数理解析研究所講究録
第 1557 巻 2007 年 187-210

Electricity markets analysis and design*

Alexander Vasin[1,3], Polina Vasina[2].

[1] *Lomonosov Moscow State University*

[2] *Dorodnicyn Computing Center of the RAS*

[3] *New Economic School, Moscow*

Abstract.

This paper considers Nash equilibria of the unique-price supply function auction for a homogeneous good. We discuss different estimates and indices of the market power with respect to an electricity market and show that standard criteria of the market competitiveness are too soft for this market. We obtain the more strict conditions that provide a sufficiently small deviation of the market price from the Walrasian price. The second part studies the problem of multiple Nash equilibria for the network supply function auction in the electricity market. We show that, under typical parameters of the market, its equilibria may be approximated by the equilibria of the market without transmission losses. This result permits to reduce the number of possible equilibria and simplifies the analysis of the market.

1. Introduction.

An important economic tendency of the last 30 years was the development of markets for electricity and natural gas in several countries. The creation of a market includes forming of several private generating companies, and determination of the market mechanism for their interaction with consumers. In many existing wholesale markets, the most important part of this mechanism is a regular supply function or a double auction that determines the market price and the production volume for each company. Typically this auction is organized as a unique price auction (though some studies show that Vickrey auction might be the more efficient form of the interaction, see Vasin, Vasina (2005).

* This work was supported by the NES research program, the RFBR, grant 02-01-00610, and by the grant 1815.2003.1 of President of Russian Federation. We thank Fuad Aleskerov for useful comments.

Creation of the market structure concerns the following problem. On one hand, in order to reduce the market power and prevent a large increase of the market price over the competitive equilibrium price, it seems reasonable to split the generation sector into many small companies. On the other hand, the scale effect and the reliability of the electricity supply (that is very important for Russia) require creation of sufficiently large generating companies. Thus an important question is what minimal degree of the splitting provides the sufficiently small deviation of the market price from the competitive equilibrium price.

Our previous study (Vasin and Vasina, 2005) shows that stable rational behavior of agents at the supply function auction corresponds to the Cournot equilibrium outcome. So the question about splitting implies the following theoretical problems.

The first one is evaluation of the Cournot price deviation from the Walrasian price under given market structure and available information on the parameters of the market. It is important to discuss the known indices of the market competitiveness (in particular, Concentration ratio and Herfindahl-Hircshman index, see Hircshman, 1963, Tirole, 1997) in context of such evaluation. In Section 3 we obtain an estimate of deviation of the Cournot price from the Walrasian price depending on the demand elasticity and the share of the largest company in the market. We also discuss standard criteria of the market competitiveness related to Concentration ratio and Herfindahl-Hircshman index and show that they are too soft for the electricity market. We obtain the more strict conditions that provide a sufficiently small deviation of the market price from the Walrasian price.

Another important problem relates to the network structure of electricity and gas markets. Below we show that in context of the imperfect competition study, the losses under transmission are not so important since the loss coefficient usually does not exceed 0.1. However, transmission capacity constraints essentially influence the properties of the market in many cases. Our study (2005) shows that, even for the simplest network market with two nodes, there exist 5 possible variants of the Nash Equilibrium (NE below), moreover, 3 NE may coexist under some parameters. This makes the analysis of the market a complicated problem. Below we develop an approach to reduce the number of possible equilibria under consideration. We employ two ideas. First, we show that some equilibria are incompatible, and provide a simple rule that distinguishes one of three variants as a possible NE under given parameters of the market. Then we show that an equilibrium of any market with losses may be approximated by some equilibrium of the similar market without losses. Thus we reduce to 3 the number of possible variants of NE for a two-node market and show that at most two NE may coexist for a market without losses. We also give an example where two NE coexist, so in general it is impossible to improve the result.

2. Survey of literature.

The problem of imperfect competition in the markets for homogeneous goods (gas, electricity etc) is widely discussed in the literature. For the empirical investigation see Sykes and Robinson (1987). The corresponding theoretical models consider a local market without network structure. Static one-period models (Baldick et al. (2000), Green (1992), Klemperer and Mayer (1989)) describe a sealed bid unique-price auction as a normal form game and characterize its Nash equilibria. The latter paper studies a model of competition via arbitrary supply functions set by producers. For a given demand function they show that for any price above the Walrasian one there exists the corresponding Nash equilibrium. Green and Newbery (1992) consider a symmetric duopoly with linear supply and demand functions and obtain the explicit expressions for computation of the Nash equilibrium. Baldick et al. (2000) generalize their result for an asymmetric oligopoly. Abolmasov and Kolodin (2002) and Dyakova (2003) apply this approach for a study of the electricity markets in two Russian regions. They use affine approximations of the actual supply functions.

Let us note that the assumption on the affine structure of supply functions does not correspond neither to the actual cost structure of generating companies, nor to the rules of supply functions auctions. Typically every producer can make a bid corresponding to the non-decreasing step supply function. The project of the Russian wholesale electricity market permits up to 3 steps in a bid of one firm for each hour (see The Model of the Russian Wholesale Market). The step structure of a bid approximately corresponds to the actual structure of variable costs of generating companies. Usually every such company owns several generators with limited capacities and approximately fixed marginal costs. The main part of these costs is the fuel costs.

Our previous paper Vasin, Vasina (2005) studies properties of Nash equilibria for the supply function auction, where a bid is a non-decreasing step function. We start with investigation of the local market. We show that there exists a unique Nash equilibrium in the Cournot model for any non-increasing demand function with the non-decreasing demand elasticity under mild assumptions on the demand asymptotics as the price tends to infinity. We develop a descriptive method for computation of the Cournot outcome under any affine demand function and piece-wise constant marginal costs of producers. In the general case, we obtain an explicit upper estimate of the deviation of the Cournot outcome from the Walrasian outcome proceeding from the demand elasticity and the maximal share of one producer in the total supply at the Walrasian price.

Amir (1996) and Amir and Lambson (2000) study existence and uniqueness of the Nash equilibrium in the Cournot model for logconvex and logconcave inverse demand functions. (Note that $D^{-1}(v)$ is concave (convex) if $p\,|\,D'(p)\,|$ increases (decreases) in p.) Thus, the first property is stronger than increasing of the demand elasticity while the second may hold or not hold in our case. A typical example of the demand function with increasing elasticity that does not meet the both properties is the demand for a necessary good with the low elasticity for low prices and the high elasticity for high prices, such that consumers prefer some substitute.

