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Existence Theorems of Two Families of Vector Generalized Quasi-Optimization Problems with Applications

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ABSTRACT

In this paper, we apply Himmelberg's fixed point theorem to establish existence theorems of two families of vector generalized quasi-optimization problems. We apply our results to establish existence theorems of systems of generalized vector-quasi-equilibrium problems. Systems of weak loose quasi-saddle point problem.

1 Introduction

Recently, Lin [7] considered simultaneous vector quasi-equilibrium problem and proved existence results for its solution. By using these results, he derived existence results for a solution of vector quasi-saddle point problem.

In the recent past, systems of scalar (vector) equilibrium problems, systems of scalar (vector) generalized equilibrium problems, systems of scalar (vector) quasi-equilibrium problems, and systems of scalar (vector) generalized quasi-equilibrium problems are used as tools to solve Nash equilibrium problem (for vector-valued functions) and Debreu type equilibrium problem(for vector-valued functions), respectively, see for example [1, 2, 3, 4, 5] and references therein.

Very recently, Ansari et al. [6] considered systems of simultaneous generalized vector quasi-equilibrium problem and proved existence results for its solution by scalarization method. By using these results, they derived existence existence results of a solution of system of vector quasi-saddle point problem.

Let I be any index set. For each $i \in I$, let E_i, V_i and Z_i be real locally convex topological vector spaces (in short, t.v.s.). For each $i \in I$, let $X_i \subset E_i$ be a nonempty convex set and $Y_i \subset V_i$ a nonempty convex set. Let $X = \prod_{i\in I} X_i$ and $Y = \prod_{i\in I} Y_i$. For each $i \in I$, let $S_i : X \times Y \multimap X_i$ be a multivalued map with nonempty values and $T_i : X \times Y \multimap Y_i$ be a multivalued map with nonempty values. Let $C_i : X \times Y \multimap Z_i$ be a multivalued map such that for each $(x, y) \in X \times Y$, $C_i(x, y)$ is a cone and $intC_i(x, y) \neq \emptyset$. Let $F_i : X \times Y \times X_i \multimap Z_i$ be a multivalued map with nonempty values and $G_i : X \times Y \times Y_i \multimap Z_i$ be a multivalued map with nonempty values.

Throughout this paper, we use these notation unless otherwise specified.

We first consider two families of vector generalized quasi-optimization problems :
Find a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $F_i(\bar{x}, \bar{y}, \bar{x}_i) \cap wMin_{C_i(\bar{x},\bar{y})} F_i(\bar{x}, \bar{y}, S_i(\bar{x}, \bar{y})) \neq \emptyset$ and $G_i(\bar{x}, \bar{y}, \bar{y}_i) \cap wMin_{C_i(\bar{x},\bar{y})} G_i(\bar{x}, \bar{y}, T_i(\bar{x}, \bar{y})) \neq \emptyset$.

For the special case of above problems is systems of simultaneous generalized vector quasi-equilibrium problem for multivalued maps.
Find a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $F_i(\bar{x}, \bar{y}, x_i) \cap (-intC_i(\bar{x}, \bar{y})) = \emptyset$ for all $x_i \in S_i(\bar{x}, \bar{y})$ and $G_i(\bar{x}, \bar{y}, y_i) \cap (-intC_i(\bar{x}, \bar{y})) = \emptyset$ for all $y_i \in T_i(\bar{x}, \bar{y})$.
If F_i and G_i are single-valued maps. will be reduced to find a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $f_i(\bar{x}, \bar{y}, x_i) \notin (-intC_i(\bar{x}, \bar{y}))$ for all $x_i \in S_i(\bar{x}, \bar{y})$ and $g_i(\bar{x}, \bar{y}, y_i) \notin (-intC_i(\bar{x}, \bar{y}))$ for all $y_i \in T_i(\bar{x}, \bar{y})$.
This problem is a generalization of in Ansari et al. [5].

In section 4, we consider the following systems of weak loose quasi-saddle point problem.

