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The Smith Isomorphism Question: A review and new results

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In 1960, P. A. Smith [Smi60] raised an isomorphism question:

Smith Isomorphism Question. *Whether the two tangential* G*-modules at two fixed points of an arbitrary smooth* G*-action on a sphere with exactly two fixed points are isomorphic to each other?*

Following [Pet82], two real G-modules V and W are called *Smith equivalent* if there exists a smooth action of G on a sphere S such that $S^G = \{x, y\}$ for two points x and y at which $T_x(S) \cong V$ and $T_y(S) \cong W$ as real G-modules.

Let $RO(G)$ denote the real representation ring of G. Define the *Smith set* $Sm(G)$ to be

$$Sm(G) = \{[V] - [W] \in RO(G) \mid V \text{ and } W \text{ are Smith equivalent}\}.$$

The Smith Isomorphism Question can be restated as follows.

Smith Isomorphism Question. *Is it true that* $Sm(G) = 0$*?*

It is easy to show that the answer is affirmative if G is a group such that each element has the order 1, 2 or 4.

In nineteen sixties, the first breakthrough was due to M. F. Atiyah and R. Bott [AB68, Theorem 7.15]:

Theorem 1 (Atiyah-Bott). *If* $G = C_p$*,* p *an odd prime, then* $Sm(G) = 0$.

Shortly thereafter, J. Milnor [Mil66, Theorem 12.11] extended their result:

Theorem 2 (Milnor). *If* G *is a compact group and the action semi-free, then* $Sm(G) = 0$.

In nineteen seventies, by using the G-signature theorem, C. U. Sanchez [San76] obtained a stronger result:

Theorem 3 (Sanchez). *Let* X *be a rational-homology sphere supporting an action of* C_n *(*n *odd) as a group of diffeomorphisms with only two fixed points* x *and* y *and satisfying the condition that*

> *for every proper subgroup* H *of* C_n*, either* $F(H, X) = \{x, y\}$ *or* $F(H, x, X) = F(H, y, X)$ *holds.*

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Then $T_x(X) \cong T_y(X)$.

In fact, by Sanchez Theorem and Smith theory, we obtain the following Corollary.

Corollary 4. *In either of the following cases,* $Sm(G) = 0$.

(1) G *is a group with odd-prime-power order.*

(2) G *is a group with* $|G| = pq$, *where* p *and* q *are odd primes.*

By using G-equivariant surgery, T. Petrie [Pet79], [PR85] obtained the first counterexample to the question:

Theorem 5 (Petrie). *If* G *is an odd order finite abelian group with at least four non-cyclic Sylow subgroups, then* $Sm(G) \neq 0$.

In nineteen eighties, S. E. Cappell and J. L. Shaneson [CS82] gave first counterexamples to the question for G a cyclic group:

Theorem 6 (Cappell-Shaneson). *If* G *is the cyclic group of order* $4m$ *such that* $m \geq 2$ *then* $Sm(G) \neq 0$.

By character theory, we have $Sm(D_6) = 0$ and $Sm(C_6) = 0$. So, C_8 is the smallest group with $Sm(G) \neq 0$.

T. Petrie and his collaborators obtained a lot of results about s-Smith equivalence, see [Pet83], [PR84], [Cho85], [CSu85], [Suh85], [Cho88]. K. H. Dovermann and T. Petrie [DP85] constructed non-isomorphic Smith equivalent representations of odd order cyclic groups.

Theorem 7 (Dovermann-Petrie). *Let* G *be an odd-order cyclic group such that the order of* G *has at least* 3 *prime divisors. If there exist real* G*-modules* A *and* B *satisfying the following conditions, then* $Sm(G) \neq 0$.

(0) $A \ncong B$,

(1) $A^g = B^g = 0$ *for each* $g \in G$ *which generates a subgroup of prime power index in* G,

(2) $\dim A^K = \dim B^K$ *whenever* $|G/K|$ *is divisible by at most* 3 *distinct primes,*

(3) $\mathrm{Res}^G_P A \cong \mathrm{Res}^G_P B$ *whenever* $|P|$ *is a prime power,*

(4) $\nu(A^P - B^P)(g) = \pm 1$ *whenever* $|P|$ *is a prime power and* $g \in G$ *generates a subgroup of prime power index in* G.

The groups exhibited in that paper were very large. As J. Ewing computed, their order were at least $10^{2812917}$. K. H. Dovermann and L. C. Washington [DW89] showed that such non-isomorphic Smith equivalent representations also exist for odd order cyclic groups of small order. For example, their orders can be $5 \cdot 11 \cdot 19 \cdot 29$ and $3 \cdot 13 \cdot 17 \cdot 23$. K. H. Dovermann and D. Y. Suh [DS92] constructed non-isomorphic Smith equivalent representations in the following cases.

