_1)-plane in each case of Proposition 3.3.

Case (A): $p_1, q_2 > 1$, $p_1 + q_1 > 1 + (2 + \sigma_1)/N$, $p_2 + q_2 > 1 + (2 + \sigma_2)/N$. Since the right hand sides of (3.10) and (3.11) are positive, we can take small α_1, $\beta_1 > 0$ satisfying (3.10) and (3.11).

Case (B): $p_1 > 1 > q_2$, $p_1 + q_1 > 1 + (2 + \sigma_1)/N$, $\alpha < N/2$.
We remark that the intersection of (3.10) and (3.11) is $(\alpha_1, \beta_1) = (N/2 - \alpha, N/2 - \beta)$. From the assumption, we can see that the intersection lies above the α_1-axis and that the boundary of (3.10) lies above the origin. For $\varepsilon_1, \varepsilon_2 > 0$, put $(\alpha_1, \beta_1) = (\varepsilon_1, \{(p_1 + q_1 - 1)N/2 - (\sigma_1 + 2)/2\}/q_1 + \varepsilon_2)$. Then there exist small constants $\varepsilon_1, \varepsilon_2 > 0$ such that (α_1, β_1) satisfy (3.10) and (3.11).

Case (C): $p_1, q_2 < 1$, $p_2q_1 - (1 - p_1)(1 - q_2) > 0$, $\alpha, \beta < N/2$.
From the assumption, we can see that the intersection lies in the first quadrant. Since $p_1, q_2 < 1$ and $p_2q_1 - (1 - p_1)(1 - q_1) > 0$, we have $(1 - p_1)/q_1 < p_2/(1 - q_2)$, that is, the angular coefficient of (3.11) is larger than that of (3.10). Hence, there exist small constants $\varepsilon_1, \varepsilon_2 > 0$ such that $(\alpha_1, \beta_1) = (N/2 - \alpha - \varepsilon_1, N/2 - \beta - \varepsilon_2)$ satisfy (3.10) and

(3.11). □

Proof of Proposition 3.3 (ii) Case (D): $p_1 > 1 > q_2$, $p_1 + q_1 > 1 + (2+\sigma_1)/N$, $\alpha < N/2$.
Put $a > 0$ such that

$$\max\left\{0, \frac{(1-p_1)N + (\sigma_1+2)}{q_1 N}\right\} < a < \frac{p_2}{1-q_2}. \tag{3.12}$$

In fact, since $q_2 < 1$, $p_2 q_1 - (1-p_1)(1-q_2) > 0$ and $\alpha < N/2$, we have

$$\begin{aligned}
&\frac{p_2}{1-q_2} - \frac{(1-p_1)N + (\sigma_1+2)}{q_1 N} \\
&= \frac{1}{Nq_1(1-q_2)}\{Nq_1p_2 - N(1-q_2)(1-p_1) - (1-q_2)(\sigma_1+2)\} \\
&= \frac{2\{p_2q_1 - (1-p_1)(1-q_2)\}}{Nq_1(1-q_2)}\left\{\frac{N}{2} - \frac{(1-q_2)(\sigma_1+2)}{2(p_2q_1 - (1-p_1)(1-q_2))}\right\} \\
&\geq \frac{2\{p_2q_1 - (1-p_1)(1-q_2)\}}{Nq_1(1-q_2)}\left(\frac{N}{2} - \alpha\right) \\
&> 0.
\end{aligned}$$

Therefore we can take $a > 0$ satysfying (3.12). Let

$$\bar{u}(x,t) = C_1(t+t_0)^{\alpha_1 - \frac{N}{2}} \exp\left(-\frac{|x|^2}{4(t+t_0)}\right), \tag{3.13}$$

$$\bar{v}(x,t) = C_2(t+t_0)^{\beta_1 - \frac{Na}{2}} \exp\left(-\frac{a|x|^2}{4(t+t_0)}\right), \tag{3.14}$$

where C_1 ,C_2, α_1, $\beta_1 > 0$, $t_0 > 1$. We can see that $(\bar{u}, \bar{v})$ are global super-solutions provided that