Vasin and Vasina (2005) consider also a model where the market price is determined from the balance of the demand and the actual supply of the sealed bid auction and producers set arbitrary non-decreasing step supply functions as their strategies. We show that, besides the

Cournot outcome, there exist other Nash equilibria. For any such equilibrium the cut price lies between the Walrasian price and the Cournot price. Vice versa, for any price between the Walrasian price and the Cournot price, there exists the corresponding equilibrium. However, we show that only the Nash equilibrium corresponding to the Cournot outcome is stable with respect to some adaptive dynamics of producers' strategies under general conditions.

This result echoes Moreno and Ubeda (2002) who obtained a similar proposition for a two-stage model where at the first stage producers choose production capacities, and at the second stage they compete by setting the reservation prices. The difference is that in our model the Cournot type equilibrium always exists under fixed production capacities since the agents set the production volumes as well as the reservation prices.

Our results differ from Klemperer and Meyer (1989) who study competition with arbitrary supply functions reported by producers. Under similar conditions, they obtain an infinite set of Nash equilibria corresponding to all prices above the Walrasian price. Our constraint that permits only non-decreasing step functions is reasonable in context of studying electricity markets. The step structure of the supply function is typical for generating companies and corresponds to the actual rules and the projects of the markets in different countries (see Hogan, 1998).

The second part of Vasin and Vasina (2005) considers a simple network market – the market with two nodes. As above, each local market is characterized by the demand function and the finite set of producers with non-decreasing marginal costs. For every producer his strategy is a reported supply function that determines his supply of the good depending on the price. The markets are connected by a transmitting line with fixed share of losses and transmission capacity. Under given strategies of producers, the network administrator first computes the cut prices for the separated markets. If the ratio of the prices is sufficiently close to one then transmission is unprofitable with account of the loss. In this case, the outcome is determined by the cut prices for the isolated markets. Otherwise the network administrator sets the flow to the market with the higher cut price (for instance market 2). This flow reduces the supply and increases the cut price at the market 1. Simultaneously it increases the supply and reduces the cut price at the market 2. If the transmitted volume does not exceed the transmission capacity, the network administrator determines this volume so that the ratio of the final cut prices corresponds to the loss coefficient. Otherwise, the administrator sets the volume to be equal to the transmission capacity. Thus, he acts as if perfectly competitive intermediaries transmit the good from one market to the other. It is easy to show that such strategy maximizes the total welfare if the reported supply functions correspond to the actual costs.

We consider Cournot competition model for this market. Our study shows that there exist three possible types of Nash equilibrium: 1) an equilibrium with zero flow between the markets and the ratio of the prices close to 1; such equilibrium is determined as if there are two separated markets; 2) an equilibrium with a positive flow and the ratio of the prices corresponding to the loss coefficient; 3) an equilibrium with a positive flow equal to the transmission capacity and the ratio of the prices exceeding the loss coefficient.

Proceeding from the first order condition, we define local equilibria of each type and show how to compute them. Then we study under what conditions the local equilibrium is a real Nash equilibrium. For the market with constant marginal costs and affine demand functions, we determine the set of Nash equilibria depending on the parameters. One interesting finding is that, in the symmetric case with equal parameters of the local markets and a small loss coefficient, the local equilibrium corresponding to the isolated markets is not a Nash equilibrium, but there exist two asymmetric Nash equilibria with a positive flow of the good.

Then we consider a standard network auction of supply functions (with unique nodal prices) and generalize the results obtained for the local auction: stable Nash equilibria correspond to the Cournot outcomes.

3. Evaluation of the market power and Cournot competition.

According to the previous results, the expected outcome of the unique-price supply function auction under rational behavior of agents corresponds to the Cournot equilibrium. Hence it is reasonable to consider deviation of the Cournot price p^* from the Walrasian price $\widetilde{p}$ as a measure of the market inefficiency related to the market power of the agents. Below we obtain an estimate of this deviation depending on the demand elasticity and the share of the largest company in the market. We also discuss the known market indices with respect to analysis of the supply function auction at the electricity market.

Consider a market with a homogenous good and a finite set of producers A. Each producer a is characterized by his cost function $C^a(v)$ with the non-decreasing marginal cost for $v \in [0, V^a]$, where V^a is his production capacity. The precise form of $C^a(v)$ is his private information. The practically important case is where the marginal cost is a step function: $C^a(0) = 0$, $C^{a'}(v) = c_i^a$ for $v \in \left(\sum_{l=0}^{i-1} V_l^a, \sum_{l=0}^{i} V_l^a\right)$, $V_0^a = 0$, $i = 1,..,m$, $\sum_{l=1}^{m} V_l^a = V^a$. Consumers' behavior is characterized by the demand function $D(p)$, which is continuously differentiable, decreases in p, tends to 0 as p tends to infinity, and is known to all agents.

Recall basic definitions.

Combination $(\widetilde{v}^a, a \in A)$ of production volumes is a *Walrasian equilibrium* (WE) and $\widetilde{p}$ is a *Walrasian price* of the local market if, for any a, $\widetilde{v}^a \in S^a(\widetilde{p}) \overset{def}{=} \underset{v^a}{Arg\max}(v^a \widetilde{p} - C^a(v^a))$,

$$\sum_a \widetilde{v}^a = D(\widetilde{p}).$$

Note. The *theoretical supply function* $S^a(p)$ determines the (generally non-unique) optimal production volume of the firm a under a given price p. Formally, it is a non-decreasing closed upper semi-continuous point-set mapping with convex values. A trivial result is that the unique Walrasian price exists under the specified assumptions on the demand function.

For a game $\Gamma = < A, X^a, f^a(x), x \in X, a \in A >$ with the set of players A, the set X^a of strategies and the payoff function f^a for each player a, strategy combination $x^* = (x^{a*}, a \in A)$

is a *Nash equilibrium* (NE) if $f^a(x^*) \ge f^a(x^* \parallel x^a)$ for any a, $x^a \in X^a$. Existence of NE for the models under consideration is established below.