Theorem 8 (Dovermann-Suh). *If* G *is a group with real* G*-modules* A *and* B *as in Theorem 7, then* $\mathrm{Sm}(G \times C_{2^k}) \neq 0$.

Theorem 9 (Dovermann-Suh). *If* G *is a finite abelian group with at least* 3 *non-cyclic Sylow subgroups, with real* G*-representations* A *and* B *satisfying the following conditions, then* $\mathrm{Sm}(G) \neq 0$.

(0) $A \ncong B$,

(1) $A^K = B^K = 0$ *whenever* $|G/K|$ *is a prime power,*

(2) $\dim A^K = \dim B^K$ *for all* $K \subset G$,

(3) $\mathrm{Res}^G_P A \cong \mathrm{Res}^G_P B$ *whenever* $|P|$ *is a prime power.*

A finite group G is an *Oliver group* if and only if G never admits a normal series

$$P \lhd H \lhd G$$

such that $|P|$ and $|G/H|$ are prime powers and H/P is a cyclic group. For a finite group G, the following three claims are equivalent ([Oli75], [LM98]).

(1) G is an Oliver group.

(2) G has a smooth one-fixed-point action on a sphere.

(3) G has a smooth fixed-point-free action on a disk.

For an element $g \in G$, let (g) denote the conjugacy class of g in G. The union $(g)^{\pm} = (g) \cup (g^{-1})$ is called the *real conjugacy class* of g in G. Let a_G denote the number of the real conjugacy classes $(g)^{\pm}$ in G such that the order of g is not a prime power.

In 1996, in the case where G is an Oliver group, E. Laitinen [LP99, Appendix] lighted the question again with an conjecture.

Laitinen Conjecture. *If* G *is an Oliver group with* $a_G \geq 2$, *then* $\mathrm{Sm}(G) \neq 0$.

E. Laitinen and K. Pawałowski proved two theorems [LP99]:

Theorem 10 (Laitinen-Pawałowski). *If* G *is a finite perfect group with* $a_G \geq 2$, *then* $\mathrm{Sm}(G) \neq 0$.

Theorem 11 (Laitinen-Pawałowski). *If* $G \cong A_n$, $SL(2,p)$ *or* $PSL(2,q)$ *with* $a_G \geq 2$, *where* n *is a natural number and* p *and* q *are primes, then* $\mathrm{Sm}(G) \neq 0$.

A real G-module V is called a *gap module* if it satisfies

(1) $\dim V^P > 2 \dim V^H$ for any subgroup $P \subset G$ of prime power order and any subgroup $H \subset G$ with $P \subsetneq H$, and

(2) $V^N = 0$ for any normal subgroup $N \subset G$ such that $|G/N|$ is a prime power.

A finite group G is called a *gap group* if G admits a gap module. We refer to [MSY00], [Sum01] and [Sum04] for more information about gap groups. Let $P\Sigma L(2,27)$ denote the splitting extension of $PSL(2,27)$ by the group $Aut(\mathbb{F}_{27})$. K. Pawałowski and R. Solomon [PaS02] answered the Smith isomorphism question in various cases:

Theorem 12 (Pawałowski-Solomon). *In either of the following cases,* $Sm(G) \neq 0$ *holds.*

(1) G *is a finite Oliver group of odd order (thus* $a_G \geq 2$*, and* G *is a gap group).*

(2) G *is a finite Oliver group with a cyclic quotient of order* pq *for two distinct odd primes* p *and* q *(thus* $a_G \geq 2$*, and* G *is a gap group).*

(3) G *is a finite non-solvable gap group with* $a_G \geq 2$*, and* $G \ncong P\Sigma L(2,27)$.

Theorem 13 (Pawałowski-Solomon). *In either of the following cases, if* $a_G < 2$ *then* $Sm(G) = 0$.

(1) G *is a finite non-abelian simple group.*

(2) $G \cong PSL(n,q)$ *or* $SL(n,q)$ *for any* $n \geq 2$ *and any prime power* q.

(3) $G \cong PSp(2n,q)$ *or* $Sp(2n,q)$ *for any* $n \geq 1$ *and any prime power* q.

(4) $G \cong A_n$ *or* S_n *for any* $n \geq 2$.

