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FINITE GROUPS POSSESSING SMITH EQUIVALENT, NONISOMORPHIC REPRESENTATIONS

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1. Introduction

Throughout this paper, we assume that groups are always finite groups, group actions are smooth and representations mean real representations.

In 1960, Paul A. Smith [63] posted the following problem:

Problem. *Let G be a finite group which acts on a homotopy sphere with just two fixed points. Then are the tangential spaces over the fixed points isomorphic as representations or not?*

We call two representations which are obtained as the tangential spaces over fixed points from a finite group action on a sphere with just two fixed points are Smith equivalent.

Atiyah-Bott [1] proved that Smith equivalent representations are always isomorphic for a cyclic group of prime order. According to Sanchez [60], they are always isomorphic for a cyclic group of odd prime power order. By character theory, we obtain they are also always isomorphic for the symmetric group on three letters and a cyclic group of order $2, 4, 6$. On the other hand, Cappell-Shaneson proved that there exist Smith equivalent representations which are not isomorphic for a cyclic group of order $4q$ for $q \geq 2$ ([6, 7, 8]). For different classes of finite groups, many related results about this problem were obtained by Petrie, Dovermann, Suh, etc. [37, 57, 58, 59, 17, 19, 21, 64, 9, 10, 22] before 1990. After that, Laitinen and Pawałowski [36] obtained that there exists a pair of Smith equivalent nonisomorphic representations for a perfect group whose Laitinen number is greater than or equal to 2. Here a real conjugacy class means $(g)^{\pm} := (g) \cup (g^{-1})$ and the Laitinen number a_G of G is a number of all real conjugacy classes of G represented by elements not of prime power order. We assume that the identity is of prime power order. Pawałowski and Solomon [54] showed there exists a pair of Smith equivalent,

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nonisomorphic representations for more groups. Most recently, Morimoto [42, 43] presented the concerning results for groups including $\mathrm{Aut}(A_6)$ and $P\Sigma L(2, 27)$.

We show that there exists a pair of Smith equivalent, nonisomorphic representations for groups of the other classes. This report is including a joint work with Krzysztof Pawałowski.

Theorem 1.1. *Suppose that G is a nonsolvable group with $a_G \geq 2$. If two Smith equivalent representations are always isomorphic, then G is isomorphic to* $\mathrm{Aut}(A_6)$.

Theorem 1.2. *There exists a solvable Oliver group G with $a_G \geq 2$ which possesses a pair of two Smith equivalent, nonisomorphic representations.*

2. Representations and real conjugacy classes

In this section, we recall a necessary condition for which two representations become Smith equivalent.

Let G be a finite group and let $RO(G)$ be the real representation ring of G. For convenience, we define subgroups of $RO(G)$. We denote by $PO(G)$ the subgroup of $RO(G)$ of G consisting of the differences $U - V$ of representations U and V such that $\dim U^G = \dim V^G$ and $\mathrm{Res}^G_P(U) \cong \mathrm{Res}^G_P(V)$ for any subgroup P of G of prime power order. We note that in [54], $PO(G)$ is denoted by $IO(G, G)$. Similarly, we denote by $\overline{PO}(G)$ the subgroup of $RO(G)$ of G consisting of the differences $U - V$ of representations U and V such that $\dim U^G = \dim V^G$ and $\mathrm{Res}^G_P(U) \cong \mathrm{Res}^G_P(V)$ for any subgroup P of G of odd prime power order and order $2, 4$. By a theorem of Sanchez [60], the difference of two Smith equivalent representations lies in $\overline{PO}(G)$ and the difference of two 2-proper Smith equivalent representations lies in $PO(G)$. The concept of 2-proper is considered by Petrie. We will write the definition of 2-proper Smith equivalence in the section 3.

The rank of $PO(G)$ is equal to maximum of 0 and the Laitinen number a_G minus 1. Moreover the rank of $\overline{PO}(G)$ is equal to the rank of $PO(G)$ plus the number of all real conjugacy classes represented by 2-elements of order ≥ 8. Now, let H be a normal subgroup of G. We denote by $PO(G, H)$ the subgroup of $RO(G)$ consisting of the differences $U - V$ of representations U and V such that $U^H \cong V^H$ as representations over G/H, and $\mathrm{Res}^G_P(U) \cong \mathrm{Res}^G_P(V)$ for any subgroup P of prime power order. Again, we note that in [54], $PO(G, H)$ is denoted by $IO(G, H)$. It holds that $PO(G) = PO(G, G)$. Let b_G be the number of all real conjugacy classes in G/H which are sent from real conjugacy classes of G represented by elements not of prime power order by the surjection $G \to G/H$. Then the rank of $PO(G, H)$ is equal to $a_G - b_{G/H}$ (See [54]).

