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Axiomatic Re-examination of the Maximin Principle in Arrow-Dasgupta Economy *

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Abstract

The maximin principle is challenged about equity when applied to intergenerational distribution. To test the principle for equity, we try to characterize the maximin consumption path with egalitarian axioms. It is characterized with Hammond Equity or its extension in both the choice functional approach and the relational approach.

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1 Introduction

The maximin principle is challenged about equity when applied to intergenerational distribution. Rawls (1971) and Solow (1974) have pointed out

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that a direct application of the maximin principle brings a peculiar result. One of the most fundamental features of the intergenerational distribution of utilities or savings is that the first generation is the worst-off because s/he has no ancestor that will leave them heritage. Therefore, the optimal path for the maximin principle is maximizing the first generation's utility. It implies that just saving is no saving. That's why Rawls refuses to apply the principle to intergenerational distribution and insists that each generation has a paternalistic concern to payoffs of its immediate descendant. It is formalized by Arrow (1973), Dasgupta (1974a, b) and Riley (1976) in the optimal growth context. They made a dynamic model with paternalistic utility functions and showed that the consumption path and the utility path optimal for the maximin principle forms a saw-tooth shape. It seems to have two problems: inequality and time-inconsistency. We focus the former.

To test the maximin principle for equity, we try to characterize the maximin path with egalitarian axioms. Our model is the same as Arrow (1973)'s. We assume a utility of each generation depends on the consumption level of its immediate descendant, and characterize the optimal path for the maximin principle in this model. Our approach to social welfare is both binary relational and choice functional.

As the result, the optimal path is characterized with Group Equity, Weak Pareto, and Efficiency in the relational approach, and Inclusion of Hammond Superior Path, Exclusion of Pareto Inferior Path, α, and δ^* in the choice functional approach. Group Equity is an extension of Hammond Equity, which is well-known equity axiom and often used to characterize the maximin principle in intra-generational distribution. Inclusion of Hammond Superior Path requires a social planner to choose more equal consumption path in the sense of Hammond (1976) when the less one is chosen. We will show the detail and the definition latter.

2 Arrow-Dasgupta Economy

Consider one private-good, non-overlapping, infinite horizon economy. Let $\mathbb{N}$ be the set of nonnegative integers, each element of which is used to represent a generation or time period. To simplify the problem of externalities, we assume that each time period consists of one generation, and each generation consists of one representative individual. The private good can be either consumed or used as capital which bears a return. k_t denotes the accumulated capital at the beginning of time period $t \in \mathbb{N}$. In that period a fraction c_t is consumed and the remainder $k_t - c_t$ is used in production.

Each unit used in production brings γ units of the good at the end of the period, and are transferred to the next period $t+1$. Hence

$$k_{t+1} = \gamma(k_t - c_t) \tag{1}$$

We generally assume that the economy is productive, so that

$$\gamma > 1 \tag{2}$$

A feasibility condition for production is naturally assumed. For all $t \geq 0$

$$k_t \geq 0 \tag{3}$$

where $k_0 > 0$ is given initially. A feasibility condition for consumption is naturally assumed. That is, any individual live any longer without consumption. Hence, for all $t \geq 0$

$$c_t \geq 0. \tag{4}$$

Now we describe our subject to find a consumption path that satisfies Rawlsian maximin principle for intergenerational justice. For the convenience of description, we adopt the following notation: let x_t and p_t be a real number and $l^\infty = \{X = (x_0, ..., x_t, ...)| \sup_t x_t < \infty\}$, $l_+^\infty = \{X \in l^\infty | \forall t : x_t \geq 0\}$, and $l_{++}^1 = \{P = (p_0, ..., p_t, ...)| \forall t : p_t > 0 \wedge \sum_{t=0}^{\infty} p_t < \infty\}$. Denote a consumption path by the capital letter, e. g., $C = (c_0, c_1, ...)$. ${}_{rep}(c_1, ..., c_n)$ presents the path $(c_1, ..., c_n, c_1, ..., c_n, c_1, ...)$ which consists of $(c_1, ..., c_n)$ repeated infinitely many times. By the feasibility condition, consumption paths ought to be chosen from the set $\{C \in l_+^\infty | given\ k_0,\ \forall t \geq 0 : 0 \leq k_{t+1} = \gamma(k_t - c_t)\}$. It is convenient, however, to use the following equivalent form:

$$\mathcal{C} = \{C \in l_+^\infty | PC \leq k_0\},$$

where $PC = \lim_{T\to\infty} \sum_{t=0}^{T} p_t c_t$, and $p_t = \gamma^{-t}$.