Find $\bar{x} = (\bar{x}_i)_{i\in I} \in X$ and $\bar{y} = (\bar{y}_i)_{i\in I} \in Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $L_i(\bar{x}_i, \bar{y}_i) \cap wMax_{C_i(\bar{x},\bar{y})} L_i(S_i(\bar{x}, \bar{y}), \bar{y}_i) \neq \emptyset$ and $L_i(\bar{x}_i, \bar{y}_i) \cap wMin_{C_i(\bar{x},\bar{y})} L_i(\bar{x}_i, T_i(\bar{x}, \bar{y})) \neq \emptyset$, where $L_i : X_i \times Y_i \multimap Z_i$.

In this paper, we prove existence theorems of two families of vector generalized quasi-optimization problems by Himmelberg's fixed point theorem. Then we apply our results to study existence theorem of systems of weak loose quasi-saddle point problem and systems of generalized vector quasi-equilibrium problems. These results improved and generalized some main results in [5].

2 Preliminaries

Throughout this paper, all topological spaces are assumed to be Hausdorff.

Definition 2.1. Let Z be a real t.v.s., D a convex cone in Z with $intD \neq \emptyset$, and A a nonempty subset of Z. Let y_1, $y_2 \in A$, we denote $y_1 \leq y_2$, if $y_2 - y_1 \in D$; $y_1 < y_2$, if $y_2 - y_1 \in intD$.

A point $\bar{y} \in A$ is called a vector minimal point of A if for any $y \in A$, $y - \bar{y} \notin -D \setminus \{0\}$. A point $\bar{y} \in A$ is called a weakly vector minimal point of A if for any $y \in A$, $y - \bar{y} \notin -intD$. The set of vector minimal(resp. weakly vector minimal) points if A is denoted by $Min_D A$ (resp. $wMin_D A$).

3 Existence Results for a Solution of Two Families of Vector Generalized Quasi-Optimization Problems

Theorem 3.1. For each $i \in I$, let S_i be a continuous compact multivalued maps with nonempty closed convex values and T_i be a continuous compact multivalued maps with nonempty closed convex values. For each $i \in I$, assume the following conditions are satisfied :

(i) $C_i(x, y)$ is a closed convex pointed cone with apex at the origin and $intC_i(x, y) \neq \emptyset$;

(ii) the map $W_i : X \times Y \multimap Z_i$ defined by $W_i(x, y) = Z_i \setminus intC_i(x, y)$ is u.s.c. ;

(iii) F_i is a continuous multivalued map with nonempty compact values such that for any fixed $(x, y) \in X \times Y$, $F_i(x, y, u_i)$ is properly quasiconvex in u_i ; and

(iv) G_i is a continuous multivalued map with nonempty compact values such that for any fixed $(x, y) \in X \times Y$, $G_i(x, y, v_i)$ is properly quasiconvex in v_i.

Then there exists a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $F_i(\bar{x}, \bar{y}, \bar{x}_i) \cap wMin_{C_i(\bar{x},\bar{y})} F_i(\bar{x}, \bar{y}, S_i(\bar{x}, \bar{y})) \neq \emptyset$
and $G_i(\bar{x}, \bar{y}, \bar{y}_i) \cap wMin_{C_i(\bar{x},\bar{y})} G_i(\bar{x}, \bar{y}, T_i(\bar{x}, \bar{y})) \neq \emptyset$.

In particular, if for each $i \in I$, for all $x \in X$ and $y \in Y$, $F_i(x, y, x_i) \subset C_i(x, y)$ and $G_i(x, y, y_i) \subset C_i(x, y)$. Then there exists a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $F_i(\bar{x}, \bar{y}, x_i) \bigcap (-intC_i(\bar{x}, \bar{y})) = \emptyset$ for all $x_i \in S_i(\bar{x}, \bar{y})$ and $G_i(\bar{x}, \bar{y}, y_i) \bigcap (-intC_i(\bar{x}, \bar{y})) = \emptyset$ for all $y_i \in T_i(\bar{x}, \bar{y})$.