Cournot competition. Consider a model of Cournot competition for the given market. Then a strategy of each producer a is his production volume $v^a \in [0, V^a]$. Producers set these values simultaneously. Let $\vec{v} = (v^a, a \in A)$ denote a strategy combination. The market price $p(\vec{v})$ equalizes the demand with the actual supply: $p(\vec{v}) = D^{-1}(\sum_{a \in A} v^a)$. The payoff function of producer a determines his profit $f^a(\vec{v}) = v^a p(\vec{v}) - C^a(v^a)$. Thus, the interaction in the Cournot model corresponds to the normal form game $\Gamma_C = \left\langle A, [0, V^a], f^a(\vec{v}), \vec{v} \in \bigotimes_{a \in A} [0, V^a], a \in A \right\rangle$, where $[0, V^a]$ is a set of strategies $a \in A$.

Combination $(v^{a*}, a \in A)$ of production volumes is a *Cournot equilibrium* (CE) if it is a NE in the game Γ_C.

Let $(v^{a*}, a \in A)$ denote Nash equilibrium production volumes and $p^* = D^{-1}(\sum_{a \in A} v^{a*})$ be the corresponding price. A necessary and sufficient condition for this collection to be a Nash equilibrium is that, for any a,

$$p^* \in Arg \max_{p \in \left[D^{-1}\left(\sum_{b \neq a} v^{b*} + V^a \right), D^{-1}\left(\sum_{b \neq a} v^{b*} \right) \right]} \left\{ \left(D(p) - D(p^*) + v^{a*} \right) p - C^a \left(D(p) - D(p^*) + v^{a*} \right) \right\}$$

Then the F.O.C. for Nash equilibrium is

$$v^{a*} \in (p^* - C^{a'}(v^{a*})) \,|\, D'(p^*) \,|, \text{ for any } a \text{ s.t. } C^{a\prime}(0) < p^* \qquad (1)$$

$$v^{a*} = 0 \text{ if } C^{a'}(0) \ge p^*, \qquad (2)$$

where $C^{a'}(v) = [C_-^{a'}(v), C_+^{a'}(v)]$ in the break points of the marginal cost function.

In particular, $C_+^{a'}(V^a) = \infty$.

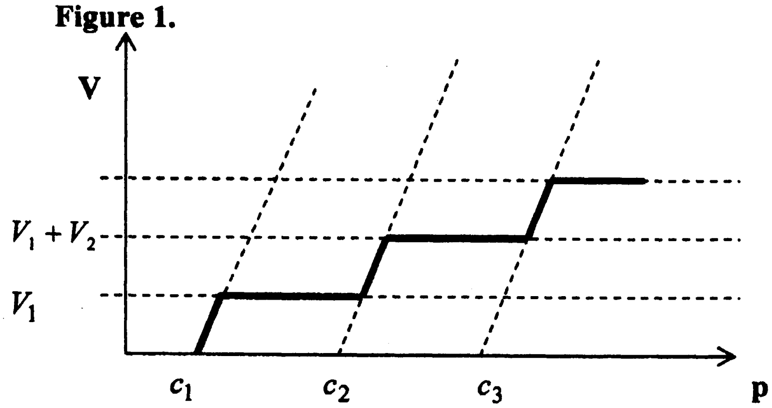
Combination $(p^*, v^{a*}, a \in A)$ is called a local Cournot equilibrium if it meets the necessary conditions (1), (2).

Let us define the Cournot supply function $S_C^a(p)$ of a producer a for $p > 0$ as a solution of the system (1),(2). This function determines the optimal production volume of producer a if p is a Cournot equilibrium price. The function is uniquely defined for any cost function C^a. In particular, consider the case with piece-wise linear cost functions and affine demand function $D(p) = \max(0, \overline{D} - dp)$. Then

$$S_C^a(p) = \begin{cases} 0, \quad p < c_1^a, \\ (p - c_1^a)d \quad \text{if } (p - c_1^a)d < V_1^a, \\ V_1^a \quad \text{if } \quad (p - c_2^a)d < V_1^a < (p - c_1^a)d, \\ (p - c_2^a)d \text{ if } \ V_1^a \le (p - c_2^a)d \le V_1^a + V_2^a, \\ V_1^a + V_2^a \quad \text{if } (p - c_2^a)d < V_1^a + V_2^a < (p - c_3^a)d, \\ \dots\dots\dots\dots\dots\dots \\ V^a \quad \text{if } \quad (p - c_m^a)d > V^a. \end{cases} .$$

Figure 1 shows a typical form of this function. The Cournot price p^* is determined by the equation $\sum_a S_C^a(p^*) = D(p^*)$.

Figure 1.



Our paper Vasin, Vasina (2005) provides the following estimate of the Cournot price deviation from the Walrasian price, proceeding from the demand elasticity $e(p) = p|D'(p)|/D(p)$ and the maximal share of one firm in the total production at the Walrasian equilibrium.

Proposition 1. *Let* $e(p) \geq \bar{e}$ *for any* $p \geq \tilde{p}$, $\max_a S^{a+}(\tilde{p})/S^{+}(\tilde{p}) \leq 1/m$, *and* $en > 1$. *Then*

$\tilde{p}/p^{*} \geq 1 - 1/(\bar{e}n)$, $\sum_a v^{a*}/D(\tilde{p}) \geq (1 - 1/(\bar{e}n))^{\bar{e}}$.

The given upper bound of deviation from the Walrasian price may be inconvenient for practical use since the shares of firms and the demand elasticity at the Walrasian price are typically unobservable, while the actual values under rational behavior of agents correspond to the Cournot outcome. Below we focus on the case where *D(p)* is linear in the practically important interval of prices.

Proposition 2. *Let the maximal share of one firm in the total production at the Cournot equilibrium meet inequality* $\max_a v^{a*} \Big/ \sum_{b \in A} v^{b*} \leq 1/n$ *and the demand elasticity at this price* $e^{*} = dp^{*} \Big/ (\bar{D} - dp^{*})$ *meet condition* $ne^{*} > 1$. *Then*