We denote the utility function of generation $t \in \mathbb{N}$, or often called individual t, by $W_t(C)$ when the consumption path C is attained. Then the maximin principle of justice gives a solution to the problem

$$\max_{C \in \mathcal{C}} \min_t W_t(C) \tag{5}$$

We assume that each generation has sympathy to the next generation. Generation t derives utility from her own consumption c_t and also from her immediate $n-1$ descendants' satisfaction, so that her utility function depends on the consumption stream of n generations beginning with his own, and is denoted by $W_t(C) = V_t(c_t, c_{t+1}, ..., c_{t+n-1})$. We assume that the utility function V_t is the same for all generations $t \in \mathbb{N}$, that is, $V_t() = V()$ for all t. Following the frameworks of Arrow (1973) and Dasgupta (1974a,b), we assume that W is additively separable as to t for simplicity, that the felicities ascribed by individual t to individual $t+1, ..., t+n-1$ are the same as those ascribed by individual $t+1, ..., t+n-1$ to themselves, that the felicity function is the same for all t, and that the felicity of the future generation may be discounted in the utility of the present generation. That is, for any consumption path $C = (c_0, c_1, \cdots)$,

$$W_t(C) = V(c_t, c_{t+1}, \cdots, c_{t+n-1}) = \sum_{i=0}^{n-1} \beta_i U(c_{t+i}) \tag{6}$$

where $\beta_0 = 1$ and $\beta_{i+1} \leq \beta_i$ for $1 \leq i < n-1$. We also assume that

$$\gamma^i \beta_i < \gamma^j \beta_j \quad (0 \leq i < j \leq n).$$

as Arrow (1973) did. This assumption requires that each generation obtains more utility if she bequeathes capital to the next generation than that if she consumes it by herself. Although the utility of the next generation is discounted by β, the total utility will go up if the in crease in production is included. The utility function U is assumed to satisfy the followings: (a) $U : \mathbb{R}_+ \rightarrow \mathbb{R}$ is differentiable, and strictly concave. (b) $U'(c) > 0$. (c) $\lim_{c \to 0} U(c) = -\infty$. Then, the maximin principle of justice gives a solution to the problem

$$\max_{C \in \mathcal{C}} \min_{t} W_t(C) = V(c_t, c_{t+1}, \cdots, c_{t+n-1}) = \sum_{i=0}^{n-1} \beta_i U(c_{t+i}) \tag{7}$$

Now we present Arrow's theorem on the maximin path when sympathy to $n-1$ future generations prevails in the economy. Let $\hat{c}$ be the consumption level for which the capital will be maintained intact, that is,

$$\hat{c} = \frac{\gamma - 1}{\gamma} k_0. \tag{8}$$

For any t such that $c_t = \hat{c}$, $k_t = k_0$. In other words the constant consumption $\hat{c}$ will cause k_t to remain constant at the initial stock k_0. Let $c_0^R, c_1^R, \cdots, c_n^R$

be the solution to the problem

$$\max_{c_0,c_1,\cdots,c_{n-1}} V(c_0,c_1,\cdots,c_{n-1}) = \sum_{i=0}^{n-1}\beta_i U(c_i) \tag{9}$$

$$\text{s.t.} \quad \sum_{i=0}^{n-1}\gamma^{-i}c_i = \hat{c}\sum_{i=0}^{n-1}\gamma^{-i}. \tag{10}$$

Theorem 1 (Arrow(1973)) *Suppose $\gamma^i\beta_i < \gamma^j\beta_j$ for any $i,j \in \mathbb{N}$ with $0 \le i < j \le n-1$. Then the feasible consumption path which maximizes $\min_t W_t$ is given by $C^R =_{rep} (c_0^R, c_1^R, \cdots, c_{n-1}^R)$. For this path $c_i^R < c_j^R$ and $W_i(C^R) < W_j(C^R)$ for $0 \le i < j \le n-1$.*

The result can be illustrated in figure 1.

