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THE GROUP CONFIGURATION THEOREM AND ITS APPLICATIONS

BYUNGHAN KIM

The group configuration theorem for stable theories [6] plays important roles in solving deep problems in geometric stability theory. The theorem roughly says that one can get the canonical non-trivial type-definable homogeneous space (i.e. a group with its transitive action on a set, all type-definable) from a group configuration, a certain geometrical configuration, in stable theories. Recently fruitful achievements of the generalization of the theorem into the context of simple theories were made. In their topical paper [1], I. Ben-Yaacov, E. Tomasic and F. O. Wagner generalize the group configuration theorem by obtaining an invariant group from the group configuration in simple theories. However the group they produce does not completely fit into the first-order context. On the other hand, T. de Piro, B. Kim and J. Millar succeed in getting the canonical hyperdefinable group from the group configuration *under 4-amalgamation* in simple theories [5]. The element of the group is a hyperimaginary, an equivalence class of a type-definable equivalence relation, and the group operation is type-definable, hence the group belongs to the domain of the standard first-order logic. The former result is for *all* simple theories but the group obtained is non hyperdefinable, where as the latter producing the desirable hyperdefinable group has a pay-off of an assumption of generalized amalgamation.

In this small note, we will review the latter result of de Piro, Kim and Millar, together with the notions around generalized amalgamation. (There is a nice expository paper on the former result appeared in the Bulletin of Symbolic Logic [2].) Kim recently continue the construction and complete the group configuration theorem [13]. Namely, under 4-amalgamation, he is able to construct a hyperdefinable homogeneous space *equivalent to* the given group configuration. This will be reviewed too. Next, we will speak about its applications. In particular we mainly pay our attentions to the open problem whether pseudolinearity implies linearity, which is known to be true for stale theories.

We assume that the reader is familiar with basics of simplicity theory [19]. Throughout the paper, T is a complete simple theory. We

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work in a saturated model $\mathcal{M}$ of T with hyperimaginaries, and $a, b, ...$ are (possibly infinitary) hyperimaginaries, M, N are small elementary submodels. (Note that tuples from $\mathcal{M}^{eq}$ are also hyperimaginaries). As usual, $a \equiv_A b$ ($a \equiv^L_A b$) means a, b have the same type (Lascar strong type, resp.) over A. We point out that usually $\mathrm{bdd}(a)$ denotes the *set* of all *countable* hyperimaginaries definable over a [19, 3.1.7]. Here, depending on the context, it can be either a specific sequence which linearly orders the set $\mathrm{bdd}(a)$; or, since a sequence of hyperimaginaries is again a hyperimaginary (of a large arity), a fixed hyperimaginary interdefinable with the sequence.

1. Generalized type-amalgamation

As well-known, in [14], B. Kim and A. Pillay prove the following form of type-amalgamation (or the independence theorem) for all simple theories.

Type-Amalgamation 1.1. If $a_1 \mathop{\smile\!\!\!\!\!\!\mid}_B a_2$, $d_i \mathop{\smile\!\!\!\!\!\!\mid}_B a_i$ $(i = 1, 2)$, and $d_1 \equiv^L_B d_2$, then there is d such that $d \equiv^L_{Ba_i} d_i$ and $\{d, a_1, a_2\}$ is B-independent.

Before Kim and Pillay's work, the original type-amalgamation is stated and proved to be held in some simple algebraic structures in a couple of papers by Hrushovski [8][10]. In particular, the one stated in [8] (which is written earlier than [14] but published later) is as follows.

Type-Amalgamation 1.2. Suppose that there are complete types $r_i(x_i)$ $(i = 1, 2, 3)$ and $r_{jk}(x_{jk})$ $(1 \leq j < k \leq 3)$, all over a set B, where x_i is possibly an infinite set of variables, such that

(1) $x_j \cup x_k \subset x_{jk}$ and $r_j \cup r_k \subset r_{jk}$, and

$r_{jk}(x_{jk})$ says

(2) x_j and x_k are B-independent,

(3) x_{jk} is as a set $\mathrm{bdd}(x_j x_k B)$.