We refer to the articles [PR84], [CS85], [DPS85], [MaP85], [Paw00] for survey of related results. K. Pawałowski and R. Solomon [PaS02, Theorem A.3] pointed out that $Aut(A_6)$ is a non-solvable Oliver group such that $a_G = 2$. In 2006, M. Morimoto [Mor07a] gave a counterexample to Laitinen Conjecture:

Theorem 14 (Morimoto). *If* $G = Aut(A_6)$ *then* $a_G = 2$*, and* $Sm(G) = 0$.

K. Pawałowski and T. Sumi [PaS07] claim $Sm(G) \neq 0$ for many Oliver groups G such that $a_G \geq 2$ and G is not a gap group, although only the sketchiest ideas of proofs are given. Let G^{nil} denote the smallest normal subgroup N of G such that G/N is nilpotent.

Announce 15 (Pawałowski-Sumi). *Let* G *be a finite Oliver group such that* G/G^{nil} *is isomorphic to neither* p*-group for a prime* p*,* $C_2 \times P$ *for an odd prime* p *and a* p*-group* P*, nor* $P_2 \times C_3$ *for a 2-group* P_2 *such that all elements of* P_2 *are self-conjugate:* $(g) = (g^{-1})$. *Then* $Sm(G) \neq 0$.

Announce 16 (Pawałowski-Sumi). *If a finite Oliver group* G *has an element of order* pqr *for distinct primes* p*,* q *and* r*, then* $Sm(G) \neq 0$.

Announce 17 (Pawałowski-Sumi). *Let* G *be a finite Oliver group with non-trivial center. If the order of* G^{nil} *is divisible by at least three primes, then* $Sm(G) \neq 0$.

Many authors have studied the Smith equivalence for various finite groups. But the Smith sets $\mathrm{Sm}(G)$ were rarely determined. In particular, when G is a non-solvable, non-perfect group, Smith set $\mathrm{Sm}(G)$ was not determined except the case $\mathrm{Sm}(G) = 0$. We have interested in the group $S_5 \times C_2$, because it is not a gap group, but it's subgroup $A_5 \times C_2$ is a gap group.

For a prime p, let $G^{\{p\}}$ denote the smallest normal subgroup H such that the order of G/H is a power of p (possibly 1). Let $\mathcal{P}(G)$ denote the set of all subgroups of G of prime power order (possibly 1). Define $\mathcal{L}(G)$ by

$$\{H \le G \mid H \ge G^{\{p\}} \text{ for some prime } p\}.$$

A real G-module V is said to be $\mathcal{L}(G)$-*free* if $V^L = 0$ for any $L \in \mathcal{L}(G)$. Define $\mathrm{RO}(G)^{\mathcal{L}}_{\mathcal{P}}$ to be the set

$$\{[V] - [W] \in \mathrm{RO}(G) \mid V \text{ and } W \text{ are } \mathcal{L}(G)\text{-free and } \mathrm{Res}^G_P V \cong \mathrm{Res}^G_P W \text{ for all } P \in \mathcal{P}(G)\}.$$

Announce 18. *The following equalities hold for* $G = S_5 \times C_2$ *and* $K = A_5 \times C_2$.

(1) $\mathrm{Sm}(K) \cong \mathbb{Z}^2$ *and* $\mathrm{Sm}(G) \cong \mathbb{Z}$.

(2) $\mathrm{Ind}^G_K(\mathrm{Sm}(K)) = \mathrm{Sm}(G)$.

Here the map $\mathrm{Ind}^G_K : \mathrm{RO}(K) \to \mathrm{RO}(G)$ *is the induction homomorphism:*

$$[V] \longmapsto [\mathbb{R}[G] \otimes_{\mathbb{R}[K]} V].$$

By means of GAP [GAP06], The complex character of $G = S_5 \times C_2$ is as in Table 1,

	1a	2a	2b	2c	3a	6a	2d	2e	4a	4b	6b	6c	5a	10a
ξ_{1C}	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ξ_{2C}	1	−1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1
ξ_{3C}	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1
ξ_{4C}	1	1	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1
ξ_{5C}	4	4	−2	−2	1	1	0	0	0	0	1	1	−1	−1
ξ_{6C}	4	−4	−2	2	1	−1	0	0	0	0	1	−1	−1	1
ξ_{7C}	4	4	2	2	1	1	0	0	0	0	−1	−1	−1	−1
ξ_{8C}	4	−4	2	−2	1	−1	0	0	0	0	−1	1	−1	1
ξ_{9C}	5	5	1	1	−1	−1	1	1	−1	−1	1	1	0	0
ξ_{10C}	5	−5	1	−1	−1	1	1	−1	−1	1	1	−1	0	0
ξ_{11C}	5	5	−1	−1	−1	−1	1	1	1	1	−1	−1	0	0
ξ_{12C}	5	−5	−1	1	−1	1	1	−1	1	−1	−1	1	0	0
ξ_{13C}	6	6	0	0	0	0	−2	−2	0	0	0	0	1	1
ξ_{14C}	6	−6	0	0	0	0	−2	2	0	0	0	0	1	−1

Table 1: The complex character of $G = S_5 \times C_2$

and the complex character of $K = A_5 \times C_2$ is as in Table 2.