(3.15)
$$\begin{cases} \alpha_1 - N/2 - 1 > p_1(\alpha_1 - N/2) + q_1(\beta_1 - Na/2) - \sigma_1/2, \quad \text{and} \\ \beta_1 - Na/2 - 1 > p_2(\alpha_1 - N/2) + q_2(\beta_1 - Na/2) - \sigma_2/2, \end{cases}$$

for small $C_1, C_2 > 0$ and large $t_0 > 1$. And (3.15) is equivalent to

$$(p_1 - 1)\alpha_1 + q_1\beta_1 < (p_1 + aq_1 - 1)N/2 - (\sigma_1+2)/2, \quad \text{and} \tag{3.16}$$

$$p_2\alpha_1 + (q_2 - 1)\beta_1 < (p_2 + aq_2 - a)N/2 - (\sigma_2+2)/2. \tag{3.17}$$

We remark that the intersection of

$$(p_1-1)\alpha_1+q_1\beta_1=(p_1+aq_1-1)N/2-(\sigma_1+2)/2, \quad \text{and}$$
$$p_2\alpha_1+(q_2-1)\beta_1=(p_2+aq_2-a)N/2-(\sigma_2+2)/2.$$

is $(\alpha_1,\beta_1)=(N/2-\alpha, Na/2-\beta)$. From the assumption $\alpha<N/2$, we see that the intersection lies above the α_1-axis. From $a>\{(1-p_1)N+(\sigma_1+2)\}/q_1N$, we can easily see that the boundary of (3.16) lies above the origin. Hence, we can prove the existence of (α_1,β_1) satisfying (3.16) and (3.17) in the same way as in Case (B).

Case (E): $p_1, q_2\le 1$, $p_2q_1-(1-p_1)(1-q_2)>0$, $\alpha,\beta<N/2$
Putting $a>0$ satisfying

$$\max\left\{\frac{1-p_1}{q_1},\frac{2\beta}{N}\right\}<a<\frac{p_2}{1-q_2}, \tag{3.18}$$

we can prove in the same way as in Case (C). In fact, since $q_2<1$, $p_2q_1-(1-p_1)(1-q_2)>0$ and $\alpha<N/2$, we have

$$\begin{aligned}
&\frac{p_2}{1-q_2}-\frac{2\beta}{N}\\
&=\frac{p_2N\{p_2q_1-(1-p_1)(1-q_2)\}-(1-p_1)p_2(\sigma_1+2)-(1-p_1)(1-q_2)(\sigma_2+2)}{(1-q_2)\{p_2q_1-(1-p_1)(1-q_2)\}}\\
&=\frac{p_2N\{p_2q_1-(1-p_1)(1-q_2)\}-(1-p_1)p_2(\sigma_1+2)-p_2q_1(\sigma_2+2)}{(1-q_2)\{p_2q_1-(1-p_1)(1-q_2)\}}\\
&+\frac{p_2q_1(\sigma_2+2)-(1-p_1)(1-q_2)(\sigma_2+2)}{(1-q_2)\{p_2q_1-(1-p_1)(1-q_2)\}}\\
&=\frac{2p_2N}{1-q_2}\left(\frac{N}{2}-\alpha\right)+\frac{\sigma_2+2}{1-q_2}\\
&>0,
\end{aligned}$$

and since $p_1,q_2\le 1$, $p_2q_1-(1-p_1)(1-q_2)>0$, we have $(1-p_1)/q_1< p_2/(1-q_2)$. Therefore, we can take $a>0$ satisfying (3.18). □

Proof of Proposition 3.3 (iii) Let $a=\frac{p_2}{1-q_2}$. Put

$$\bar{u}(x,t)=C_1\langle x\rangle^{-2\delta_1}\exp(kt), \tag{3.19}$$

$$\bar{v}(x,t)=C_2\langle x\rangle^{-2\delta_2}\exp(akt), \tag{3.20}$$

where C_1 ,C_2, $k > 0$. We can see that $(\bar{u}, \bar{v})$ are global super-solutions for large $k > 0$.