$$\frac{p^{*}}{\tilde{p}} - 1 \leq \frac{1}{(e^{*}n - 1)}. \tag{3}$$

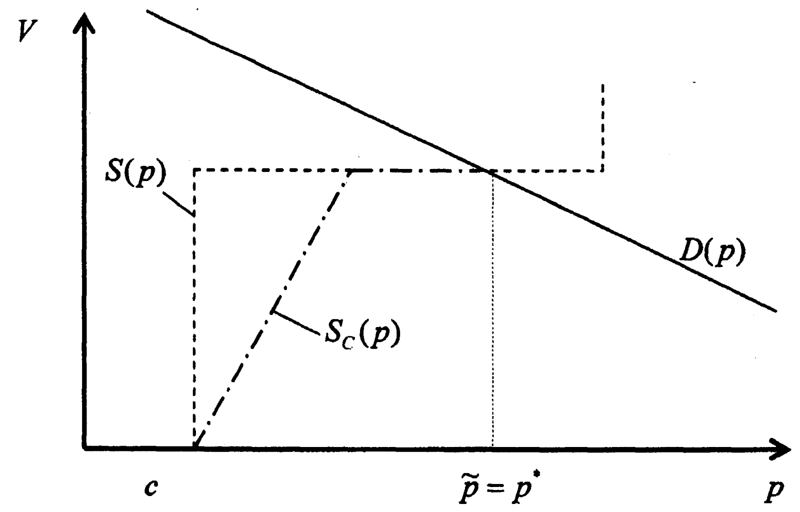
This estimate becomes a strict equality for a symmetric oligopoly with a fixed marginal cost $c = \tilde{p}$ *and unbinding capacity constraints. This case also gives the maximal possible loss of the total welfare:* $w^{*} \Big/ \tilde{w} = 1 - \left(1 - \frac{D(p^{*})}{D(c)}\right)^{2} = 1 - \frac{1}{(n+1)^{2}}$.

Proof. Consider a fixed profile $(v^{a*}, a \in A)$ of production volumes at the Cournot equilibrium. First, we determine the maximal deliation of the Walrasian price $\tilde{p}$ from the Cournot price p^{*} under this profile. Note the following relation betmeen the Cournot and Walrasian supply functions: for any p and a, $S^{a}_{-}(p - S^{a}_{C}(p)/d) \leq S^{a}_{C}(p)$. In particular, for any a, $S^{a}_{-}(p^{*} - v^{a*}/d) \leq v^{a*}$, $S^{a}_{C}(p^{*}) > v^{a*}$. Hence, for $\bar{a} = \arg\max_a v^{a*}$ and otherwise $\bar{p} = p^{*} - v^{\bar{a}*}/d$, $S_{-}(\bar{p}) \leq S_{C}(p^{*}) = D(p^{*}) < D(\bar{p})$. Thus $\tilde{p} \geq \bar{p}$. Moreover, $\tilde{p} = \bar{p}$ under sufficiently large $S_{+}^{\bar{a}}(\bar{p})$, and the maximal deviation is

$$p^{*}\big/\tilde{p} - 1 = \frac{v^{\bar{a}*}/d}{p^{*} - v^{\bar{a}*}/d} = \frac{1/n}{dp^{*}/D(p^{*}) - 1/n} = \frac{1}{e(p^{*})n - 1}.$$ Q.E.D.

Note. Another interesting example that provides the same ratio between the Cournot and the Walrasian prices is where a large firm with the market share $1/n$, a fixed marginal cost c and unbounded capacity interacts with the competitive fringe with lower costs and maximal capacity $V_F = \left(1 - \frac{1}{n}\right)D(p^{*})$. Then $\tilde{p} = c$, the Cournot price meets the same condition $d(p^{*} - \tilde{p}) = (\bar{D} - dp^{*})\big/ n$. The lower bound for deviation of the Cournot price from the Walrasian price under given conditions is 0. Figure 2 provides the corresponding example.

Figure 2.



Now consider the following regulation problem. Assume that under transition to the deregulated market a state regulated monopoly that provided electricity for some region is splitted in n companies with the same constant marginal cost $c^{a} = c$. How large should be n in order to prevent the increase of the market price more than 50% of the cost? It depends on the demand elasticity. Consider a moderate estimate of this value for the electricity market: $e(p^{*}) = 0.2$. Proceeding from proposition 2, we obtain $n = 15$. Even if we take the more favorable value $e(c)=0.2$ then we obtain $n=\frac{c}{(p^*-c)e(c)}-1=9$. Thus a standard assumption that the 20% barrier for the largest company provides a sufficiently competitive market (see, for instance, Dyakova, 2003) seems to fail in this case.

Now consider another popular measure of the market competitiveness – the Herfindahl-Hircshman index (Hirschman, 1963) $HHI=\sum_a (y_a \cdot 100)^2$ where $y_a = {V^a}/{V}$ is the market share of company a.

Proposition 3. *Under any fixed* $HHI = {10^4}/{n}$ *and market elasticity* $e(p^*)=e^*$ *for the Cournot outcome, such that* $e^*\sqrt{n}>1$ *, the ratio of the Walrasian and the Cournot prices meets the inequality:*

$$p^*/\tilde{p} - 1 \le (e^*\sqrt{n}-1)^{-1}. \tag{4}$$

This estimate becomes a strict equality in the case where a large firm with the market share ${1}/{\sqrt{n}}$*, a fixed marginal cost* c *and unbounded capacity interacts with the competitive fringe with lower costs and maximal capacity* $V_F = \left(1-\frac{1}{\sqrt{n}}\right)D(p^*)$.

Under the symmetric oligopoly with the same HHI, the price deviation meets (3) as an equality.

Proof. Under the given HHI, the share of one firm in the total production does not exceed $n^{-1/2}$. So the inequality (4) follows from Proposition 2. The latter two statements of the proposition relate to the examples considered in the note and proposition 2 respectively.

Consider again the regulation problem. Now, the question is: how low should be the *HHI* in order to prevent the increase of the market price more than 50% of the cost? The US government agencies propose (see Report of Office of Economic, 2000) that $HHI \le 1000$ $(n \ge 10)$ means that no firm obtains the market power, and the market is sufficiently competitive. Consider favorable demand electricity $e(c) = 0.2$. Under the symmetric oligopoly, we obtain the desirable result. However, in the case of a large firm with the competitive fringe, $p^*/c - 1 = \frac{1}{(\sqrt{n}+1)e(c)} > 1$, that is, the market price exceed the marginal cost more than two times.

Note2. The value $e(c) = 0.2$ means that $e(p^*) \geq 0.5$ in the latter example. For the demand with the constant elasticity $e(p) = 0.2$, the result would be essentially worser.

4. Empirical study.

In this section we compute NE of the unique-price auction for several variants of the electricity market in the Central economic region of Russia. The paper by Dyakova (2003) based on the data from the RAO UES provides the following values of marginal costs and production capacities of the generating companies in this region. (See Table 1.)

Table 1.

