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# Price and Quantity Competition in an Oligopoly with Discrete Choices

Shin Kawai* and Tadashi Minagawa†

*Graduate school of Economics,*

*Nagoya University,*

*Furo-cho, Chikusa-ku, Nagoya, 464-8601, Japan.*

**Abstract**

This paper investigates the price and quantity competition in an oligopoly with capacity constraints: i.e., a Bertrand-Edgeworth competition. We assume that consumers take into account the probability of purchase given prices, quantities and the number of consumers. In the preceding Bertrand-Edgeworth competition models, two rationing rules, *surplus-maximizing* and *proportional* rationing, have been proposed. We consider a consumer's rational behavior instead of these rules, and show that there is a range of a continuum of symmetric Nash equilibrium in pure strategy above marginal cost.

*JEL classification:* L13, D45

*Keywords:* Bertrand-Edgeworth, Oligopoly, Rationing Rules

*Tel.: +81-52-789-5456. Fax.: +81-52-789-5456. E-mail: skawai@soec.nagoya-u.ac.jp
†Tel.: +81-52-789-2384. Fax.: +81-52-789-4924. E-mail: minagawa@soec.nagoya-u.ac.jp

# 1 Introduction

This paper investigates the price and quantity competition in an oligopoly with capacity constraints; Bertrand-Edgeworth competition. We assume that consumers take into account not only the prices but also the probability of purchase at each retail store. The probability of purchase is introduced as a function of the quantity and the number of consumers as in Barro and Romer (1987). The equilibrium number of consumer is determined by the interaction among consumers. In the preceding Bertrand-Edgeworth competition models, two rationing rules have been proposed; the efficient rationing rule and the proportional rationing rule. We consider a consumer's rational behavior instead of these rules, and show that there is a range of a continuum of symmetric Nash equilibrium above marginal cost with pure strategy.

The Cournot Competition is a quantity competition in which the equilibrium price is uniquely determined above marginal cost. Bertrand criticized Cournot. If firms compete in prices, the equilibrium price equals to the marginal cost. Thus, the competitive price will be appeared even under oligopoly. This is the famous Bertrand Paradox. There are two main keywords to solve this paradox: the product differentiation and the capacity constraints. We focus on the latter. The representative model of capacity constraints is the Bertrand-Edgeworth competition. This model needs the rationing rule since firms face the residual demand when they raise the prices more than the other firms' prices. As noted above, two leading rationing rules have been proposed. However, these rationing rules are merely used as a substitute for a complete analysis of consumer behavior (Tirole 1988).

There are two main purposes in this paper. One is to characterize a rationing by introducing the interactions among consumers. There are homogenous consumers with reservation utility for an indivisible good v. Consumers take into account the probability of purchase at each retail store as a function of quantity and the number of consumers. From these assumptions, the following expected surplus fūnction at firm i is derived:

$$V_i(p_i, q_i, n_i) = \lambda_i(q_i, n_i)(v - p_i).$$

Another main purpose is to show that there exists a continuum of Nash equilibria in pure-strategy in the case that the total capacity level equals to the total number of consumers. There is an existence problem in the Bertrand-Edgeworth competition. Edgeworth pointed out that pure-strategy equilibria may not exist unless demand is highly elastic. With either rationing rule, the only possible candidates for pure-strategy equilibria are uniquely determined under some conditions (Tirole 1988; Vives 1999). In the case of discrete choices with homogenous reservation utility v, the efficient rationing rule and the proportional rationing rule coincide with each other. Equilibrium price is uniquely determined at v, a monopoly price level, in the case of the total capacity level equals to the total number of consumers. Our results show that the rationing by introducing the interactions among consumers leads to a quite different conclusion.

## 2 Model

Consider a Bertrand-Edgeworth (B-E) competition in the retail market for an Indivisible Good. Suppose that there are capacity constraints. There are $N$ consumers. Let $N$ be a large number. In the standard B-E model, a common downward sloping demand functions are given. A contingent demand function is derived from the function with a rationing rule. In our paper, on the other hand, we derived a contingent demand function from consumers' discrete choices. Suppose that they are all identical and have a reservation utility $v(> 0)$ for a good.