We are now in a position to prove the global existence theorems.

Proof of Theorems 2.1(i), 2.2 and 2.3. Let T^* be the maximal existence time of the classical solutions for (1.1)–(1.2). From the local existence theorem in Section 3, it is clear that $T^* \neq 0$. Assume $T^* < \infty$. If the initial data (u_0, v_0) are sufficiently small, then the solutions (u, v) are estimated above by the super-solutions in Proposition 3.3. Using Theorem 3.1, we can extend the solutions (u, v) with new initial data $(u(T^*), v(T^*))$ to time $T^{**} > T^*$. This contradicts the maximality of T^*. Hence $T^* = \infty$. □

Proof of Theorem 2.1 (ii). The constants C_1 and $C_2 > 0$ in Proposition 3.3 (iii) have no restriction. Hence, the argument as above works for arbitrary initial data in $I^{\delta_1} \times I^{\delta_2}$. □

4. Preliminaries to nonexistence theorems

In this section, we prepare several estimates for the solutions. To show them, we introduce the system of integral equations associated to (1.1) and (1.2):

$$u(t) = S(t)u_0 + \int_0^t S(t-s)|\cdot|^{\sigma_1} u(s)^{p_1} v(s)^{q_1} ds, \tag{4.1}$$

$$v(t) = S(t)v_0 + \int_0^t S(t-s)|\cdot|^{\sigma_2} u(s)^{p_2} v(s)^{q_2} ds, \tag{4.2}$$

where

$$S(t)f(x) = (4\pi t)^{-\frac{N}{2}} \int_{R^N} \exp\left(-\frac{|x-y|^2}{4t}\right) f(y) dy.$$

The following lemma is a well-known estimate for the heat equations.

LEMMA 4.1. *Let u and v be solutions of the system* (1.1) *and* (1.2). *There exists $C > 0$ such that*

$$u(x,t) \geq C(1+t)^{-\frac{N}{2}} \exp\left(-\frac{|x|^2}{2t}\right), \qquad (t>0),$$

$$v(x,t) \geq C(1+t)^{-\frac{N}{2}} \exp\left(-\frac{|x|^2}{2t}\right), \qquad (t>0).$$

Moreover, we can add logarithmic growth to the bounds in the critical case.

LEMMA 4.2. ([4]) *Let u and v be solutions of the system* (1.1) *and* (1.2). *Assume that*

$$u(x,t) \geq C_1(1+t)^{-\frac{N}{2}} \exp\left(-\frac{|x|^2}{t}\right), \qquad (t>0),$$

$$v(x,t) \geq C_2(1+t)^{m} \exp\left(-\frac{C_3|x|^2}{t}\right), \qquad (t>t_0),$$

where C_1, C_2, $C_3 > 0$, $t_0 \geq 0$ and $m \in \boldsymbol{R}$. If m and σ_1 satisfy

$$-\frac{Np_1}{2} + mq_1 + \frac{\sigma_1+2}{2} = -\frac{N}{2}, \quad \sigma_1 > \max(-2,-N),$$

then there exist constants C_4, $C_5 > 0$ and $t_1 > t_0$ such that

$$u(x,t) \geq C_4(1+t)^{-\frac{N}{2}} \log(1+t) \exp\left(-\frac{C_5|x|^2}{t}\right), \qquad (t>t_1).$$

PROOF. See Proposition 1 in [4]. □

The following two lemmas are for the sublinear case.