Then there is a complete type $r_{123}(x_{123}) \supseteq r_{12} \cup r_{13} \cup r_{23}$ over B saying that $\{x_1, x_2, x_3\}$ is B-independent, and x_{123} is $\mathrm{bdd}(x_1 x_2 x_3 B)$.

It is not difficult to see that the two statements 1.1 and 1.2 are equivalent. However this equivalence no longer holds if the index increases.

Generalized Type-Amalgamation 1.3. For B-independent $A = \{a_1, ..., a_{n-1}\}$ and $d_i \mathop{\smile\!\!\!\!\!\!\mid}_B A_i$ where $A_i = A \setminus \{a_i\}$ for $i = 1, .., n-1$, whenever $d_i \equiv^L_{BA_{ij}} d_j$ where $A_{ij} = A_i \cap A_j$, then there is d such that $d \equiv^L_{BA_i} d_i$ and $\{d, a_1, ..., a_{n-1}\}$ is B-independent.

Generalized Type-Amalgamation 1.4. Let W be a collection of subsets of $\{1, ..., n\} = u_n$, closed under subsets. For each $w \in W$, complete type $r_w(x_w)$ over B is given where x_w is possibly an infinite set of variables. Suppose that

(1) for $w \subseteq w'$, $x_w \subseteq x_{w'}$ and $r_w \subseteq r_{w'}$.

Moreover for any $a_w \models r_w$,

(2) $\{a_{\{i\}} | i \in w\}$ is B-independent,

(3) a_w is as a set $\mathrm{bdd}(\bigcup_{i \in w} a_{\{i\}} B)$ (and the map $a_w \to x_w$ is a bijection).

Then there is a complete type $r_{u_n}(x_{u_n})$ over B such that (1),(2),(3) hold for all $w \in W \cup \{u_n\}$.

An example shows that the two propositions do not coincide even when $n = 4$.

Example 1.5. In the random graph M in $\mathcal{L} = \{R\}$, choose distinct $a_i, b_i, c_i \in M$ and imaginary elements $d_i = \{b_i, c_i\}$ ($i = 1, 2, 3$). We additionally assume that $R(a_1, c_3) \wedge R(a_2, b_3) \wedge \neg R(a_1, b_3) \wedge \neg R(a_2, c_3)$, and $\mathrm{tp}(a_1 a_2; b_3 c_3) = \mathrm{tp}(a_2 a_3; b_1 c_1) = \mathrm{tp}(a_1 a_3; b_2 c_2)$. Now it follows that for $\{i, j, k\} = \{1, 2, 3\}$, $d_i \equiv^L_{a_j} d_k$. But it is easy to see that $\mathrm{Lstp}(d_1/a_2 a_3)$, $\mathrm{Lstp}(d_2/a_1 a_3)$, and $\mathrm{Lstp}(d_3/a_1 a_2)$ have no common realization. Namely, M does not satisfy 1.3 for $n = 4$ (and larger).

However, due to elimination of weak imaginaries (and elimination of hyperimaginaries) of the random graph, for any hyperimaginary B, $\mathrm{bdd}(B) = \mathrm{dcl}(A)$ for a set A in a home sort. Hence to check 1.4, it suffices to examine the amalgamation in the home-sort. It follows that M satisfies 1.4 for every n. The reader may wonder why the above arrangement of a_i, b_i, c_i does not raise a trouble as before. If we put $r_{\{i\}} = \mathrm{tp}(a_i)$ ($i = 1, 2, 3$), and $r_{\{4\}} = \mathrm{tp}(b_i c_i)$, then we should let $r_{\{1,4\}} = \mathrm{tp}(a_1; b_3 c_3) = \mathrm{tp}(a_1; b_2 c_2)$, $r_{\{3,4\}} = \mathrm{tp}(a_3; b_2 c_2) = \mathrm{tp}(a_3; b_1 c_1)$ (note that $\mathrm{acl}(d_i) = \mathrm{dcl}(b_i c_i)$). But then $r_{\{2,4\}}$ must be either $\mathrm{tp}(a_2; b_1 c_1)$ or $\mathrm{tp}(a_2; b_3 c_3)$, which are distinct!, i.e. the arrangement does not give a compatible system of types $r_w(x_w)$.