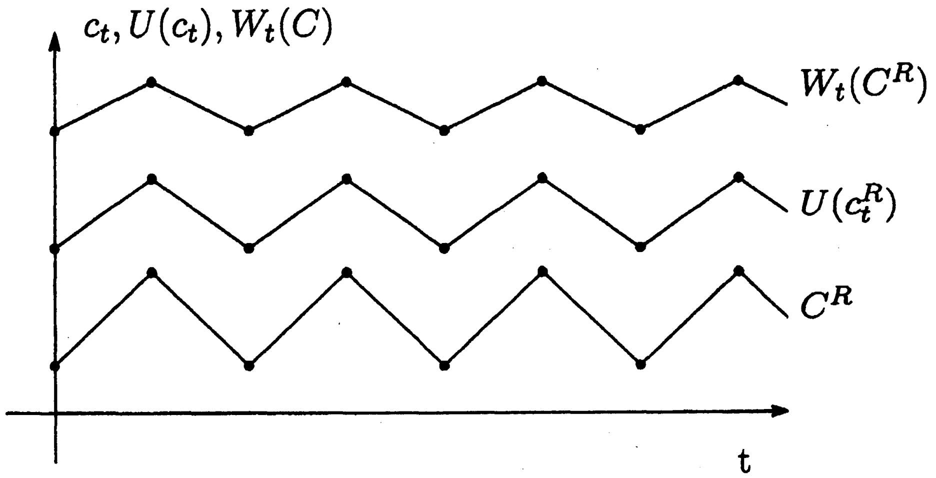

Figure 1:

3 Binary Relational Approach to AD Economy

In this section, we introduce a framework and results in Suga and Udagawa(2006a). In the paper, an intergenerational justice is formalized as a binary relation over a set of consumption paths. The binary relation whose maximal path coincides with the maximin consumption path is characterized by Group Equity and other axioms. Proof of each proposition is omitted (see Suga and Udagawa 2006a).

3.1 Axioms for the Maximin Principle

We will give a characterization of the maximin principle in the Arrow-Dasgupta economy. An intergenerational preference ordering on $\mathcal{C}$ is assumed to be represented by a complete, reflexive, and transitive binary relation $\succsim$. The strict preference ($\succ$) and indifference ($\sim$) relations are defined as follows:

$$C \sim C' \iff C \succsim C' \wedge C' \succsim C$$
$$C \succ C' \iff C \succsim C' \wedge \neg(C' \succsim C)$$

Now we define some axioms for the characterization of the maximin principle in the Arrow-Dasgupta economy. For any intergenerational preference $\succsim$ a consumption path C^* is called maximal if there exists no C such that $C \succ C^*$. The first axiom is a condition to require the existence of the maximal path.

Axiom 1 *Effectiveness : For any $P \in l^1_{++}$ there exists a consumption path C^* that is maximal in $\mathcal{C} \in \{C \in l^\infty_+ | PC \leq k_0\}$.*

The second axiom is the weak efficiency condition, which is familiar to the axiomatic approach in social choice.

Axiom 2 *Weak Pareto : For any C and $C' \in \mathcal{C}$, if $W_t(C) > W_t(C') \forall t \in \mathbb{N}$, then $C \succ C'$.*

The third axiom is Hammond equity principle, which is also well-known in the theories of fairness. This axiom demonstrates that the social preference of the consumption paths whose welfare levels differ from each other only for the two generations should be determined by the preference of the less favored.

Axiom 3 *Hammond equity : For any C and $C' \in \mathcal{C}$, if there exist $t^1, t^2 \in \mathbb{N}$ such that $W_{t^1}(C') \geq W_{t^1}(C) \geq W_{t^2}(C) \geq W_{t^2}(C')$ and $W_t(C) = W_t(C')$ for all $t \neq t^1, t^2$, then $C \succsim C'$.*