LEMMA 4.3. *Let $0 \leq q_2 < 1$, $\sigma_2 > \max(-2,-N)$ and define*

$$\bar{v}(x,t) = \widetilde{C} t^{\frac{\sigma_2+2}{2(1-q_2)}} \left(S(t)u_0(x)^{\varepsilon}\right)^{\frac{p_2}{\varepsilon(1-q_2)}}.$$

for $\widetilde{C}$, $\varepsilon > 0$. If $\widetilde{C}$ and ε are sufficiently small, then $\bar{v}(x,t)$ is a subsolution for the problem:

$$v_t - \Delta v = |x|^{\sigma_2} u^{p_2} v^{q_2}, \qquad x \in \boldsymbol{R}^N, \ t>0,$$

$$v(x,0) = v_0(x), \qquad x \in \boldsymbol{R}^N.$$

PROOF. Let $k > \max\{(\sigma_2 + N)/N, 1\}$ and $0 < \varepsilon < \min(1, p_2/\{(1 - q_2)k\})$. It suffices to prove that

$$\bar{v}(x,t) \leq \int_0^t S(t-s)|x|^{\sigma_2} (S(s)u_0(x))^{p_2} \bar{v}(x,s)^{q_2} ds.$$

By Jensen's inequality, we have

$$\begin{aligned} &\int_0^t S(t-s)|x|^{\sigma_2} (S(s)u_0(x))^{p_2} \bar{v}(x,s)^{q_2} ds \\ &\geq \widetilde{C}^{q_2} \int_0^t s^{\frac{q_2(\sigma_2+2)}{2(1-q_2)}} S(t-s)|x|^{\sigma_2} (S(s)u_0(x)^{\varepsilon})^{\frac{p_2}{\varepsilon(1-q_2)}} ds. \end{aligned} \tag{4.3}$$

Using the inverse Hölder inequality and Jensen's inequality again, we have for $k > 1$,

$$\begin{aligned} &S(t-s)|x|^{\sigma_2} (S(s)u_0(x)^{\varepsilon})^{\frac{p_2}{\varepsilon(1-q_2)}} \\ &\geq \{S(t-s)|x|^{\frac{\sigma_2}{1-k}}\}^{1-k}\{S(t-s)\,(S(s)u_0(x)^{\varepsilon})^{\frac{p_2}{k\varepsilon(1-q_2)}}\}^k \\ &\geq \{C_1(t-s)^{\frac{\sigma_2}{2(1-k)}}\}^{1-k}\{S(t-s)\,(S(s)u_0(x)^{\varepsilon})\}^{\frac{p_2}{\varepsilon(1-q_2)}} \\ &= C_1^{1-k}(t-s)^{\frac{\sigma_2}{2}} (S(t)u_0(x)^{\varepsilon})^{\frac{p_2}{\varepsilon(1-q_2)}}. \end{aligned} \tag{4.4}$$

Substituting (4.4) into (4.3), we obtain

$$\begin{aligned} &\int_0^t S(t-s)|x|^{\sigma_2} (S(s)u_0(x))^{p_2} \bar{v}(x,s)^{q_2} ds \\ &\geq \widetilde{C}^{q_2} C_1^{1-k} (S(t)u_0(x)^{\varepsilon})^{\frac{p_2}{\varepsilon(1-q_2)}} \int_0^t s^{\frac{q_2(\sigma_2+2)}{2(1-q_2)}} (t-s)^{\frac{\sigma_2}{2}} ds \\ &\geq \widetilde{C}^{q_2} C_1^{1-k} C_2 t^{\frac{\sigma_2+2}{2(1-q_2)}} (S(t)u_0(x)^{\varepsilon})^{\frac{p_2}{\varepsilon(1-q_2)}} \\ &= \widetilde{C}^{q_2-1} C_1^{1-k} C_2 \bar{v}(x,t) \\ &\geq \bar{v}(x,t) \end{aligned}$$

for sufficiently small $\widetilde{C} > 0$. This completes the proof. □

LEMMA 4.4. *Let $0 \leq q_2 < 1$, and $\sigma_2 > (-2, -N)$ and let u and v be solutions of the system (1.1) and (1.2). Then there exist constants C_1, $C_2 > 0$ such that*

$$v(x,t) \geq C_1 t^{\frac{\sigma_2+2}{2(1-q_2)}} (1+t)^{-\frac{p_2 N}{2(1-q_2)}} \exp\left(-\frac{C_2|x|^2}{t}\right), \qquad (t > 0).$$