The quasi-linear utility function of the representative consumer is defined:

$$u(x, m) = vx + m, \tag{1}$$

where $x \in \{0, 1\}$ and $m > 0$ denote the number of an indivisible good and the residual income respectively. At the begging, they choose whether to enter or not the market. After they enter the market, there are two steps. The first step is to choose a store. Suppose that it is prohibitively expensive to change the store. As in Stiglitz (1989), this assumption seems proper for the goods that is not so durable like milk , eggs and breads. It is consumed one per week or day. Hence, budget constraint at each firm $i$ becomes $p_i x + m = y$, where $y$ denotes income. Substituting the budget constraint into $m$ in the utility function at firm $i$:

$$U_i(x, p_i, y) = (v - p_i)x + y. \tag{2}$$

Suppose that the representative consumer can observe the number of supply $q_i$ and shoppers $n_i$ at firm $i$ and that he takes into account the probability of purchase $\lambda_i$ at firm $i$ as a function of $q_i$ and $n_i$:

$$\lambda_i(q_i, n_i) = \begin{cases} 1, & \text{if } 0 \le n_i < q_i, \\ \frac{q_i}{n_i}, & \text{if } q_i \le n_i \le N. \end{cases} \tag{3}$$

Then, the expected utility function at firm $i$ can be defined:

$$\begin{aligned} \bar{V}_i(p_i, q_i, n_i) &= \lambda_i(q_i, n_i)U_i(1, p_i, y) + (1 - \lambda_i(q_i, n_i))U_i(0, p_i, y) \\ &= \lambda_i(q_i, n_i)(v - p_i) + y. \end{aligned} \tag{4}$$

The utility when the consumer doesn't enter the market is $y$. Then it is useful to introduce the expected surplus:

$$\begin{aligned} V_i(p_i, q_i, n_i) &= \bar{V}_i(p_i, q_i, n_i) - y \\ &= \lambda_i(q_i, n_i)(v - p_i). \end{aligned} \tag{5}$$

From (3), it also can be written as,

$$V_i(p_i, q_i, n_i) = \begin{cases} (v - p_i), & \text{if } 0 \le n_i < q_i, \\ \frac{q_i}{n_i}(v - p_i), & \text{if } q_i \le n_i \le N. \end{cases} \tag{6}$$

The function $V_i$ appears in Figure 1(a). Since the utility is $y$ when a consumer doesn't enter the market, all consumers participate the market if $(v - p_i)$ is positive. Thus the sum of the number of shoppers at each firm equals to the number of consumers $N$. If there are two firms, $n_1 + n_2 = N$, Functions $V_1$ and $V_2$ can be depicted at once (Figure 1(b)). From Figure 1(b), it can be seen the

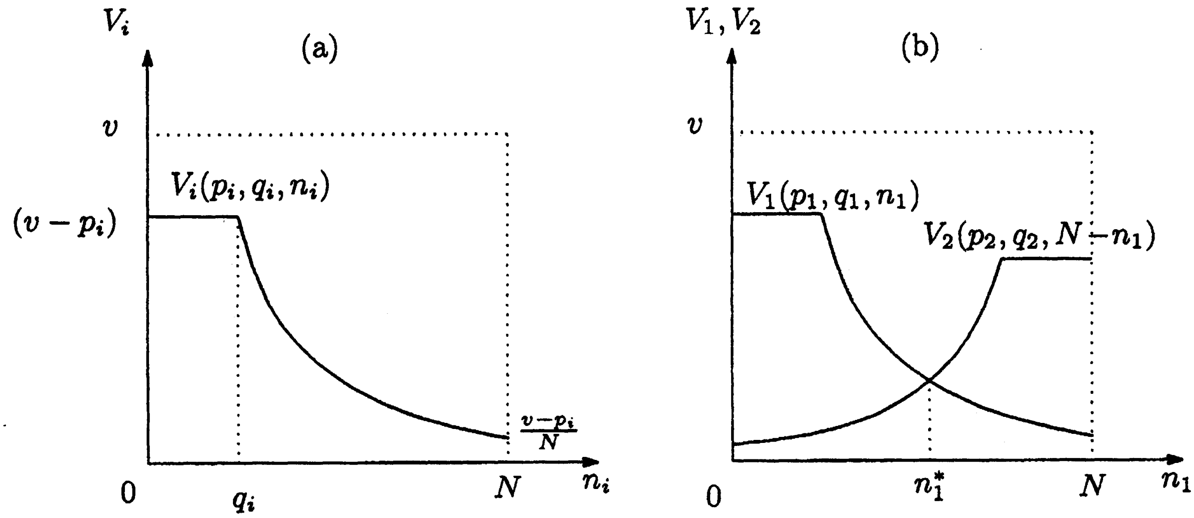


Figure 1: Expected Surplus and the Equilibrium Number of Consumers

equilibrium number of consumers at firm $i$ for $i = 1, 2$. More precisely, it can be derived by solving the following equation for $n_i$:

$$V_i(p_i, q_i, n_i) = V_j(p_j, q_j, N - n_i). \tag{7}$$

From (7), we can derive the contingent demand for firm 1 is given by

if $q_1 + q_2 \ge N$,

$$n_1(p_1; p_2, q_1, q_2) = \begin{cases} \left(\frac{v-p_1}{v-p_2}\right) q_1, & \text{if } p_1 < p_2, \\ \left(\frac{q_1}{q_1+q_2}\right) N, & \text{if } p_1 = p_2, \\ N - \left(\frac{v-p_2}{v-p_1}\right) q_2, & \text{if } p_1 > p_2, \end{cases} \tag{8}$$

if $q_1 + q_2 < N$,

$$n_1(p_1; p_2, q_1, q_2) = \begin{cases} \left(\frac{v-p_1}{v-p_2}\right) q_1, & \text{if } p_1 \leq a_1, \\ \left(\frac{(v-p_1)q_1}{(v-p_1)q_1+(v-p_2)q_2}\right) N, & \text{if } a_1 < p_1 \leq b_1, \\ N - \left(\frac{v-p_2}{v-p_1}\right) q_2, & \text{if } p_1 > b_1, \end{cases} \tag{9}$$

where

$$a_1 = v - \frac{(v-p_2)(N-q_2)}{q_1}, \quad \text{and} \quad b_1 = v - \frac{(v-p_2)q_2}{N-q_1}. \tag{10}$$

When the quoted prices are the same in the case $q_1 + q_2 > N$, demand is split in proportion to the supplies of the firms.

In a parallel fashion the contingent demand for firm 2, $n_2(p_2; p_1, q_2, q_1)$ can be given.

# 3 Equilibrium

There are two firms. Firm $i$ sets price $p_i$ and quantity $q_i$ with 0 marginal cost. Suppose that there is the capacity constraint $q_i \leq k_i$ where $k_i$ denotes the capacity level of firm $i$. Further, suppose that $k_i < N$ for $i = 1, 2$, and firms take $k_i$ as given. At prices and capacity levels $(p_1, p_2, k_1, k_2)$, the quantity chosen by firm $i$ would be $q_i = \min\{k_i, n_i(p_1; p_2, k_1, k_2)\}$, and its payoff $\pi_i(p_i, p_j, k_i, k_j) = p_i q_i$.

**PROPOSITION 3.1** *If $k_1 + k_2 > N$, then an equilibrium in pure strategy does not exist.*

**proof** *In the case $k_1 + k_2 > N$, from* (8), *the profits of firm 1 can be written as*

$$\pi_1(p_1, p_2, k_1, k_2) = \begin{cases} p_1 k_1, & \text{if } p_1 \leq p_2, \\ p_1 \left(\frac{k_1}{k_1+k_2}\right) N, & \text{if } p_1 = p_2, \\ p_1 \left(N - \left(\frac{v-p_2}{v-p_1}\right) k_2\right), & \text{if } p_1 > p_2. \end{cases} \tag{11}$$

*Notice that the profit is not continuous at* $p_1 = p_2$. *The response function of firm 1 when* $p_1 > p_2$ *can be derived as*

$$\tilde{R}_1(p_2, k_2) = v - \sqrt{\frac{v(v-p_2)k_2}{N}}. \tag{12}$$

*It can be seen that* $\tilde{R}_1(p_2, k_2) > p_2$ *when* $p_2 < \tilde{p}_2$, *where* $\tilde{p}_2 = (1 - k_2/N)v$.

*The optimal strategy for firm 1 when* $p_2 \geq \tilde{p}_2$ *is to set price* $p_1$ *less than but infinitely close to* $p_2$. *This strategy is the same as Bertrand competition. However, we can easily seen that* $\tilde{R}_1(0, k_2) > 0$ *and hence firm 1 can earn some positive profit when* $p_2 = 0$. *The similar argument applies to firm 2. Therefore, there is no equilibrium.*

Proposition 3.1 has an analogy with that Edgeworth pointed out. This result is caused by the capacity constraint rather than by the rationing rules. Thus, this paper doesn't focus on this proposition.