	1a	2a	3a	6a	2b	2c	5a	10a	5b	10b
δ_{1C}	1	1	1	1	1	1	1	1	1	1
δ_{2C}	1	−1	1	−1	1	−1	1	−1	1	−1
δ_{3C}	3	3	0	0	−1	−1	A	A	$\widehat{A}$	$\widehat{A}$
δ_{4C}	3	3	0	0	−1	−1	$\widehat{A}$	$\widehat{A}$	A	A
δ_{5C}	3	−3	0	0	−1	1	A	$-A$	$\widehat{A}$	$-\widehat{A}$
δ_{6C}	3	−3	0	0	−1	1	$\widehat{A}$	$-\widehat{A}$	A	$-A$
δ_{7C}	4	4	1	1	0	0	−1	−1	−1	−1
δ_{8C}	4	−4	1	−1	0	0	−1	1	−1	1
δ_{9C}	5	5	−1	−1	1	1	0	0	0	0
δ_{10C}	5	−5	−1	1	1	−1	0	0	0	0

Table 2: The complex character of $K = A_5 \times C_2$

where $\omega = \exp \frac{2\pi\sqrt{-1}}{5}$, $A = -\omega - \omega^4 = \frac{1-\sqrt{5}}{2}$, $\widehat{A} = -\omega^2 - \omega^3 = \frac{1+\sqrt{5}}{2}$.

By Morimoto's Surgery Theory ([Mor95] [Mor98]), we can prove that $RO(G)^{\mathcal{L}}_{\mathcal{P}} = Sm(G)$ and $RO(K)^{\mathcal{L}}_{\mathcal{P}} = Sm(K)$. Let $\{\xi_i, 1 \le i \le 14\}$ be the $\mathbb{Z}$-basis of $RO(G)$ such that the complification of ξ_i is ξ_{iC}, and $\{\delta_i, 1 \le i \le 8\}$ the $\mathbb{Z}$-basis of $RO(K)$ such that the complification of δ_i is δ_{iC}. By calculation, a $\mathbb{Z}$-basis of $RO(G)^{\mathcal{L}}_{\mathcal{P}}$ is $\{\mathbf{y}\}$, where $\mathbf{y} = 2\xi_5 - 2\xi_6 + 2\xi_7 - 2\xi_8 - \xi_9 + \xi_{10} - \xi_{11} + \xi_{12} - \xi_{13} + \xi_{14}$, and the $\mathbb{Z}$-basis of $RO(K)^{\mathcal{L}}_{\mathcal{P}}$ is $\{\mathbf{x}_1, \mathbf{x}_2\}$, where $\mathbf{x}_1 = \delta_3 - \delta_5 - 2\delta_7 + 2\delta_8 + \delta_9 - \delta_{10}$, $\mathbf{x}_2 = \delta_4 - \delta_6 - 2\delta_7 + 2\delta_8 + \delta_9 - \delta_{10}$. Since the equalities

$$
\begin{aligned}
\mathrm{Ind}_K^G \delta_1 &= \xi_1 + \xi_4, & \mathrm{Ind}_K^G \delta_2 &= \xi_2 + \xi_3,\\
\mathrm{Ind}_K^G \delta_3 &= \xi_{13}, & \mathrm{Ind}_K^G \delta_4 &= \xi_{13},\\
\mathrm{Ind}_K^G \delta_5 &= \xi_{14}, & \mathrm{Ind}_K^G \delta_6 &= \xi_{14},\\
\mathrm{Ind}_K^G \delta_7 &= \xi_5 + \xi_7, & \mathrm{Ind}_K^G \delta_8 &= \xi_6 + \xi_8,\\
\mathrm{Ind}_K^G \delta_9 &= \xi_9 + \xi_{11}, & \mathrm{Ind}_K^G \delta_{10} &= \xi_{10} + \xi_{12},
\end{aligned}
$$

hold, we obtain $\mathrm{Ind}_K^G(\mathbf{x}_1) = \mathrm{Ind}_K^G(\mathbf{x}_2) = -\mathbf{y}$, which determines the induction map $\mathrm{Ind}_K^G : Sm(K) \longrightarrow Sm(G)$.

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