PROOF. Fix arbitrary $s > 0$, and apply Lemma 4.3 to $U(t) = u(t+s)$ and $V(t) = v(t+s)$. Then, we have

$$V(x,t) \geq Ct^{\frac{\sigma_2+2}{2(1-q_2)}} \left(S(t)U(x,0)^{\varepsilon}\right)^{\frac{p_2}{\varepsilon(1-q_2)}} .$$

Putting $s = t$ and using Lemma 4.1, we obtain

$$\begin{aligned} v(x,2t) &\geq Ct^{\frac{\sigma_2+2}{2(1-q_2)}} \left(S(t)u(x,t)^{\varepsilon}\right)^{\frac{p_2}{\varepsilon(1-q_2)}} \\ &\geq Ct^{\frac{\sigma_2+2}{2(1-q_2)}} (1+t)^{-\frac{p_2 N}{2(1-q_2)}} \left\{(4\pi t)^{-\frac{N}{2}} \int \exp\left(-\frac{|x-y|^2}{4t} - \frac{\varepsilon|y|^2}{2t}\right) dy\right\}^{\frac{p_2}{\varepsilon(1-q_2)}} \\ &\geq Ct^{\frac{\sigma_2+2}{2(1-q_2)}} (1+t)^{-\frac{p_2 N}{2(1-q_2)}} \exp\left(-\frac{C|x|^2}{t}\right). \end{aligned}$$

This completes the proof. □

5. PROOF OF THEOREM 2.1 : NONEXISTENCE

In this section we prove Theorem 2.1 (i). For Theorem 2.1 (ii) and (iii), see [12] .

NECESSARY CONDITION FOR THE GLOBAL EXISTENCE Assume that (u, v) are global solutions for (1.1) and (1.2). Since $p_1 < 1$, $q_2 < 1$ and $p_2 q_1 - (1-p_1)(1-q_2) > 0$, we can take a positive constant $k > 0$ such that $(1-q_2)/p_2 < k < q_1/(1-p_1)$. For this k, fix positive constants r_1, $r_2 > 0$ satisfying

$$\begin{aligned} &r_2 = kr_1, \\ &r_1 < \min\{1-p_1, \ p_2\}, \\ &r_2 < \min\{1-q_2, \ q_1\}, \\ &r_1\sigma_1 < \frac{N(q_1 - k(1-p_1))}{k}, \\ &r_2\sigma_2 < \frac{N(kp_2 - (1-q_2))}{k}. \end{aligned}$$

For $\varepsilon > 0$, define the cut off function

$$\rho_\varepsilon(x) = \begin{cases} \varepsilon^{\frac{N}{2}} \exp\left(-\dfrac{1}{1-\varepsilon|x|^2}\right) & (|x| < \varepsilon^{-\frac{1}{2}}) \\ 0 & (|x| \geq \varepsilon^{-\frac{1}{2}}), \end{cases}$$

and set

$$F_\varepsilon(t) = \int_{\boldsymbol{R}^N} u(x,t)^{r_1} \rho_\varepsilon(x)dx, \tag{5.1}$$

$$G_\varepsilon(t) = \int_{\boldsymbol{R}^N} v(x,t)^{r_2} \rho_\varepsilon(x)dx. \tag{5.2}$$

Then the following inequalities hold.

LEMMA 5.1. *Let $p_1 < 1$, $q_2 < 1$ and $\sigma_j > -N$ $(j = 1, 2)$. Then there exist constants C_1, C_2, C_3, $C_4 > 0$ such that*

$$F_\varepsilon'(t) \geq -C_1 \varepsilon F_\varepsilon(t) + C_2 \varepsilon^{-\frac{\sigma_1}{2}} F_\varepsilon(t)^{-\frac{(1-p_1)-r_1}{r_1}} G_\varepsilon(t)^{\frac{q_1}{r_2}}, \tag{5.3}$$

$$G_\varepsilon'(t) \geq -C_3 \varepsilon G_\varepsilon(t) + C_4 \varepsilon^{-\frac{\sigma_2}{2}} F_\varepsilon(t)^{\frac{p_2}{r_1}} G_\varepsilon(t)^{-\frac{(1-q_2)-r_2}{r_2}}. \tag{5.4}$$