Proof. For each $i \in I$, since S_i and T_i are compact, there exist compact subsets $D_i \subseteq X_i$ and $M_i \subset Y$ such that $S_i(X \times Y) \subseteq D_i$ and $T_i(X \times Y) \subseteq M_i$. For each $i \in I$ and for all $(x, y) \in X \times Y$, define two multivalued maps $\Phi_i : X \times Y \multimap D_i$ and $\Psi_i : X \times Y \multimap M_i$ by

$$\Phi_i(x, y) = \{u_i \in S_i(x, y) : F_i(x, y, u_i) \cap wMin_{C_i(x,y)} F_i(x, y, S_i(x, y)) \neq \emptyset\}$$

and

$$\Psi_i(x, y) = \{v_i \in T_i(x, y) : G_i(x, y, v_i) \cap wMin_{C_i(x,y)} G_i(x, y, T_i(x, y)) \neq \emptyset\}.$$

Since $S_i : X \times Y \multimap X_i$ is a compact multivalued map with nonempty closed values, S_i has nonempty compact values. Since $F_i : X \times Y \times X_i \multimap Z_i$ is u.s.c. with compact values, $F_i(x, y, S_i(x, y))$ is a nonempty compact set for each $i \in I$, $\emptyset \neq Min_{C_i(x,y)} F_i(x, y, S_i(x, y)) \subset wMin_{C_i(x,y)} F_i(x, y, S_i(x, y))$.
Then there exists $k_i \in wMin_{C_i(x,y)} F_i(x, y, S_i(x, y))$ such that $k_i \in F_i(x, y, u_i)$ for some $u_i \in S_i(x, y)$. Therefore, $\Phi_i(x, y) \neq \emptyset$ for each $i \in I$ and for all $(x, y) \in X \times Y$. Suppose there exist some $(x, y) \in X \times Y$ and some $i \in I$ such that $\Phi_i(x, y)$ is not a convex subset of $S_i(x, y)$. Then there exist $v_i^1, v_i^2 \in \Phi_i(x, y)$ and $t \in [0, 1]$ such that

$$tv_i^1 + (1 - t)v_i^2 \notin \Phi_i(x, y). \quad (1)$$

We have $v_i^1 \in S_i(x, y)$, $v_i^2 \in S_i(x, y)$,

$$F_i(x, y, v_i^1) \cap wMin_{C_i(x,y)} F_i(x, y, S_i(x, y)) \neq \emptyset.$$

and $F_i(x, y, v_i^2) \cap wMin_{C_i(x,y)} F_i(x, y, S_i(x, y)) \neq \emptyset$.
Thus, there exists $a_i^1 \in F_i(x, y, v_i^1)$ such that for each $b_i \in F_i(x, y, S_i(x, y))$,

$$b_i - a_i^1 \notin -intC_i(x,y) \tag{2}$$

and there exists $a_i^2 \in F_i(x,y,v_i^2)$ such that for each $b_i \in F_i(x,y,S_i(x,y))$,

$$b_i - a_i^2 \notin -intC_i(x,y). \tag{3}$$

Since $S_i : X \times Y \multimap X_i$ is a multivalued map with nonempty convex values,

$$tv_i^1 + (1-t)v_i^2 \in S_i(x,y). \tag{4}$$

By (1) and (4), we have

$$F_i(x,y,tv_i^1 + (1-t)v_i^2) \cap wMin_{C_i(x,y)}F_i(x,y,S_i(x,y)) = \emptyset. \tag{5}$$

Then for each $c \in F_i(x,y,tv_i^1 + (1-t)v_i^2)$, there exists $d_c \in F_i(x,y,S_i(x,y))$ such that $d_c - c \in -intC_i(x,y)$. (6)

By (2), (3) and conditions (iii), there exists $z_{a_i^1 a_i^2} \in F_i(x,y,tv_i^1+(1-t)v_i^2)$ such that either $a_i^1 - z_{a_i^1 a_i^2} \in C_i(x,y)$ (7)

or $a_i^2 - z_{a_i^1 a_i^2} \in C_i(x,y)$. (8)