The example also says 1.3 is not preserved in the interpreted theories while 1.4 is. It is generally agreed that 1.4 is the correct definition of generalized amalgamation.

Definition 1.6. We say T has *n-complete amalgamation (n-CA)* if 1.4 holds for n. We simply say T has *n-amalgamation* if 1.4 holds for n with $W = \mathcal{P}(u_n)^- = \mathcal{P}(u_n) \setminus \{u_n\}$.

Clearly k-CA implies n-CA for $k \geq n$. Note that 4-CA and 4-amalgamation are equivalent. For each $n \geq 3$, there are examples having n-CA but not having $(n+1)$-CA [16]. Stable theories satisfy n-CA over models, but not in general over algebraically closed sets [5].

This unsatisfactory phenomenon leads to define the so-called *model-n-CA*, a variation of n-CA, which all stable theories have. We omit the description, but for the detail, see [5] or [15].

In this note, as we will concentrate our attentions to 4-amalgamation, we restate it in the similar manner of 1.3 which seems helpful to conceptualize.

4-Amalgamation 1.7. Let $\{i,j,k\} = \{1,2,3\}$. Suppose that a_0-independent $\{a_1, a_2, a_3\}$ and $d_i \mathop{\smile\!\!\!\!\!\!|}_{a_0} a_j a_k$ such that $a_0 \subseteq a_i, d_i$, all boundedly closed, are given. Let $\overline{a_i d_j}, \overline{a_i a_j}$ be some enumerations of $\mathrm{bdd}(a_i d_j)$, $\mathrm{bdd}(a_i a_j)$, respectively. If $\overline{d_j a_i} \equiv_{a_i} \overline{d_k a_i}$, then there is $d(\mathop{\smile\!\!\!\!\!\!|}_{a_0} a_1 a_2 a_3)$ with $d \equiv_{a_0} d_i$, and enumerations $\overline{da_i}$, such that for $i < j$,

$$\overline{da_i}\ \overline{da_j} \equiv_{\overline{a_i a_j}} \overline{d_k a_i}\ \overline{d_k a_j}.$$

Before closing this section, we point out the notion of n-simplicity, initially introduced by A. Kolesnikov [16] and further modifications were made in [15]. Recall that, 1.1 is proved by the use of the following fact.

Fact 1.8. *(T simple.) Assume that $I = \langle a_n |\ n \in \omega \rangle$ is a Morley sequence over b. If $c \mathop{\smile\!\!\!\!\!\!|}_b a_0$, then there is $c' \equiv_{ba_0} c$ such that I is $c'b$-indiscernible and $c' \mathop{\smile\!\!\!\!\!\!|}_b I$.*

The property 1.8 proved in [12] is indeed a special case of type-amalgamation(= 3-amalgamation). In other words, the particular amalgamation property implies full 3-amalgamation. Thus it is natural to ask whether a higher dimensional variation of 1.8 can imply generalized amalgamation. Indeed Kolesnikov proved in [16] that the following property, a particular case of 1.3 for $n = 4$, implies it.

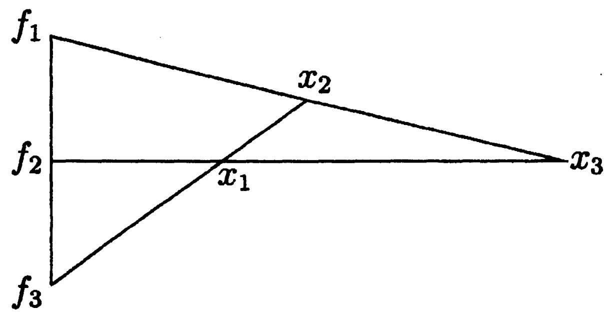
Property 1.9. Assume that $I = \langle a_n |\ n \in \omega \rangle$ is a Morley sequence over b. If $c \mathop{\smile\!\!\!\!\!\!|}_b a_0 a_1$ and $a_0 \equiv^L_{bc} a_1$, then there is $c' \equiv_{ba_0a_1} c$ such that I is $c'b$-indiscernible and $c' \mathop{\smile\!\!\!\!\!\!|}_b I$.