PROOF. Multiplying (1.1) by $u^{r_1-1}\rho_\varepsilon$, and integrating over $\boldsymbol{R}^N$ with respect to x, we obtain the desired inequality (5.3). Indeed, integration by parts implies that

$$\int_{\boldsymbol{R}^N} \rho_\varepsilon u^{r_1-1} u_t dx = \frac{1}{r_1}\frac{d}{dt} F_\varepsilon(t),$$

$$\begin{aligned} \int_{\boldsymbol{R}^N} \rho_\varepsilon u^{r_1-1} \Delta u dx &\geq -\int_{\boldsymbol{R}^N} \nabla\rho_\varepsilon \cdot u^{r_1-1} \nabla u dx \\ &= -\frac{1}{r_1}\int_{\boldsymbol{R}^N} \nabla\rho_\varepsilon \cdot \nabla(u^{r_1}) dx \\ &= \frac{1}{r_1}\int_{\boldsymbol{R}^N} u^{r_1} \Delta\rho_\varepsilon dx \\ &\geq -\frac{C\varepsilon}{r_1} F_\varepsilon(t). \end{aligned}$$

Here, we have used the property of ρ_ε that there exists a constant $C > 0$ depending only on N such that $\Delta\rho_\varepsilon \geq -C\varepsilon\rho_\varepsilon$. The normal and inverse Hölder inequalities also imply that

$$\int_{\boldsymbol{R}^N} \rho_\varepsilon |x|^{\sigma_1} u^{r_1-(1-p_1)} v^{q_1} dx$$

$$\geq \left(\int_{|x|<\varepsilon^{-\frac{1}{2}}} \rho_\varepsilon v^{r_2} dx \right)^{\frac{q_1}{r_2}} \left(\int_{|x|<\varepsilon^{-\frac{1}{2}}} \rho_\varepsilon |x|^{\frac{r_2\sigma_1}{r_2-q_1}} u^{\frac{r_2(r_1-(1-p_1))}{r_2-q_1}} dx \right)^{\frac{r_2-q_1}{r_2}}$$

$$\geq G_\varepsilon^{\frac{q_1}{r_2}} \left(\int_{|x|<\varepsilon^{-\frac{1}{2}}} \rho_\varepsilon u^{r_1} dx \right)^{\frac{r_1-(1-p_1)}{r_1}} \left(\int_{|x|<\varepsilon^{-\frac{1}{2}}} \rho_\varepsilon |x|^{-\frac{r_1 r_2 \sigma_1}{r_1 q_1 - r_2(1-p_1)}} dx \right)^{-\frac{r_1 q_1 - r_2(1-p_1)}{r_1 r_2}}$$

$$= C\varepsilon^{-\frac{\sigma_1}{2}} F_\varepsilon(t)^{-\frac{(1-p_1)-r_1}{r_1}} G_\varepsilon(t)^{\frac{q_1}{r_2}}.$$

Multiplying (1.2) by $v^{r_2-1}\rho_\varepsilon$, and integrating over $\boldsymbol{R}^N$ with respect to x, we can also get (5.4). □

Setting

$$\widetilde{F_\varepsilon}(t) = F_\varepsilon^{\frac{1-p_1}{r_1}}(t),$$
$$\widetilde{G_\varepsilon}(t) = G_\varepsilon^{\frac{1-q_2}{r_2}}(t),$$

we simplify the inequalities (5.3) and (5.4).