We introduce a new axiom of equity among groups in order to describe a fairness requirement that treats groups of generations equally if these are regarded equal in utility profile. As an auxiliary step, we review the lexicographic ordering R^L on the Euclidean n-space E^n. For every $v \in E^n$, let $i(v)$ denote the i-th smallest element, ties being broken arbitrarily, so that we have

$$v_{1(v)} \leq v_{2(v)} \leq \cdots \leq v_{n(v)}$$

We may then define three binary relations P^L, I^L and R^L on E^n by

$$v^1 P^L v^2 \Longleftrightarrow \exists r \leq n : \begin{cases} \forall i \in \{1, 2, \ldots, r-1\} : v^1_{i(v^1)} = v^2_{i(v^2)} \\ \textit{and} \\ v^1_{r(v^1)} > v^2_{r(v^2)} \end{cases}$$

and

$$v^1 R^L v^2 \Longleftrightarrow v^1 P^L v^2 \quad \textit{or} \quad v^1 I^L v^2 \quad \textit{for all } v^1, v^2 \in E^n.$$

We are now in the position of defining an axiom called group equity. Take any two groups G_1, G_2 which consist of finite number n of successive generations. For any consumption path C, we have two n-dimensional vectors $\{W_t(C)\}_{t\in G_1}$ and $\{W_t(C)\}_{t\in G_2}$. Let $i(G)$ be the index of the i-th smallest component of C_G, so that we have

$$c_{1(G)} \leq c_{2(G)} \leq \cdots \leq c_{n(G)}.$$

When $\{W_t(C)\}_{t\in G_1} P^L \{W_t(C)\}_{t\in G_2}$ holds, we say that $\{W_t(C)\}_{t\in G_1}$ *leximin dominates* $\{W_t(C)\}_{t\in G_2}$. With this notation we define an extension of Hammond equity in the case of sympathy to n future generations.

Axiom 4 *Group Equity : For any C and $C' \in \mathcal{C}$, $C \succsim C'$ if there exist t^r and t^p such that*

1. $W_t(C') \geq W_t(C)$ $(t = t^r - (n-1), \ldots, t^r)$,
2. $W_t(C) \geq W_t(C')$ $(t = t^p - (n-1), \ldots, t^p)$,
3. $W_t(C) = W_t(C')$ *(otherwise), and*
4. $\{W_t(C)\}_{t=t^r-(n-1),\ldots,t^r}$ *leximin dominates* $\{W_t(C)\}_{t=t^p-(n-1),\ldots,t^p}$,

then $C \succsim C'$, where $W_t(C) = W_0(C)$ for $t < 0$.

This axiom is a new requirement for equity among groups of successive generations, which is applied to the case that a transfer from one generation to the other occurs. Such transfer increases the welfare of the transferred generation, as well as of the adjacent generation before this. This also decreases welfare of the deprived generation as well as of the adjacent generation before that. The reason why we need this type of requirement is that a change of consumption in some generation under the feasibility of the economy brings increase of utility on a group of successive generations and decrease on another group, which does not satisfy the conditions presupposed in Hammond equity.

3.2 Characterization of the Maximin Principle under Sympathy

In this subsection, we characterize the maximin principle in the Arrow-Dasgupta economy where each generation has sympathy to the immediate $n-1$ future generations. The following lemmas show the important role of group equity axiom.

Lemma 1 *Suppose that $\succsim$ satisfies effectiveness, group equity and weak Pareto. If C^* is the maximal consumption path of $\succsim$, then at C^* generation 0 has the minimum level of welfare, that is, $W_0(C^*) = \min_t W_t(C^*)$.*

Lemma 2 *Suppose that $\succsim$ satisfies effectiveness, group equity and weak Pareto. If C^* is the maximal consumption path of $\succsim$, then at C^*, in any interval with the length n, there is at least one generation whose welfare is the minimum of $\{W_t(C^*)\}_t$, that is, $\forall t^0 : \min_{t^0 \leq t < t^0+n} W_t(C^*) = \min_t W_t(C^*)$.*

Lemma 3 *Suppose that $\succsim$ satisfies effectiveness, group equity and weak Pareto,and that $\gamma^i\beta_i < \gamma^j\beta_j$ for any $i, j \in \mathbb{N}$ with $0 \leq i < j \leq n-1$. Then $c^*_{nl} < c^*_{nl+k}$ and $W_{nl}(C^*) < W_{nl+i}(C^*)$ for any $l, n \in \mathbb{N}_+$, $0 \leq i < n$.*

Lemma 4 *Suppose that $\succsim$ satisfies effectiveness, group equity, and weak Pareto. Then, generation 0 in the maximal consumption path C^* for $\succsim$ has the largest welfare among all feasible consumption paths where generation 0 has the least welfare among all the generations. That is,*

$$W_0(C^*) = \max_{C \in \mathcal{D}_0} W_0(C)$$

where $\mathcal{D}_0 = \{C \in l^\infty_+ | C \text{ is feasible and } W_0(C) = \min_t W_t(C)\}$.