But as 1.4 is the correct notion of amalgamation, not 1.3, 1.9 has to be modified appropriately indicating a special case of 4-amalgamation. The modified property, which we call *2-simplicity*, is equivalent to 4-amalgamation as Kolesnikov's idea goes through in this context [15]. But the question remains whether it keeps holding for larger n. Surprisingly, it is not unless *n-simplicity* for $n \geq 3$ should be defined in terms of *finite* Morley sequences rather than infinite ones. For details, see [5] or [15].

2. The group configuration theorem

Definition 2.1. By a *group configuration* we mean a 6-tuple of hyperimaginaries $C = (f_1, f_2, f_3, x_1, x_2, x_3)$ over a hyperimaginary e such that, for $\{i, j, k\} = \{1, 2, 3\}$,

(1) $f_i \in \mathrm{bdd}(f_j, f_k; e)$,
(2) $x_i \in \mathrm{bdd}(f_j, x_k; e)$,
(3) all other triples and all pairs from C are independent over e.



If it has the property that $\mathrm{bdd}(f_i; e) = \mathrm{bdd}(\mathrm{Cb}(x_j x_k / f_i e); e)$, we call such C a *bounded quadrangle.* In particular, we call $(f_1', f_2', f_3', x_1, x_2, x_3)$ where $f_i' = \mathrm{Cb}(x_j x_k / e f_i)$, an *induced bounded quadrangle from* C over e. Now if additionally $f_i \in \mathrm{bdd}(x_j, x_k; e)$, we call the the group configuration C over e *principal.* We say two group configurations $C = (f_1, f_2, f_3, x_1, x_2, x_3)$ over e and $C' = (f_1', f_2', f_3', x_1', x_2', x_3')$ over e' are *equivalent* (over d) if for some $d \supseteq ee'$, $C \mathop{\smile\!\!\!\!\!\mid}_e d, C' \mathop{\smile\!\!\!\!\!\mid}_{e'} d$ and each pair of $(f_i, f_i'), (x_i, x_i')(i = 1, 2, 3)$ is interbounded over d. Its transitive closure is an equivalence relation among the group configurations.

The reason why the configuration is said to be a *group configuration* is that it is canonically obtained from a given hyperdefinable homogeneous space. More precisely, let $((G, \circ), X, .)$ be a hyperdefinable homogeneous space (i.e. the hyperdefinable action $.$ of the group G on the set X is transitive) over e. We say $a \in X$ is *generic* (over e), if for $g \in G$ with $g \mathop{\smile\!\!\!\!\!\mid}_e a$, $g . a \mathop{\smile\!\!\!\!\!\mid}_e g$ holds. For notational convenience, we suppress e to $\emptyset$. Similarly to the group case, if $x (\in X)$ is independent with generic $f \in G$, then $f . x$ is generic. Hence a generic element of X exists. Moreover generic $f (\in G)$ is generic with respect to X as well. Namely, for $y (\in X) \mathop{\smile\!\!\!\!\!\mid} f$, $y \mathop{\smile\!\!\!\!\!\mid} f . y$ holds. We have the following.

Observation 2.2. *A hyperdefinable homogeneous space (G, X) (over $\emptyset$) is given. We can choose $f_2, f_3 \in G$ and $x_1 \in X$, all generic, such that $\{f_2, f_3, x_1\}$ is independent. Then $C = (f_1, f_2, f_3, x_1, x_2, x_3)$ forms a group configuration where $f_1 = f_2 \circ (f_3)^{-1}$, $x_2 = f_3 . x_1$, $x_3 = f_2 . x_1$. Note that f_i, x_i $(i = 1, 2, 3)$ are all generic. We call C, a* group configuration obtained from *the homogeneous space* (G, X).