LEMMA 5.2. *Let $p_1 < 1$, $q_2 < 1$ and $\sigma_j > -N$ $(j = 1, 2)$. Then there exist constants C_5, C_6, C_7, $C_8 > 0$ such that*

$$\widetilde{F_\varepsilon}'(t) \geq -C_5\varepsilon \widetilde{F_\varepsilon}(t) + C_6 \varepsilon^{-\frac{\sigma_1}{2}} \widetilde{G_\varepsilon}(t)^{\frac{q_1}{1-q_2}},$$
$$\widetilde{G_\varepsilon}'(t) \geq -C_7\varepsilon \widetilde{G_\varepsilon}(t) + C_8 \varepsilon^{-\frac{\sigma_2}{2}} \widetilde{F_\varepsilon}(t)^{\frac{p_2}{1-p_1}}.$$

From the phase field argument in [9], we get upper bounds of $F_\varepsilon(t)$ and $G_\varepsilon(t)$ as follows:

PROPOSITION 5.3. *Let $p_1 < 1$, $q_2 < 1$ and $\sigma_j > -N$ $(j = 1, 2)$.*
(i) *There exist constants $A > 0$ and $B > 0$ such that*

$$\widetilde{F_\varepsilon}(t) \leq A\varepsilon^{\alpha(1-p_1)}, \tag{5.5}$$

$$\widetilde{G_\varepsilon}(t) \leq B\varepsilon^{\beta(1-q_2)}, \tag{5.6}$$

for all $t > 0$ *and* $\varepsilon > 0$, *where* α *and* β *are defined in* (2.1).
(ii) (*upperbounds*) *There exist constants* $A > 0$ *and* $B > 0$ *such that*

$$F_\varepsilon(t) \le A\varepsilon^{\alpha r_1}, \tag{5.7}$$

$$G_\varepsilon(t) \le B\varepsilon^{\beta r_2}, \tag{5.8}$$

for all $t > 0$ *and* $\varepsilon > 0$.

PROOF OF THEOREM 2.1(i). We consider the case $\alpha \ge N/2$. Lemmas 4.1, 4.2, 4.4, and the definition of F_ε in (5.1) give lower bounds of $F_\varepsilon(\varepsilon^{-1})$:

$$F_\varepsilon(\varepsilon^{-1}) \ge \begin{cases} C_5\varepsilon^{\frac{Nr}{2}}, & (\alpha > \frac{N}{2}), \\ C_6\varepsilon^{\frac{Nr}{2}}\log(1+\varepsilon^{-1}), & (\alpha = \frac{N}{2}). \end{cases} \tag{5.9}$$

Indeed, in the critical case $\alpha = N/2$, we have

$$u(x,t) \le C(1+t)^{-\frac{N}{2}}\exp\left(-\frac{|x|^2}{t}\right), \qquad (t > 0),$$

$$v(x,t) \le C(1+t)^{\frac{\sigma_2+2-p_2N}{2(1-q_2)}}\exp\left(-\frac{C|x|^2}{t}\right), \qquad (t > 1)$$

from Lemmas 4.1 and 4.4. Applying Lemma 4.2, we have

$$u(x,t) \le C(1+t)^{-\frac{N}{2}}\log(1+t)\exp\left(-\frac{|x|^2}{t}\right), \qquad (t > t_0) \tag{5.10}$$

for some $t_0 > 1$. Substituting (5.10) into (5.1), we obtain (5.9). This contradicts (5.7) for small $\varepsilon > 0$. This completes the proof.□

6. PROOFS OF THEOREMS 2.2 AND 2.3 : NONEXISTENCE

In this section we prove Theorems 2.2 (i) and 2.3 (i). In order to prove the theorems, it suffices to show the following propositions.

PROPOSITION 6.1. *Let* $p_1 > 1$, $q_2 < 1$. *If* $\alpha \ge N/2$, *then no nontrivial global solutions exist.*

PROPOSITION 6.2. *Let* $p_1 > 1$. *If* $p_1 + q_1 \le 1 + (2+\sigma_1)/N$, *then no nontrivial global solutions exist.*

Necessary condition for the global existence Assume that (u, v) are global solutions for (1.1) and (1.2). For $\varepsilon > 0$, define

$$F_\varepsilon(t) = \int_{\mathbf{R}^N} u(x,t)^r \rho_\varepsilon(x)dx, \tag{6.1}$$

where $r > 0$ satisfying $r\sigma_1 < \frac{N}{p_1 - 1}$.