These lemmas shows the next theorem in Suga and Udagawa (2006a). It claims that the maximin path in the economy with sympathy is supported by the ethical preference satisfying the above four axioms.

Theorem 2 *Suppose that $\succsim$ satisfies effectiveness, group equity and weak Pareto,and that $\gamma^i\beta_i < \gamma^j\beta_j$ for any $i, j \in \mathbb{N}$ with $0 \leq i < j \leq n-1$. Let C^* be the maximal consumption path of $\succsim$. Then, $c^*_t = c^R_t$ for all $t \in \mathbb{N}$ and $C^* =_{rep} (c^R_0, c^R_1, \cdots, c^R_{n-1})$.*

The next corollary shows that the later the generation comes, the higher the level of consumption becomes for the generations in between those with the minimum level.

Corollary 1 *Suppose that* $\succsim$ *satisfies effectiveness, group equity and weak Pareto,and that* $\gamma^i\beta_i < \gamma^j\beta_j$ *for any* $i, j \in \mathbb{N}$ *with* $0 \leq i < j \leq n-1$. *Let* C^* *be the maximal consumption path of* $\succsim$. *Then,* $c^*_{t-1} < c^*_t$ *and* $W_{t-1}(C^*) \leq W_t(C^*)$ *for* $t \neq 0, n, 2n, \ldots$.

When $n = 1$, that is, each generation cares about herself, her utility function is given by $W_t(C) = W(c_t) = U(c_t)$. In this economy we have constant path if the social preference satisfies the three axioms except group equity. Let c^R be the solution to the problem $\max_{C\in\mathcal{C}} \min_t W(c_t)$. Then we have the following result.

Corollary 2 *Suppose that* $\succsim$ *satisfies effectiveness, Hammond equity and weak Pareto. The maximal consumption path* C^* *of* $\succsim$ *is a constant path, that is* $C^* =_{rep} (\hat{c})$.

4 Choice Functional Approach to AD Economy

In this section, we introduce a framework and results in Suga and Udagawa(2006b). In the paper, an intergenerational justice is formalized as a choice function mapping any feasible set of consumption paths to a proper subset. The choice function whose value coincides with a set containing only the maximin consumption path is characterized by Inclusion of Hammond Superior Path and other axioms. Proof of each proposition is omitted (see Suga and Udagawa 2006b).

At first, define a choice function G which maps any set S of feasible consumption paths to a non-empty subset of itself given a common utility function, e.g., $G(S, W)(\neq \emptyset) \subset S$. Similarly, we define a Rawlsian Choice Function G^R which selects a set of maximin paths C^R for any set of feasible consumption paths given a common utility function, e.g., $C^R \in G^R(S, W)$. It may not be well-defined in general, but discussions in latter shows that so is it in this case.

4.1 Axioms

In this subsection we define some axioms to characterize the maximin principle in this simple dynamic economy.[1] As an auxiliary step, we define two binary relations on the set of consumption paths ℓ_+^∞. The first is strict Paretian relation $\succ^P$, which is given by:

$$C^1 \succ^P C^2 \iff \forall t : W_t(C^1) > W_t(C^2).$$

The second is Hammond equity relation $\succeq^H$, which is defined by:

$$C^1 \succeq^H C^2 \iff \exists t^1, t^2 : \text{(i)} W_{t^1}(C^1) \leq W_{t^2}(C^1), \text{(ii)} W_{t^1}(C^1) \geq W_{t^1}(C^2),$$
$$\text{(iii)} W_{t^2}(C^1) \leq W_{t^2}(C^2), \text{and (iv)} W_t(C^1) = W_t(C^2) \forall t \neq t^1, t^2.$$

Strict Hammond equity relation is given in the usual way as follows:

$$C^1 \succ^H C^2 \iff C^1 \succeq^H C^2 \wedge \neg C^2 \succeq^H C^1.$$

Now we provide three kinds of axioms, e.g., an inclusion of some paths in a choice set, an exclusion of some paths from a choice set, and a consistency of a choice set for an expansion and a contraction of the feasible set. The first and second axioms are requirements from the view point of the Pareto criterion. The former axiom requires that, if a path is Pareto superior to the path which is in the choice set, then the superior path must also be included in the choice set. The latter requires that, if a path is Pareto inferior to the path which is feasible, then the inferior path must be excluded from the choice set.