	Generator	Marginal cost (Rub/mwth)	Capacity (bln kwth per year)
Mosenergo:			
	G1	0	5
	G2	75	10
	G3	80	10
	G4	85	25
	G5	90	10
	G6	100	5
	G7	165	10
Rosenergoatom:			
		12.5	125.4
GC1:			
	1	0	16
	2	60	2
	3	112	3
	4	125	2
	5	150	16
	6	200	2
	7	255	2
	8	340	10
GC2:			
	1	95	2.5
	2	110	2.5
	3	120	4

4	128	13
5	135	6
6	145	2
7	162	15

GC3:

1	0	3.5
2	100	2.5
3	120	21
4	150	3.5
5	170	4.5
6	200	4.5
7	215	3

We consider several demand functions $D(p) = N - \gamma p$ corresponding to the consumption and the price in 2000:

γ	0.1	0.2	0.4	0.6
N	279.9	316.1	388.4	460.7

For the local market model, we find the Cournot outcome and Vickrey outcome for two variants of the market structure:

a) 5 independent companies,

b) 3 independent companies (Mosenergo, Rosenergoatom and UGC including all the rest generators).

For each slope ratio γ, we evaluate the deviations of the Cournot price from the Walrasian price.

Table 2. Walrasian ($\tilde{p}$) and Cournot (p^*) prices for the electricity market in the Central economic region of Russia. The cases with 5 and 3 generating companies.

γ \ p	$\tilde{p}$	$p_5^*/\tilde{p}$	$p_3^*/\tilde{p}$
0,1	135	4.24	5.65
0,2	150	2.45	3.10
0,4	172.5	1.56	1.87
0,6	219.67	1.15	1.34

Our results show that deviation of the Cournot price from the Walrasian price strongly depends on the slope of the demand curve and practically interesting values $\gamma = 0.1 - 0.2$,.

5. Network markets and the problem of multiple Cournot equilibria.

Consider two local markets connected by a transmitting line. Every local market $l = 1,2$ is characterized by the finite set A^l of producers, $|A^l| = n_l$, the cost functions $C^a(v), a \in A^l$, and demand function $D^l(p)$, in the same way as the local market in Section 3: each cost function is a private information of agent a, the demand function and other market parameters are a common knowledge. Let $k \in (0,1)$ be the loss coefficient that shows the share of the lost good (in particular, the electric power) under transmission from one market to the other, Q is the maximal amount of the transmitted good. Consider Cournot competition in this model. Then each producer sets $v^a \in [0, V^a]$. The interaction is as follows.

1. Each firm finds out its cost function.
2. Simultaneously and independently each firm reports the auctioneer its strategy.
3. For a given strategy combination nodal cut-off prices $\tilde{c}^l$ and transmitted volume q are determined as follows.

Let $\bar{c}^l(v), l = 1,2,$ denote the cut-off prices for isolated markets, such that $D^l(\bar{c}^l) = \sum_{a \in A} v_l^q$, $\lambda = (1-k)^{-1}$. If $\lambda^{-1} \le \bar{c}^2(\bar{v})/\bar{c}^1(\bar{v}) \le \lambda$ then $q = 0$, the final prices are $\tilde{c}^l(\bar{v}) = \bar{c}^l(\bar{v}), l = 1,2$, that is, the markets stay isolated. If $\bar{c}^2(\bar{v})/\bar{c}^1(\bar{v}) > \lambda$ then q (the transmitted volume from market 1 to market 2) is a solution of the system

$$D^2(\tilde{c}^2) = \sum_{A^2} v^a + \bar{q}; \tag{5}$$

$$D^1(\tilde{c}^1) = \sum_{A^1} v^a - \bar{q}; \tag{6}$$

$\tilde{c}^2 = \lambda \tilde{c}^1$, until $\bar{q} > Q$.

The unique solution of the system exists because the involved functions are monotonous and continuous. If $\bar{q} > Q$ then $q = Q$, $\tilde{c}^i$ are determined from (5),(6) with $\bar{q} = Q$, $\tilde{c}^2 > \lambda \tilde{c}^1$. The capacity constraint is binding in this case. The case $\bar{c}^1(\bar{v})/\bar{c}^2(\bar{v}) > \lambda$ is treated in the symmetric way. This section aims to study these 3 types of Nash equilibria of the auction.

a). The first-order conditions for the type a) outcome with prices

$$p_1^*,\ p_2^* \quad \text{s.t.} \quad \lambda^{-1} < p_2^*/p_1^* < \lambda, \tag{7}$$

to be a Cournot equilibrium are quite similar to the conditions (1), (2) for the local market:

$$v^a* \in (p_i^* - C^{a'}(v^a*)) \mid D^{i}{}'(p_i^*) \mid, \text{for any } a \in A^i \text{ s.t. } C^a{}'(0) < p_i^*$$

$$v^a* = 0 \text{ if } C^a{}'(0) \geq p_i^*,$$

where $C^a{}'(v) = [C_-^a{}'(v), C_+^a{}'(v)]$ in the break points of the marginal cost function.

Besides that,

$$\sum_{A^i} v^a* = D(p_i^*), \quad i = 1, 2.$$

b). For the type b) outcome with

$$q \in (0, Q),\ \lambda p_1^* = p_2^*,$$

the first-order conditions of the Cournot equilibrium are obtained in a similar way. Note that, for any small change of the strategy v^a*, producer $a \in A^1$ stays in the market with the demand function

$$D^1(p_1(\bar{v})) + \lambda(D^2(\lambda p_1(\bar{v})) - \sum_{A^2} v^a)$$

where the price $p_1(\bar{v})$ meets equation $\sum_{A^1} v^a = D^1(p_1) + \lambda(D^2(\lambda p_1) - \sum_{A^2} v^a))$.