Without lost of generality, we may assume that (7) is true, then by (5), there exists $d_{z_{a_i^1 a_i^2}} \in F_i(x,y,S_i(x,y))$ such that

$$d_{z_{a_i^1 a_i^2}} - z_{a_i^1 a_i^2} \in -intC_i(x,y). \tag{9}$$

By (7), $z_{a_i^1 a_i^2} - a_i^1 \in -C_i(x,y)$ and (9), we have

$$\begin{aligned} d_{z_{a_i^1 a_i^2}} - a_i^1 = (d_{z_{a_i^1 a_i^2}} - z_{a_i^1 a_i^2}) + (z_{a_i^1 a_i^2} - a_i^1) &\in (-intC_i(x,y)) + (-C_i(x,y)) \\ &\subset -intC_i(x,y). \end{aligned} \tag{10}$$

By (2) and (10), we have a contraction. Therefore, for each $i \in I$ and for all $(x,y) \in X \times Y$, $\Phi_i(x,y)$ is a convex subset of $S_i(x,y)$.

For each $(x,y,u_i) \in \overline{Gr(\Phi_i)}$, there exists $(x^\alpha, y^\alpha, u_i^\alpha) \in Gr\Phi_i$ and $(x^\alpha, y^\alpha, u_i^\alpha) \to (x,y,u_i)$. One has $u_i^\alpha \in S_i(x^\alpha, y^\alpha)$ and

$$F_i(x^\alpha, y^\alpha, u_i^\alpha) \cap wMin_{C_i(x^\alpha,y^\alpha)}F_i(x^\alpha, y^\alpha, S_i(x^\alpha, y^\alpha)) \neq \emptyset. \tag{11}$$

Since $u_i^\alpha \in S_i(x^\alpha, y^\alpha)$ and S_i is u.s.c. with closed values, $u_i \in S_i(x,y)$. By (11), there exists $\{b_i^\alpha\}$ in Z_i such that

$$b_i^\alpha \in F_i(x^\alpha, y^\alpha, u_i^\alpha) \cap wMin_{C_i(x^\alpha,y^\alpha)}F_i(x^\alpha, y^\alpha, S_i(x^\alpha, y^\alpha)) \text{ for each } \alpha. \tag{12}$$

Let $K = \{(x^\alpha, y^\alpha, u_i^\alpha) : \alpha \in \Lambda\} \cup \{(x,y,u_i)\}$. Then K is a compact set. By conditions (iii), $F_i(K)$ is a compact set in Z_i. By (12), there exists a subnet $\{b_i^\beta\}$ of $\{b_i^\alpha\}$ such that $b_i^\beta \to b_i \in F_i(K)$.

Since $b_i^\beta \in F_i(x^\beta, y^\beta, u_i^\beta)$ and F_i is closed, $b_i \in F_i(x,y,u_i)$. Since $b_i^\beta \in F_i(x^\beta, y^\beta, u_i^\beta)$, $b_i \in F_i(x,y,u_i)$.

We need to show $b_i \in wMin_{C_i(x,y)}F_i(x,y,S_i(x,y))$.
For each $c_i \in F_i(x,y,S_i(x,y))$, we have $d_i \in S_i(x,y)$ such that

$$c_i \in F_i(x,y,d_i).$$

Since S_i is l.s.c. and $d_i \in S_i(x,y)$, there is a net $\{d_i^\beta\}$ such that $d_i^\beta \in S_i(x^\beta, y^\beta)$ and $d_i^\beta \to d_i$. Since F_i is l.s.c., and $c_i \in F_i(x,y,d_i)$, there is a net $\{c_i^\beta\}$ such that

$$c_i^\beta \in F_i(x^\beta, y^\beta, d_i^\beta) \text{ and } c_i^\beta \to c_i. \quad (13)$$

By (12) and (13), $c_i^\beta - b_i^\beta \notin -intC_i(x^\beta, y^\beta)$

$$\Leftrightarrow b_i^\beta - c_i^\beta \in Z_i \setminus intC_i(x^\beta, y^\beta) = W_i(x^\beta, y^\beta)$$