Axiom 5 *$G(.,.)$ satisfies Inclusion of Pareto Superior Path(IP) iff*

$$\forall C^1, C^2 \in \ell_+^\infty, \forall S \subset \ell_+^\infty : [[C^1 \succ^P C^2 \wedge C^1 \in S \wedge C^2 \in G(S, W)] \rightarrow C^1 \in G(S, W)].$$

Axiom 6 *$G(.,.)$ satisfies Exclusion of Pareto Inferior Path(EP) iff*

$$\forall C^1, C^2 \in \ell_+^\infty, \forall S \subset \ell_+^\infty : [[C^1 \succ^P C^2 \wedge C^1 \in S] \rightarrow C^2 \notin G(S, W)].$$

The third and fourth axioms are requirements from the view point of the Hammond equity criterion. The former axiom requires that, if a path is Hammond superior to the path which is in the choice set, then the superior path must also be included in the choice set. The latter requires that, if a path is Hammond inferior to the path which is feasible, then the inferior path must be excluded from the choice set.

[1] For the choice theoretic framework of the social choice theory, see, for example, Sen (1970) and Suzumura (1986).

Axiom 7 *$G(.,.)$ satisfies Inclusion of Hammond Superior Path(IH) iff*

$$\forall C^1, C^2 \in \ell_+^\infty, \forall S \subset \ell_+^\infty : [[C^1 \succeq^H C^2 \wedge C^1 \in S \wedge C^2 \in G(S,W)] \to C^1 \in G(S,W)].$$

Axiom 8 *$G(.,.)$ satisfies Exclusion of Hammond Inferior Path(EH) iff*

$$\forall C^1, C^2 \in \ell_+^\infty, \forall S \subset \ell_+^\infty : [[C^1 \succeq^H C^2 \wedge C^1 \in S] \to C^2 \notin G(S,W)].$$

The last two axioms are conditions of the consistency for the choice sets. The fifth axiom is a requirement that any path in the choice set for a larger opportunity set is also included in the choice set for a smaller opportunity set if the path is still feasible. So, it is also called as set-contraction condition.

Axiom 9 *$G(.,.)$ satisfies Condition α iff*

$$\forall S^1, S^2 \subset \ell_+^\infty, S^1 \subset S^2 : \forall C^1 \in S^1 : [C^1 \in G(S^2, W) \to C^1 \in G(S^1, W)].$$

The sixth axiom is a requirement that, if two consumption paths are compared and both are to be chosen, then they must be equally evaluated in the larger feasible set.

Axiom 10 *$G(.,.)$ satisfies Condition δ^* iff*
$\forall S \subset \ell_+^\infty : \forall C^1, C^2 \in S : [[\{C^1, C^2\} = G(\{C^1, C^2\}, W)] \to [C^1 \in G(S,W) \iff C^2 \in G(S,W)]].$

4.2 Main Theorem

We are in the position to provide our main theorem about the characterization of the Rawlsian choice function, $G^R(.,.)$.

Theorem 3 *Suppose that $G(.,.)$ satisfies EP, IH, α, and δ^*. Then, $G(F(k_0,\gamma),W) = C^R(F(k_0,\gamma),W)$.*

To prove this theorem we need some lemmas.

Lemma 5 *Suppose that $G(.,.)$ satisfies EP, IH, α, and δ^*. Then generation 0 has the least welfare in all generations in any chosen path, that is, $W_0(C) = \min_t W_t(C)$ for any $C \in G(F(k_0,\gamma),W)$.*

Lemma 6 *Suppose that $G(.,.)$ satisfies EP, IH, α, and δ^*. Then, generation 0 in $C \in G(F(k_0,\gamma),W)$ has the largest welfare among all feasible consumption paths where generation 0 has the least welfare among all the generations. That is,*

$$W_0(C) = \max_{C \in \mathcal{D}_0} W_0(C)$$

for any $C \in G(F(k_0,\gamma),W)$ where $\mathcal{D}_0 = \{C \in l_+^\infty | C \text{ is feasible and } W_0(C) = \min_t W_t(C)\}$.