Thus,

$$v^a* \in (p_1^* - C^a{}'(v^a*)) \mid D^1{}'(p_1^*) + \lambda^2 D^2{}'(\lambda p_1^*) \mid$$

for any

$$a \in A^1 \quad \text{s.t.} \quad C^a*(0) < p_1^*, v^a* = 0 \quad \text{if} \quad C^a{}'(0) \geq p_1^*.$$

Similarly, producers in the market 2 face the demand $D^2(\lambda p_1) + 1/\lambda(D^1(p_1) - \sum_{A^1} v^a)$, and

$$v^{a*} \in (\lambda p_1^* - C^{a\prime}(v^{a*}))\,|\,D^{2\prime}(\lambda p_1^*) + D^{1\prime}(p_1^*)/\lambda^2\,|$$

for any

$$a \in A^2 \quad \text{s.t.} \quad C^{a\prime}(0) < p_2^*, v^{a*} = 0 \quad \text{if} \quad C^{a\prime}(0) \geq p_2^*.$$

c). Finally, if the capacity constraint is binding

$$q = Q, \quad \lambda p_1^* < p_2^*,$$

then the F.O.C.s are

$$v^{a*} \in (p_i^* - C^{a'}(v^{a*}))\,|\,D^{i\prime}(p_i^*)\,|, \text{for any } a \in A^i \text{ s.t. } C^{a'}(0) < p_i^*$$

$$v^{a*} = 0 \text{ if } C^{a'}(0) \geq p_i^*,$$

$$\sum_{A^1} v^{a*} = D(p_1^*) + Q$$

$$\sum_{A^2} v^{a*} = D(p_2^*) - \lambda Q.$$

Thus, even in the simplest variant of the network market there exist 5 possible local equilibria. The following proposition shows that some of them are incompatible, and reduces the set of variants for examination.

Proposition 4. *Under fixed parameters of the two-node market, only one local equilibrium of the types a) or c) may exist. More precisely, if the Cournot equilibrium prices* $\overline{p}_i$, $i = 1,2$, *for separated markets meet condition (7) then only a local equilibrium of the type a) exists in the market, otherwise if* $\overline{p}_i > \lambda\overline{p}_j$ *then only a local equilibrium of the type c) with the flow from the market* j *to the market* i *may exist (but does not necesserely exist),* $i = 1,2$, $j = \{1,2\} \setminus i$.

Proof. Consider a system that determines the prices for a local equilibrium of the type c) with the flow from 1 to 2:

$$\sum_{A^1} S_c^{a1}(\overline{\overline{p}}_1) = D^1(\overline{\overline{p}}_1) + Q\,,\ \sum_{A^2} S_c^{a2}(\overline{\overline{p}}_2) = D^2(\overline{\overline{p}}_2) - Q,\ \overline{\overline{p}}_2 > \lambda\overline{\overline{p}}_1. \tag{8}$$

Let $\overline{\overline{p}}_i(Q)$, $i = 1,2$, denote a solution of the equations for any $Q \geq 0$. Note that $\overline{p}_i = \overline{\overline{p}}_i(0)$, $i = 1,2$, $\overline{\overline{p}}_1(Q) \uparrow Q$, $\overline{\overline{p}}_2(Q) \downarrow Q$. If the equilibrium a) exists then $\overline{\overline{p}}_2(Q) < \overline{p}_2 < \lambda\overline{p}_1 < \lambda\overline{\overline{p}}_1(Q)$ for any $Q>0$, so the system (8) is incompatible and, similarly, a symmetric equilibrium with the flow from 2 to 1 does not exist. Otherwise if $\overline{p}_2 > \lambda\overline{p}_1$ then a solution of (8) exists if Q is sufficiently small, that is $\overline{\overline{p}}_2(Q) > \lambda\overline{\overline{p}}_1(Q)$. A solution to the symmetric system does not exist in this case. □

This result simplifies analysis of the market. However, our paper (2005) shows that a typical case is where three different local equilibria of the types a) and b) exist, some of them are true NE, the other are not stable with respect to the large change of the production volume by some agent. As the number of local markets in the network increases, the number of possible local equilibria of the network market grows with the exponentical rate. The problem of their careful computation and analysis for the actual markets seems to be hopeless.

This section aims to develop an alternative approach to analysis of such markets. Since the loss coefficients in the actual networks are usually less then 0.1, we show that NE of a market with the losses may be approximated by the NE of the similar market without losses and evaluate the error. Then we consider a problem of the Cournot equilibrium search and analysis for a network market without losses.

Below we study these issues for the market with affine demand functions and constant marginal costs. We also assume that capacity constraints in production are not binding. Formally we assume that $D^i(p) = \overline{D}_i - dp$, $C^a(v) = c_i v$ for $a \in A^i, i = 1,2$. First let us show that the deviation of the type b) equilibrium for the market with losses from a similar equilibrium for the market without losses smoothly depends on the loss coefficient and is small for a typical value $k \le 0.1$. Under a flow from market 1 to market 2, the equilibrium price p_1^* at the market 1 meets equation

$$\sum_{A_1}(p_1^* - c_1)(d_1 + \lambda^2 d_2) + \sum_{A_2}(\lambda p_1^* - c_2)(d_1 + \lambda\, d_2) = D^1(p_1^*) + \lambda D^2(\lambda p_1^*).$$

The market without losses corresponds to $\lambda = 1$. According to the theorem on the derivative of an implicit function,

$$\left|\frac{dp_1}{d\lambda}\right| = \frac{2\lambda d_2(p_1^* - c_1)|A_1| + \left(p_1^*(d_1 + \lambda d_2) + d_2(\lambda p_1 - c_2)\right)|A_2| + d_2 \lambda p_1^* - D^2(\lambda p_1^*)}{\left(|A_1| - 1\right)\left(d_1 + \lambda^2 d_2\right) + |A_2|\lambda(d_1 + \lambda d_2)}.$$

Thus $\left|\frac{dp_1}{d\lambda}\right| \Big/ p_1 \Big|_{\lambda=1} \le \frac{2d_2|A_1| + (d_1 + d_2)|A_2|}{(d_1 + d_2)\left(|A_1| + |A_2| - 1\right)} \approx 1$ for $d_1 = d_2$; $\frac{p_1(1) - p_1(\lambda)}{p_1(1)} \approx \lambda - 1 \le 0.1$ under typical values of the loss coefficient.

A similar evaluation holds for an equilibrium of the type c) with the binding transmission capacity constraint. However, an equilibrium of the type a) (with separated markets) does not exist under $k = 0$, while for $k > 0$ a local equilibrium of this type may exist and essentially differ from any equilibrium of the network market without losses. For instance, consider a symmetric oligopoly with equal parameters for the both local markets. Then the Cournot price for each separated local market is $\overline{p} = c + \frac{(\overline{D} - dc)}{d(m+1)}$, where $m = |A_1| = |A_2|$. The Cournot price for the united market with $k = 0$ is $p^* = c + \frac{(\overline{D} - dc)}{d(2m+1)}$, that is, $\overline{p} - c \approx 2(p^* - c)$.

However, the local equilibrium with separated markets is not a true Nash equilibrium under typical values of the electricity market parameters! Let us prove this proposition.