**PROPOSITION 3.2** *If* $k_1 + k_2 = N$, *there exists a continuum of symmetric Nash equilibria* $p^*$ *in pure strategy in the range of* $[\tilde{p}^*, v]$, *where* $\tilde{p}^* = \max\{(1 - k_1/N)v, (1 - k_2/N)v\}$.

**proof** *In the case* $k_1 + k_2 = N$, *from* (8), *the profits of firm 1 can be written as*

$$\pi_1(p_1, p_2, k_1, k_2) = \begin{cases} p_1 k_1, & \text{if } p_1 \leq p_2, \\ p_1\left(N - \left(\frac{v-p_2}{v-p_1}\right) k_2\right), & \text{if } p_1 > p_2. \end{cases} \tag{13}$$

*In this case, profit is continuous at* $p_1 = p_2$.

$$R_1(p_2, k_2) = \begin{cases} v - \sqrt{\frac{v(v-p_2)k_2}{N}}, & \text{if } 0 \leq p_2 < \tilde{p}_2, \\ p_2, & \text{if } \tilde{p}_2 \leq p_2 \leq v, \end{cases} \tag{14}$$

*where* $\tilde{p}_2 = (1 - k_2/N)v$. *Firm 1's reaction function* (14) *appears in Figure 2. Similarly, firm 2's reaction function can be derived. If* $k_1 = k_2$, *then* $\tilde{p}_1 = \tilde{p}_2 =$

*$v/2$. In this case, there is a continuum of symmetric Nash equilibria in $[v/2, v]$ (Figure 4 (a)). If $k_1 < k_2$, then $\tilde{p}_1 > \tilde{p}_2$. In this case, the minimum of the equilibrium price $\tilde{p}^* = \tilde{p}_1$ as in Figure 4 (b).*