The group configuration theorem is a theorem about the reverse process. The theorem says that a given group configuration, one can construct a homogeneous space (G, X) having an equivalent group configuration. In [5], de Piro, Kim and Millar obtained the first step of the theorem. Namely, given a group configuration, a canonical group whose generic elements are equivalent to the first triple of the configuration. Then recently Kim [13] completes the group configuration theorem under 4-amalgamation.

Theorem 2.3. (The group configuration theorem) *Assume T has 4-CA. After possibly naming a model, we can assume $\emptyset = \mathrm{bdd}(\emptyset)$. Given an induced bounded quadrangle C from a group configuration over $\emptyset$, we can construct a hyperdefinable homogeneous space over $\emptyset$ such that C and a bounded quadrangle obtained from the space are equivalent.*

Recall that for any stable T, if the elements of the group configuration is finitary, then a type-definable homogeneous space having an equivalent configuration is constructible.

3. Applications and pseudolinearity

One application of 2.3 (or just the earlier version of dePiro, Kim, and Millar) is the following result. This extends the theorem [4, 3.23] that, in any modular non-trivial ω-categorical simple T, an infinite vector space over some finite field is definably recovered in $\mathcal{M}^{eq}$. Recall that T is said to be *non-trivial* if there are hyperimaginaries a_1, a_2, a_3 and A such that for $1 \le i < j \le 3$, a_i, a_j are independent over A whereas $\{a_1, a_2, a_3\}$ is dependent over A.

Theorem 3.1. *Suppose that T is modular, non-trivial, having 4-CA. Then there is a hyperdefinable infinite bounded-by-Abelian group V over a model M of SU-rank 1 generic types. Moreover for the bounded subgroup $V_0 = V \cap \mathrm{bdd}(M)$, V/V_0 forms a vector space over the division ring R of $\mathrm{bdd}(M)$-endomorphisms of V such that for $b, a_1, ..., a_n \in V$, $b \in \mathrm{bdd}(a_1 ... a_n)$ iff $b + V_0 = \alpha_1(a_1 + V_0) + ... + \alpha_n(a_n + V_0)$ for some $\alpha_i \in R$.*

Theorem 3.1 is important since it shows that from a pure logical condition of independence, we can recover a concrete algebraic structure. The theory T being *modular* simply means that the model theoretic dimension property is similar to that of linear (projective or, affine) spaces. Namely, we say T modular if $A \mathop{\smile\!\!\!\!\!\!\mid}_{A \cap B} B$ holds for any boundedly closed sets A, B. For finite dimensional case, it simply means

$$\dim(A \cup B) = \dim(A) + \dim(B) - \dim(A \cap B).$$

This is exactly the case when a space is projective or affine. Hence the group configuration theorem in stable and simple theories is the main to recover an underlying concrete algebraic structure from a structure having a pure model theoretic condition. This issue is also related to the so-called Zilber's principle. We will get back to this later.

For the rest of this section, we pay our attentions to the possible application of 2.3 to the pseudolinearity conjecture. Before stating what it is, let us recall some of necessary definitions. We also restrict our attentions to a solution set D of SU-rank 1 Lascar strong type over, for convenience, $\emptyset$. In a stable theory, D can be (strongly) minimal.

Definition 3.2. Let $k \geq 1$.

(1) We say D is *k-linear* (or *pseudolinear*) if for any two singletons $a, b \in D$ and parameters B with $SU(ab/B) = 1$, $SU(e) \leq k$ where $e = \mathrm{Cb}(ab/B)$. We say D is *linear* if it is 1-linear.

(2) We say D is *k-based* if for any indiscernible sequence $I = \langle \bar{c}_i | i \in \omega \rangle$ from D, $I \setminus I_k$ is Morley over $I_k := \{\bar{c}_i |\ i < k\}$.