By condition (ii), W_i is a closed map and then $b_i - c_i \in W_i(x,y)$. Therefore, $c_i - b_i \notin (-intC_i(x,y))$ for all $c_i \in F_i(s,y,S_i(x,y))$ and

$$b_i \in wMin_{C_i(x,y)}F_i(x,y,S_i(x,y)). \quad (14)$$

By (14) and $b_i \in F_i(x,y,u_i)$, $b_i \in F_i(x,y,u_i) \cap wMin_{C_i(x,y)}F_i(x,y,S_i(x,y))$. Since $u_i \in S_i(x,y)$, $u_i \in \Phi_i(x,y)$ and $(x,y,u_i) \in Gr\Phi_i$. Therefore, $\Phi_i : X \times Y \multimap D_i$ is a closed map for each $i \in I$, it follows that Φ_i is u.s.c..

Since Φ_i is closed, $\Phi_i(x,y)$ is a closed set for each $(x,y) \in X \times Y$ and each $i \in I$. Similarly, for each $i \in I$, Ψ_i is u.s.c. and $\Psi(x,y)$ is a closed set for each $(x,y) \in X \times Y$ and each $i \in I$.

For each $i \in I$, define the multivalued map $A_i : X \times Y \multimap D_i \times M_i$ by

$$A_i(x,y) = (\Phi_i(x,y), \Psi_i(x,y)) \text{ for all } (x,y) \in X \times Y.$$

Then for each $i \in I$, A_i is u.s.c. with nonempty compact convex values. Let $D = \prod_{i\in I} D_i$ and $M = \prod_{i\in I} M_i$. The multivalued map $A : X \times Y \multimap D \times M$ defined by $A(x,y) = \prod_{i\in I} A_i(x,y)$ is u.s.c. with nonempty compact convex values. By Himmelberg fixed point theorem [6], there exists a point $(\bar{x},\bar{y}) \in D \times M$ such that $(\bar{x},\bar{y}) \in A(\bar{x},\bar{y})$. This means for each $i \in I$, $\bar{x}_i \in S_i(\bar{x},\bar{y})$, $\bar{y}_i \in T_i(\bar{x},\bar{y})$, $F_i(\bar{x},\bar{y},\bar{x}_i) \cap wMin_{C_i(\bar{x},\bar{y})}F_i(\bar{x},\bar{y},S_i(\bar{x},\bar{y})) \neq \emptyset$ and $G_i(\bar{x},\bar{y},\bar{y}_i) \cap wMin_{C_i(\bar{x},\bar{y})}G_i(\bar{x},\bar{y},T_i(\bar{x},\bar{y})) \neq \emptyset$.
Then there exists $b \in F_i(\bar{x},\bar{y},\bar{x}_i)$ such that for each $c \in F_i(\bar{x},\bar{y},S_i(\bar{x},\bar{y}))$,

$$c - b \notin -intC_i(\bar{x},\bar{y})$$

If $F_i(x,y,x_i) \subseteq C_i(x,y)$, it is easy to see that Therefore, $F_i(\bar{x},\bar{y},x_i) \bigcap (-intC_i(\bar{x},\bar{y})) = \emptyset$ for all $x_i \in S_i(\bar{x},\bar{y})$ and $G_i(\bar{x},\bar{y},y_i) \bigcap (-intC_i(\bar{x},\bar{y})) = \emptyset$ for all $y_i \in T_i(\bar{x},\bar{y})$.

Remark 3.1. Theorem 3.1 is still true if condition (iii) is replaced by (iii)′ F_i is a continuous multivalued map with compact values and for any fixed $(x, y) \in X \times Y$, $F_i(x, y, u_i)$ is $C(x, y)$ quasiconvex in u_i.

With the same arguments as Theorem 3.1, we have the following theorem.

Theorem 3.2. In theorem 3.1, if the condition (iii) of Theorem 3.1 is replaced by

(iii′) $F_i : X \times Y \times X_i \multimap Z_i$ is a continuous multivalued map with nonempty compact values such that for any fixed $(x, y) \in X \times Y$, $F_i(x, y, u_i)$ is properly quasiconcave in u_i.