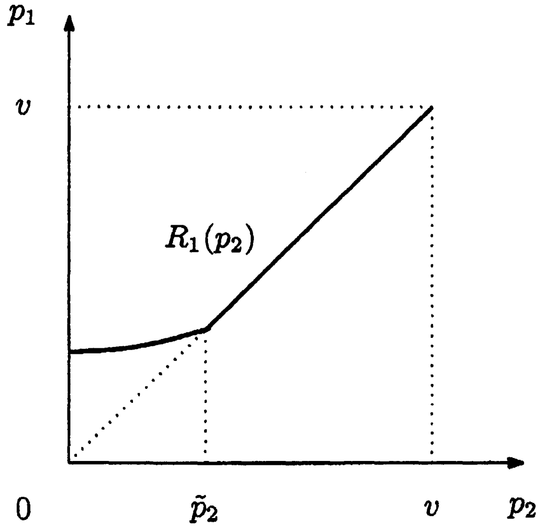


Figure 2: Reaction curve of firm 1

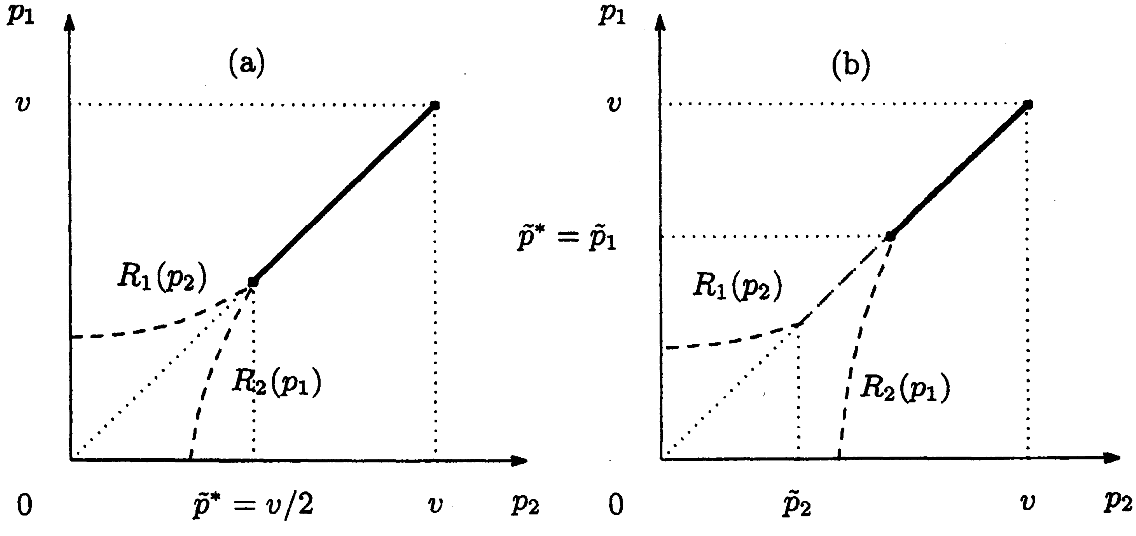


Figure 3: A Continuum of Symmetric Nash Equilibria

This is the main proposition in this paper. There is a range of equilibrium price levels. However, the realized equilibrium price level is unique even under the asymmetric capacity levels. It is also found that the equilibrium range is maximum at the same capacity levels.

**PROPOSITION 3.3** *If $k_1 + k_2 < N$, there exists a Nash equilibrium $p^*$ in pure strategy. The equilibrium price $p^*$ equals to the monopoly price level: i.e., $p^* = v$.*

**proof** *In the case $k_1 + k_2 < N$, from (9), the profits of firm 1 can be written as*

$$\pi_1(p_1, p_2, k_1, k_2) = \begin{cases} k_1, & \text{if } p_1 \leq b, \\ N - \left(\frac{v-p_2}{v-p_1}\right) k_2, & \text{if } p_1 > b_1, \end{cases} \tag{15}$$

*where $b_1 = v - \frac{(v-p_2)k_2}{N-k_1}$. It is found that $b_1$ is upward sloping linear function in $p_2$ and $b_1 = v(1 - k_2/(N - k_1))$ when $p_2 = 0$, $b_1 = v$ when $p_2 = v$. This is the reaction function of firm 1. Similarly, the reaction function of firm 2 is $b_2 = v(1 - k_1/(N - k_2))$. These curves intersect only once at $v$.*

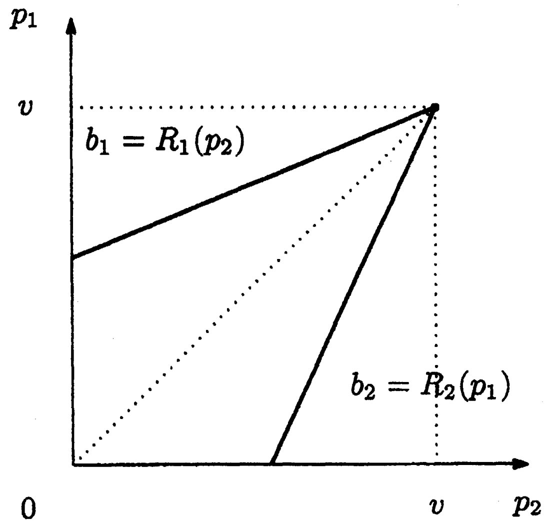


Figure 4: Symmetric Nash Equilibrium

This result is intuitively obvious under this setting. The market is completely separated and the firm has monopoly power to the residual demand.

# 4 Conclusion

This paper finds that there exists the equilibrium price levels in pure strategy. There are the ranges of the equilibrium price levels if the total capacity level equals to the number of consumers. If the total capacity level is less than the number of consumers, the equilibrium price would be monopoly one. This result is caused by the capacity constraints and the demand structure. Notice that under two leading rationing rules, proportional and efficient, the equilibrium price level becomes monopoly price if $k_1 + k_2 \leq N$. Therefore, the contribution of this paper is finding of proposition 3.2. This result is caused by introducing the strategic interaction of firms and consumers combined with capacity constraints.

The implication of Proposition 3.3 is that there exists price dispersion between several locally separating oligopoly markets in space or in time like Gas stations or service price of eggs. The equilibrium price levels has some range in these cases.

Like Kreps and Sheinkman (1983), introducing the stage of the determination of capacity levels among firms and to investigate whether the results of Proposition 3.2 are the next program of our research. It seems that the case of Proposition ?? disappear since the firm slightly reduce the capacity level and intends to have a monopoly power. However, it might be not true if the free entry is allowed. As in Davidson and Deneckere (1986), we have to investigate which rule might be occur in the long-run equilibrium.

In this paper, comparing the results between the two rationing rules and our model in the specific demand structure. Introducing the heterogenous reservation utility levels and downward sloping demand function as a limit case is another extension of our research.

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