Lemma 7 *If any path C satisfies that $\sum_{s=0}^{n-1} \gamma^{-s} c_{s+ln} \geq \sum_{s=0}^{n-1} \gamma^{-s} c_s^R$ for all $l \in \mathcal{Z}_+$ and $\sum_{s=0}^{n-1} \gamma^{-s} c_{s+l'n} > \sum_{s=0}^{n-1} \gamma^{-s} c_s^R$ for some $l' \in \mathcal{Z}_+$, then C is infeasible.*

We will provide inverse relations of Theorem 3.

Theorem 4 *Rawlsian choice function G^R satisfies IP, IH, EP, α and δ^*.*

With Theorems 3 and 4, we finally come to the following characterization theorem.

Theorem 5 *Suppose that the utility of generation t is given by*

$$W_t = \sum_{i=0}^{n-1} \rho_i U(c_{t+i}),$$

that $\gamma^i \rho_i$ increases with i for $i \in [0, n-1]$, and that ρ_i is non-increasing with i. Then the choice function $G(.,.)$ satisfies EP, IH, α and δ^*, iff it is Rawlsian, that is, $G(F(k_0,\gamma),W) = G^R(F(k_0,\gamma),W)$.

4.3 Independence of Axioms

There is no choice function satisfies EH. It is impossible to strengthen IH to EH.

Consider three consumption paths, C^1 and C^2, such that $W(C^1) = (3,2,0,0,0,...)$ and $W(C^2) = (3,1,0,0,0,...)$. On one hand, let $t^1 = 1$ and $t^2 = 0$ in the definition of $\succeq^H$ and then $C^1 \succeq^H C^2$. On the other hand, let

$t^1 = 2$ and $t^2 = 1$ in the definition of $\succeq^H$ and then $C^2 \succeq^H C^1$. So both C^1 and $C^2 \notin G(\{C^1, C^2\}, W)$. It violates the non-emptiness of the choice function.

If we drop IH, a myopic choice function satisfies the remains of axioms.

Example 1. A myopic choice function, $G(S, W) = \arg\max_{C \in S} W_0(C)$, satisfies EP, α, and δ^*.

EP: By the hypothesis of EP, $W_0(C^1) > W_0(C^2)$. So, if C^1 is feasible, $G(S, W)$ does not contains C^2 by the definition of $G(.,.)$. Therefore EP holds.
α: By the hypothesis of α and the definition of $G(.,.)$, generation 0 has the maximal welfare in C^1 among S^2. So clearly it does so among S^1 ($\subset S^2$).
δ^*: By the hypothesis of δ^* and the definition of $G(.,.)$, generation 0 has the same welfare in both C^1 and C^2 and therefore the conclusion of δ^* holds.

Now, consider two consumption paths, C^1 and C^2, such that $W(C^1) = (2, 0, 0, 0, 0, ...)$ and $W(C^2) = (1, 1, 0, 0, 0, ...)$. IH requires $C^2 \in G(\{C^1, C^2\}, W)$, but $\{C^1\} = G(\{C^1, C^2\}, W)$ by definition. Therefore, IH does not hold.

If we replace EP with a weaker condition, IP, then a trivial choice function satisfies the set of axioms.

Example 2. A trivial choice function, $G(S, W) = S$, satisfies IP, IH, α, and δ^*.

This choice set always contains all feasible consumption paths. So the conclusion of IP, IH, α, and δ^* holds for any feasible set and any utility function respectively. Therefore, this choice function satisfies IP, IH, α, and δ^*.

On the other hand, it clearly violates EP.

5 Concluding Remarks

We show the maximin consumption path in Arrow-Dasgupta economy is supported by both ethical preference relation and choice function satisfying egalitarian axioms. This result claims the saw-tooth shaped path has an egalitarian property.

However, ideas of the characterizing egalitarian axioms is different from each other in a binary relational approach and a choice functional approach. We guess the difference is caused by the position to use infeasible paths to construct an ethical preference relation and a choice function. To formulate and analyse it is our future problem.

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