Then there exists a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $F_i(\bar{x}, \bar{y}, \bar{x}_i) \cap wMax_{C_i(\bar{x},\bar{y})} F_i(\bar{x}, \bar{y}, S_i(\bar{x}, \bar{y})) \neq \emptyset$ and $G_i(\bar{x}, \bar{y}, \bar{y}_i) \cap wMin_{C_i(\bar{x},\bar{y})} G_i(\bar{x}, \bar{y}, T_i(\bar{x}, \bar{y})) \neq \emptyset$.

Corollary 3.1. If conditions (iii) and (iv) of Theorem 3.1 is replaced by (iii)′ and (iv)′ respecetiy, where

(iii)′ $f_i : X \times Y \times X_i \to Z_i$ is a continuous function such that for all $x = (x_i)_{i \in I} \in X$ and $y \in Y$, $f_i(x, y, x_i) \in C_i(x, y)$ and for any fixed $(x, y) \in X \times Y$, the map $u_i \mapsto f_i(x, y, u_i)$ is properly quasiconvex.

(iv)′ $g_i : X \times Y \times Y_i \to Z_i$ is a continuous function such that for all $x \in X$ and $y = (y_i)_{i \in I} \in Y$, $g_i(x, y, y_i) \in C_i(x, y)$ and for any fixed $(x, y) \in X \times Y$, the map $v_i \mapsto g_i(x, y, v_i)$ is properly quasiconvex.

Then there exists a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $f_i(\bar{x}, \bar{y}, x_i) \notin (-intC_i(\bar{x}, \bar{y}))$ for all $x_i \in S_i(\bar{x}, \bar{y})$ and $g_i(\bar{x}, \bar{y}, y_i) \notin (-intC_i(\bar{x}, \bar{y}))$ for all $y_i \in T_i(\bar{x}, \bar{y})$.

Corollary 3.2. In Theorem 3.1, if we assume that (i), (ii) and

(iii) $F_i : X \times Y \times X_i \multimap Z_i$ is a continuous multivalued map with nonempty compact values such that for all $x \in X$ and $y \in Y$, $F_i(x, y, x_i) \subset C_i(x, y)$, and for any fixed $(x, y) \in X \times Y$, $F_i(x, y, u_i)$ is properly quasiconvex in u_i.

Then there exists a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$ and $F_i(\bar{x}, \bar{y}, x_i) \bigcap (-intC_i(\bar{x}, \bar{y})) = \emptyset$ for all $x_i \in S_i(\bar{x}, \bar{y})$.

Corollary 3.3. In Theorem 3.1, if we assume (i) (ii) and

(iii) $G_i : X \times Y \times Y_i \multimap Z_i$ is a continuous multivalued map with nonempty compact values such that for any fixed $(x, y) \in X \times Y$, $G_i(x, y, v_i)$ is properly quasiconvex in v_i.

Then there exists a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, and $G_i(\bar{x}, \bar{y}, \bar{y}_i) \cap wMin_{C_i(\bar{x}, \bar{y})} G_i(\bar{x}, \bar{y}, T_i(\bar{x}, \bar{y})) \neq \emptyset$.

4 Applications to Systems of Loose Quasi-Saddle Point Problem and Constrained Competitive Nash-Type Equilibrium Problems

Theorem 4.1. Let I, F_i, V_i, Z_i, X_i, Y_i, X, Y, S_i and T_i be the same as in Theorem 3.1. Suppose that conditions (i), (ii) of theorem 3.1 are true. Suppose that

(iii) $L_i : X_i \times Y_i \multimap Z_i$ is a continuous multivalued map with nonempty compact values ;

(a) for any fixed $y_i \in Y_i$, $L_i(x_i, y_i)$ is properly quasiconcave in x_i ; and

(b) for any fixed $x_i \in X_i$, $L_i(x_i, y_i)$ is properly quasiconvex in y_i.