The conditions for the local equilibrium of the type a) take the form: $v^{a*} = v_i^* = (p_i^* - c_i)d$, $a \in A^i$, $p_i^* - c_i = \frac{\overline{\overline{D}}_i}{d(n_i + 1)}$ where $\overline{\overline{D}}_i = \overline{D}_i - c_i d$. Hence $f^a(\overline{v}^*) = (p_i^* - c_i)^2 d$, $a \in A^i$. The optimal price $\overline{p}_1$ for $a \in A^1$ in the joint market meets equation

$$\overline{p}_1 - c_1 = \frac{\overline{D}_1 + \lambda \overline{D}_2 - c_1(1 + \lambda^2)d - (n_1 - 1)v_1^* - \lambda n_2 v_2^*}{2d(1 + \lambda^2)}.$$

Condition (7) for existence of the local equilibrium and the conditions of its instability under the optimal deviation of agent $a \in A^1$ take the form:

$$\lambda > p_2^*/p_1^* > \lambda^{-1} \Leftrightarrow \lambda > \left(\frac{\overline{\overline{D}}_2}{d(n_2 + 1)} + c_2\right) \Big/ \left(\frac{\overline{\overline{D}}_1}{d(n_1 + 1)} + c_1\right) > \lambda^{-1}; \qquad (9)$$

$$f_1(\bar{v}^*) < f_1(\bar{v}^* \,\|\, \bar{v}_1) \Leftrightarrow \frac{2\overline{\overline{D}}_1\sqrt{1+\lambda^2}}{d(n_1+1)} < (\lambda c_2 - \lambda^2 c_1 + \frac{2\overline{\overline{D}}_1}{d(n_1+1)} + \frac{\lambda \overline{\overline{D}}_2}{d(n_2+1)}) \qquad (10)$$

Let us show that condition (10) holds under typical parameters of an electricity market, so the local equilibrium of the type a) (with separated local markets) is not a true NE. Proceeding from (9),(10) it suffices to check the following inequality:

$$\frac{2\overline{\overline{D}}_1(\sqrt{1+\lambda^2}-1)}{d(n_1+1)} < \frac{\overline{\overline{D}}_1}{d(n_1+1)} + c_1(1-\lambda^2) \text{ or}$$

$$1 < \frac{1}{e(c_1)(n_1+1)}\left(\frac{3-2\sqrt{1+\lambda^2}}{\lambda^2-1}\right). \qquad (11)$$

Consider several typical values of the loss coefficient.

Proposition 5. *For the loss coefficient* $k = 0.1$, *the separated equilibrium does not exist if* $(n+1)e(c) < 0.08$ *or* $ne(p^*) < 1.08$, *where* $p^* = \frac{c+(\overline{D}-dc)}{cd(n+1)}$ *is the local equilibrium price. For* $k = 0.05$ *this proposition holds if* $(n+1)e(c) < 0.9$ *or* $ne(p^*) < 1.9$. *In particular, if* $e(p^*) \le 0.1$ *then the equilibrium does not exist for* $n < 19$.

Proof immediately follows from inequality (11) and relations

$$p^* = c\left(1+\frac{1}{(n+1)e(c)}\right) = \frac{c}{\left(1-\frac{1}{ne(p^*)}\right)} \square$$

Proceeding from the given results we expect that in a general case the analysis of Cournot equilibria for a network market may be reduced to the analysis of a similar market without losses. For a two-node market without losses, we may consider only local equilibria of the types b) and c), and there exist at most one equilibrium of each type. It would be nice to show that only one Cournot equilibrium exists in this market in a general case. The following example shows that, unfortunately, this is not true.

Consider conditions for local equilibria b) with the united market and c) with the binding constraint and a flow from market 2 to market 1. Let $\bar{p}$ and $(\bar{p}_1, \bar{p}_2)$ denote the corresponding prices, $S_C^{i,12}$, and S_C^i be the Cournot supply functions at the market i under the united market and under the binding constraint respectively. Then

$$S_C^{1,12}(\overline{p}) + S_C^{2,12}(\overline{p}) = D^1(\overline{p}) + D^2(\overline{p}),\ \left|S_C^{2,12}(\overline{p}) - D^2(\overline{p})\right| \le Q,$$

$$S_C^2(\overline{p}_2) - D^2(\overline{p}_2) = Q = D^1(\overline{p}_1) - S_C^1(\overline{p}_1),\ \overline{p}_1 > \overline{p}_2.$$

First, let us show that these conditions are incompatible if $S_C^{1,12}(\overline{p}) = S_C^1(\overline{p})$, that is, the production capacity of the importing market is fully employed at the price $\overline{p}$. Since $S_C^{2,12}(\overline{p}) \ge S_C^2(\overline{p})$ and $S_C^{2,12}(\overline{p}) - D^2(\overline{p}) \le Q$ then $\overline{p} \le \overline{p}_2$. Moreover, $D^1(\overline{p}) - S_C^{1,12}(\overline{p}) \le Q$, so $\overline{p} \ge \overline{p}_1$ under the given condition. This contradicts to $\overline{p}_1 > \overline{p}_2$.

However, the both local equilibria may exist in the case where $S_C^{2,12}(\overline{p}) = S_C^2(\overline{p}_2) = S_{\max}^2$. Consider a symmetric oligopoly with m producers and fixed marginal costs $c^a \equiv c$ at the market 1, and $D^1(p) = \overline{D}^1 - pd$. Then the equilibrium price and the profit of each firm at the equilibrium c) are

$$\overline{p}_1 = \frac{\overline{D}_1 - Q + mdc}{d(m+1)},\ \Pr_1 = d(\overline{p}_1 - c)^2, \tag{12}$$

and similar values for the equilibrium b) are

$$\overline{p} = \frac{\overline{D}_1 + 2mdc + d\tilde{p}_2}{2d(m+1)},\ \Pr = 2d\left[\left(\overline{D}_1\big/2d + \tilde{p}_2\big/2 - c\right)/(m+1)\right]^2, \tag{13}$$

where $\tilde{p}_2$ is the competitive equilibrium and the Cournot price for the separated market 2: $\overline{D}_2 - d\tilde{p}_2 = S_{\max}^2$. So at each equilibrium the exporting market 2 supplies the same volume $S_{\max}^2$. At the equilibrium c), each firm at the market 1 produces less than at the equilibrium b). So the price is higher and the transmission capacity constraint is binding in the former local equilibrium. By a sufficiently large increase of the production volume, a firm in the market 1 may reduce the price and make the constraint unbinding (see Fig. 3).