Hence D being k-linear means that any *curve* in D^2, the rank of the space of its conjugates is bounded by k. It is well-known, in general, an infinite (rank 1, e.g. algebraically closed) field is *not* pseudolinear. For example, if we take a curve defined by

$$y = x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0,$$

where $\{a_0, \dots, a_{n-1}\}$ is algebraically independent, then the rank of the space of the conjugates of the curve is possibly $\geq n$, thus can be arbitrarily large.

Now, by similar ideas in the proof of [4, 3.6], the following can be obtained.

Theorem 3.3. (1) *The following are equivalent.*

(a) *D is k-linear.*

(b) *D is k-based.*

(2) *The following are equivalent.*

(a) *D is linear.*

(b) *D is modular, i.e. for any $A, B \subseteq D$, $A \mathop{\smile\!\!\!\!\!\!|}_{\mathrm{bdd}(A) \cap \mathrm{bdd}(B)} B$.*

As mentioned after 3.1, typical examples of linear(= modular) structures are vector spaces. Conversely, 3.1 says D being modular is the same amount of saying that it can be reduced to an underlying vector space. After having seen examples so far, one may boldly guess that there is no 'real' k-linear examples other than $k = 1$. Namely we have the following conjecture.

Pseudolinearity Conjecture 3.4. *If D is k-linear, then it is linear.*

Indeed Zilber's principle (or dichotomy) goes a little further. He conjectured that any non-trivial strongly minimal structure is either modular (so interpreting an infinite vector space), or interpreting a field (which has to be algebraically closed). As known, his conjecture was shown to be false by Hrushovki who constructed counterexamples [7]. His construction method itself created an important new area in model theory. After then, he and Zilber together suggested a famous *Zariski condition*, and under the constraint on the strongly minimal structures, they succeeded to show the dichotomy [11]. It turns out that this dichotomy plays a great role in the applications of model theory to other branches of mathematics such as geometry and number theory [9]. Extending the dichotomy to the context of general simple rank 1 set D is a big open project. Theorem 3.1 can be considered as an achievement in this direction, as it says at least for concerning *modularity*, it is to do with a concrete vector space as in stable case.

Now by the remark after 3.2, if Zilber's principle holds, then 3.4 easily follows: Nonlinearity of D implies the interpretability of a field which can not be k-linear.

But, regardless of that Zilber's principle is false, 3.4 is known to be true for stable theories [3]. The proof uses the group configuration theorem for stable theories. Let us briefly review the proof. If stable D is k-linear (for minimal such k), then easily a group configuration $C = (f_1, f_2, f_3, x_1, x_2, x_3)$ can be obtained where $\mathrm{rk}(f_i) = k$ and $\mathrm{rk}(x_i) = 1$. Then by the group configuration theorem, there is a type-definable homogeneous space (G, X) whose group configuration is equivalent to C. In particular, ranks are preserved. Namely $\mathrm{rk}(G) = k$ and $\mathrm{rk}(X) = 1$. Then by the general stable group theory [17, 1.6.25], $\mathrm{rk}(G)$ =1,2 or 3. If 2 or 3, then an infinite field is interpretable from X, which again is not k-linear (the remark after 3.2). Hence k must be 1, and 3.4 for stable theories is obtained.

When we try to mimic the ideas under 4-CA, the initial part of the proof will go through using 2.3, so from that D being k-linear we have a hyperdefinable homogeneous space (G, X) with $\mathrm{rk}(G) = k$ and $\mathrm{rk}(X) = 1$. But we do not have so far the analogous theorem to [17, 1.6.25]. In other words, problem is rather reduced to the theory of hyperdefinable groups having simple theories. So far no progress was made in this regards. But we believe that, under 4-amalgamation, one may develop finer group theory so that many important open problems including this and supersimple field conjecture (any supersimple field is pseudo algebraically closed) can be resolved.

We finally point out that 3.4 is proved to be true for any ω-categorical simple theories [18]. For an ω-categorical structure, the group constructed in [1] is definable, and for this particular case, a finer group theory exists.

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Department of Mathematics, Yonsei University, 134 Shinchon-dong, Seodaemun-gu, Seoul 120-749, South Korea

E-mail address: `bkim@yonsei.ac.kr`