Then there exists a $\bar{x} = (\bar{x}_i)_{i \in I} \in X$ and $\bar{y} = (\bar{y}_i)_{i \in I} \in Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $L_i(\bar{x}_i, \bar{y}_i) \cap wMax_{C_i(\bar{x}, \bar{y})} L_i(S_i(\bar{x}, \bar{y}), \bar{y}_i) \neq \emptyset$ and $L_i(\bar{x}_i, \bar{y}_i) \cap wMin_{C_i(\bar{x}, \bar{y})} L_i(\bar{x}_i, T_i(\bar{x}, \bar{y})) \neq \emptyset$.

Proof. For each $i \in I$, let $F_i(x, y, u_i) = L_i(u_i, y_i)$ and $G_i(x, y, v_i) = L_i(x_i, v_i)$.
Then Theorem 4.1 follows from Theorem 3.2.
If L_i is a single valued map, we have the following systems of vector quasi-saddle point problem.

Corollary 4.1. For each $i \in I$, let $S_i : X \multimap X_i$ be a continuous compact multivalued map with nonempty closed convex values and $T_i : Y \multimap Y_i$ be a continuous compact multivalued map with nonempty closed convex values. For each $i \in I$, assume the following conditions are satisfied.

(i) $C_i : X \multimap Z_i$ is a multivalued map such that for each $x \in X$, $C_i(x)$ is a closed convex pointed cone with apex at the origin and $intC_i(x) \neq \emptyset$;

(ii) the map $W_i : X \multimap Z_i$ defined by $W_i(x) = Z_i \setminus intC_i(x)$ is u.s.c. ;

(iii) $L_i : X_i \times Y_i \to Z_i$ is a continuous map such that

(a) for any fixed $y_i \in Y_i$, $L_i(x_i, y_i)$ is properly quasiconcave in x_i ; and

(b) for any fixed $x_i \in X_i$, $L_i(x_i, y_i)$ is properly quasiconvex in y_i.

Then there exists a $\bar{x} = (\bar{x}_i)_{i \in I} \in X$ and $\bar{y} = (\bar{y}_i)_{i \in I} \in Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x})$, $\bar{y}_i \in T_i(\bar{y})$, $L_i(\bar{x}_i, \bar{y}_i) - L_i(x_i, \bar{y}_i) \notin (-intC_i(\bar{x}))$ for all $x_i \in S_i(\bar{x})$. and $L_i(\bar{x}, y_i) - L_i(\bar{x}_i, \bar{y}_i) \notin (-intC_i(\bar{x}))$ for all $y_i \in T_i(\bar{y})$.

References

1. Q. H. Ansari, W. K. Chan and X. Q. Yang, The system of vector quasi-equilibrium problems with applications, Journal of Global Optimization, Vol. 29, No. 1, pp. 45-57, 2004.

2. Q. H. Ansari, S. Schaible and J. C. Yao, The system of generalized vector equilibrium problems with applications, Journal of Global Optimization, Vol. 22, pp. 3-16, 2003.

3. Q. H. Ansari and Z. Khan, System of generalized vector quasi-equilibrium problems with applications , To appear in the Proceedings of the Inter national Conference on Analysis and Discrete Structures, Edited By s. Nanda, Narosa Publication House, New Delhi, 2003.

4. Q. H. Ansari, S. Schaible and J. C. Yao, System of vector equilibrium problems and its applications, Journal of Optimization Theory and Applications, Vol. 107, pp. 547-557, 2000.

5. Q. H. Ansari, L. J. Lin, and L. B. Su. Systems of simultaneous generalized vector quasi-equilibrium problems and their applications. in Journal of Optimization Theory and Applications, 127 (2005), 27-44.

6. C. J. Himmelberg, Fixed point of compact multifunctions, Journal of Mathematical Analysis and Applications, Vol. 38, pp. 205-207, 1972.

7. L. J. Lin, Existence theorems of simultaneous equilibrium problems and generalized vector quadi-saddle points, To Appear in Journal of Global Optimization, 32 (2005), 613-632.