Figure 3.

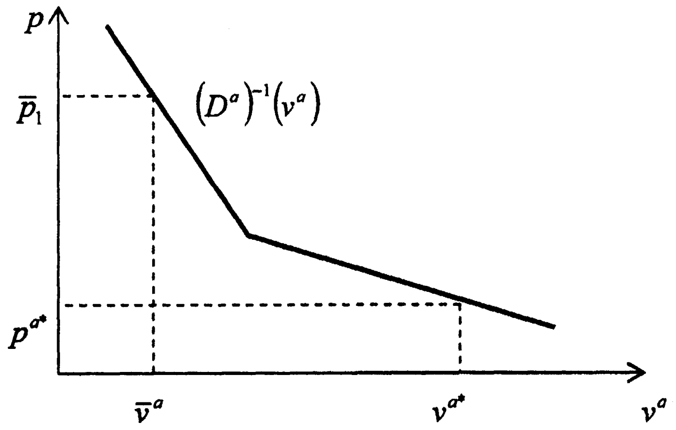


Fig. 3. The inverse demand function under fixed strategies $v^b = \bar{v}^a$ *for* $b \in A^1 \setminus \{a\}$, $v^{A_2} = S^2_{\max}$.

The demand function in this case is

$$D(p) = \overline{D}_1 + \overline{D}_2 - S^2_{\max} - 2dp - \frac{m-1}{m+1}(\overline{D}_1 - Q - cd),$$

the maximal profit for a is

$$\Pr^* = \left(\overline{D}_1 + \overline{D}_{2e} - \frac{m-1}{m+1}(\overline{D}_1 - Q - cd) - 2cd\right)^2 \Big/ 8d, \text{ where } \overline{D}_{2e} = \overline{D}_2 - S^2_{\max}.$$

As to the local equilibrium b), it may be unstable with respect to sufficiently large decrease of the production volume by some agent $a \in A^1$, such that the transmission constraint becomes binding. The demand function and the maximal profit in this case are

$$\hat{D}(p) = \overline{D}_1 - Q - \frac{m-1}{m+1}(\overline{D}_1 + \tilde{p}_2 d - 2cd)\Big/ 2d - pd,$$

$$\hat{\Pr} = \left[\overline{D}_1 - Q - \frac{m-1}{m+1}(\overline{D}_1 + \tilde{p}_2 d - 2cd)\Big/ 2d - cd\right]^2 \Big/ Ld.$$

The following proposition summarizes the results of our study.

Proposition 6. *For the given two-node market, two Cournot equilibria exist if and only if the profit values determined by (10)-(13) meet inequalities* $\Pr \ge \Pr^*$, $\Pr_1 \ge \hat{\Pr}$.

An equivalent system is:

$$\sqrt{2}(\overline{D}_1 - Q - cd) \ge \left(\overline{D}_1 + \frac{\overline{D}_{2e}(m+1)}{2} + \frac{(m-1)}{2}Q - \frac{cd}{2}(m+3)\right);$$

$$(\overline{D}_1+\overline{D}_{2e}-2cd)\geq\sqrt{2}\left(\overline{D}_1-\overline{D}_{2e}\frac{(m-1)}{2}-\frac{Q_{12}(m+1)}{2}+\frac{cd}{2}(m-3)\right).$$

Consider the following example. Let $\overline{D}_{2e}=1$, $\overline{D}_1=2.5$, $d=1$, $c_1=1$. Then we obtain the following system for Q and m:

$$\sqrt{2}\left(\frac{m+1}{2}\right)Q\geq 1.5(\sqrt{2}-1)\geq Q\left(\frac{m-1}{2}+\sqrt{2}\right).$$

In particular, for any odd $m_1=2k+1$, we obtain an interval $1.5(\sqrt{2}-1)/(k+\sqrt{2})\geq Q\geq 1.5(\sqrt{2}-1)/(k+1)\sqrt{2}$ where two Cournot equilibria exist in the market.

References

1. Abolmassov A., Kolodin D., (2002). "Market Power and Power Markets: Structural Problems of Russian Wholesale Electricity Market"// EERC final report.

2. Amir, R, (1996). "Cournot Oligopoly and the Theory of Supermodular Games"// Games and Economic Behavior, vol.15, pp.132-148.

3. Amir , R., and V. Lambson, (2000). "On the Effects of Entry in Cournot Markets"// Review of Economic Studies, vol. 67, pp. 235-254.

4. Baldick,R., Grant, R., Kahn E., (2000). "Linear Supply Function Equilibrium: Generalizations, Application, and Limitations"// POWER Working Paper PWP-078. University of California Energy Institute, August.

5. Dyakova, Yu., (2003). "Modelling of the Russian Wholesale Electricity Matrket. Master thesis"// New Economic school, Moscow.

6. Green, R., (1996). "Increasing Competition in the British Electricity Spot Market"// Journal of Industrial Economics, 44(2), 205-216.

7. Hogan, W. W., (1998). "Competitive Electricity Market Design: a Wholesale Primer"// Harvard University.

8. Green, R and D. Newbery, (1992). "Competition in the British Electricity Spot Market "// Journal of Political Economy, vol 100(5), pp. 929-953.

9. Klemperer, P. and M. Meyer, (1989). "Supply Function Equilibria in Oligopoly under Uncertainty"// Econometrica, vol. 57 (6), pp. 1243-1277.

10. Moreno, D. and L. Ubeda, (2001). "Capacity precommitment and price competition yield the Cournot outcome"// unpublished manuscript.

11. Vasin A., Vasina P., (2005). "Models of Supply Functions Competition with Application to the Network Auctions"// Final Research Report on the EERC Project R03-1011, site eerc.ru

12. Hircshman A. O., (1963). "The poternity of an index "// American Economic Preview, 54(5), 761-762.

13. Tirole J., (1997). "The theory of Industrial Organization"// MIT Press.

14. Report of Office of Economic, Electricity and Natural Gas Analysis, Office of Policy U.S. Department of Energy "Horizontal Market Power in Restructured Electricity Markets", Washington, DC, March 2000.

15. Sykes, A. and Robinson, C., (1987). "Current Choices: Good Ways and Bad to Privatize Electricity"// Policy Study no 87. London: Centre Policy Studies.

16. The Model of the Russian Wholesale Market RAO UES Draft Document. Version 2.2.