Multiplying (1.1) by $\rho_\varepsilon(x)u^{r-1}$ and integrating by parts, we have

$$F_\varepsilon(t)' \geq -C_1\varepsilon F_\varepsilon(t) + C_2\varepsilon^{-\frac{\sigma_1}{2}} t^{\frac{q_1(\sigma_2+2)-p_2q_1N}{2(1-q_2)}} F_\varepsilon(t)^{\frac{r+p_1-1}{r}} \quad (t \geq \varepsilon^{-1}),$$

where C_1 and $C_2 > 0$. Indeed, from the inverse Hölder inequality and Lemma 4.4,

$$\begin{aligned} &\int_{\mathbf{R}^N} \rho_\varepsilon |x|^{\sigma_1} u^{r+p_1-1} v^{q_1} dx \\ &\geq \left(\int_{\mathbf{R}^N} \rho_\varepsilon u^r dx\right)^{\frac{r+p_1-1}{r}} \left(\int_{\mathbf{R}^N} \rho_\varepsilon |x|^{\frac{r\sigma_1}{1-p_1}} v^{\frac{rq_1}{1-p_1}} dx\right)^{\frac{1-p_1}{r}} \\ &\geq F_\varepsilon(t)^{\frac{r+p_1-1}{r}} \cdot C\varepsilon^{-\frac{\sigma_1}{2}} t^{\frac{q_1(\sigma_2+2)-p_2q_1N}{2(1-q_2)}}. \end{aligned}$$

Putting

$$\begin{cases} \widetilde{F_\varepsilon}(s) = \varepsilon^{\frac{q_1(\sigma_2+2)+(1-q_2)(\sigma_1+2)-p_2q_1N}{2(1-p_1)(1-q_2)}r} F_\varepsilon(t), \\ s = \varepsilon t, \end{cases}$$

yields the following inequality:

$$\widetilde{F_\varepsilon}(s)' \geq -C_1\widetilde{F_\varepsilon}(s) + C_2 s^{\frac{q_1(\sigma_2+2)-p_2q_1N}{2(1-q_2)}} \widetilde{F_\varepsilon}(s)^{\frac{r+p_1-1}{r}} \qquad (s \geq 1).$$

A comparison argument and the global existence of $\widetilde{F_\varepsilon}(s)$ imply that

$$\widetilde{F_\varepsilon}(1) \leq K,$$

where $K > 0$ is independent of $0 < \varepsilon \leq 1$. Hence,

$$F_\varepsilon(\varepsilon^{-1}) \leq K\varepsilon^{-\frac{q_1(\sigma_2+2)+(1-q_2)(\sigma_1+2)-p_2q_1N}{2(1-p_1)(1-q_2)}r}, \tag{6.2}$$

for $0 < \varepsilon \leq 1$.

PROOF OF PROPOSITION 6.1. Lemmas 4.1 and 4.2, and the definition of F_ε in (6.1) give lower bounds of $F_\varepsilon(\varepsilon^{-1})$:

$$F_\varepsilon(\varepsilon^{-1}) \geq \begin{cases} C_3\varepsilon^{\frac{Nr}{2}}, & (\alpha > \frac{N}{2}), \\ C_4\varepsilon^{\frac{Nr}{2}}\log(1+\varepsilon^{-1}), & (\alpha = \frac{N}{2}), \end{cases}$$

which contradicts (6.2) for small $\varepsilon > 0$. Indeed, one can see that $\alpha \geq \frac{N}{2}$ is equivalent to

$$-\frac{q_1(\sigma_2+2)+(1-q_2)(\sigma_1+2)-p_2q_1N}{2(1-p_1)(1-q_2)} \geq \frac{N}{2}.$$

This completes the proof. □

PROOF OF PROPOSITION 6.2. Using Lemma 4.1 instead of Lemma 4.4 for the estimate of $v(x,t)$, we can prove Proposition 6.2 in the same way as the proof of Proposition 6.1. □

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