For each prime p, let $O^p(G)$ be the minimal subgroup among normal subgroups N of G with index a power of p. Let $\mathcal{L}(G)$ be the set of subgroups L of G containing $O^p(G)$ for some prime p. A representation U is said to be $\mathcal{L}(G)$-free if $\dim U^L = 0$

for any $L \in \mathcal{L}(G)$. We denote by $LO(G)$ the subgroup of $PO(G)$ consisting of the differences $U - V$ of representations U and V which are both $\mathcal{L}(G)$-free. Then it holds that

$$LO(G) \leq PO(G) \leq \overline{PO}(G) \leq RO(G)$$

and Pawałowski and Solomon showed

$$PO(G, G^{nil}) \leq LO(G),$$

where G^{nil} is the minimal subgroup among normal subgroups N of G such that G/N is nilpotent. Note that $G^{nil} = \cap_p O^p(G)$.

We denote by $QO(G)$ the subgroup of $PO(G)$ consisting of the differences $U - V$ of representations U and V such that $\mathrm{Res}^G_H U \cong \mathrm{Res}^G_H V$ for any proper subgroup H of G.

Lemma 2.1. *$PO(G) \otimes \mathbb{Q}$ is spanned by elements of $\mathrm{Ind}^G_C QO(C)$ for all cyclic subgroups C of G not of prime power order.*

Corollary 2.2. *Let C_1 and C_2 be cyclic subgroups of G not of prime power order. If C_1 and C_2 are not conjugate then*

$$\mathrm{Ind}^G_{C_1} QO(C_1) \cap \mathrm{Ind}^G_{C_2} QO(C_2) = \{0\}.$$

3. Finite group actions on spheres with exactly two fixed points

We denote by $Sm(G)$ the subset of $RO(G)$ consisting the differences of two Smith equivalent representations. A group action of a sphere Σ is 2-proper, if $\Sigma^{\langle g \rangle}$ is connected for any 2-element g of G of order ≥ 8. In accordance with Petrie's definition, two representations U and V are 2-proper Smith equivalent if there exists a 2-proper action of G on a sphere with exactly two fixed points at which tangential spaces are isomorphic to U and V respectively. We denote by $LSm(G)$ the subset of $Sm(G)$ consisting the differences of two 2-proper Smith equivalent representations. Since $LSm(G) \subset PO(G)$, $a_G \leq 1$ implies $LSm(G) = 0$.

Pawałowski and Solomon showed that if G is a gap Oliver group then $LO(G) \subseteq LSm(G)$, and moreover, if G is a gap nonsolvable group with $a_G \geq 2$ and $G \not\cong \mathrm{Aut}(A_6), P\Sigma L(2, 27)$ then $PO(G, G^{nil}) \neq 0$ and thus $LSm(G) \neq 0$. Recent works by Morimoto gave us that $Sm(\mathrm{Aut}(A_6)) = 0$ and $LSm(P\Sigma L(2, 27)) \neq 0$.

Now we recall the weak gap condition ([41]). A representation V satisfies the *weak gap condition* if it satisfies the following properties.

(1) If $P \in \mathcal{P}(G)$ and $H > P$, then $2 \dim V^H \leq \dim V^P$.
(2) If $P \in \mathcal{P}(G)$, $H > P$ and $2 \dim V^H = \dim V^P$, then $[H : P] = 2$, $\dim V^H > \dim V^K + 1$ for any $K > H$.
(3) If $P \in \mathcal{P}(G)$, $[H : P] = 2$ and $2 \dim V^H = \dim V^P$, then V^H is orientable so that $g \colon V^H \to V^H$ is orientation preserving for any $g \in N_G(H)$.

(4) If $P \in \mathcal{P}(G)$, $H > P$, $K > P$ and $2\dim V^H = 2\dim V^K = \dim V^P$, then the smallest subgroup $\langle H, K\rangle$ including H and K does not belong to $\mathcal{L}(G)$.

Here, $\mathcal{P}(G)$ is the set of all subgroups of G of prime power order.

We denote by $WLO(G)$ the subgroup of $LO(G)$ consisting of the differences $U - V$ of representations U and V such that both $U \oplus W$ and $V \oplus W$ are $\mathcal{L}(G)$-free and satisfy the weak gap condition. Note that $WLO(G) = LO(G)$ if G is a gap group.

Lemma 3.1. *It holds $WLO(G) \subseteq LSm(G)$ for an Oliver group G.*

From now on, we investigate conditions for which Oliver groups G satisfy that $WLO(G) \neq 0$.

4. A sufficient condition

We introduce a sufficient condition for Oliver groups G to hold $WLO(G) \neq 0$ by using elements of the groups.

A pair (x, y) of elements $x, y \in G$ is called *basic* if the following two condition hold.

(1) x and y are not of prime power order, and x and y are not real conjugate in G (and thus $a_G \geq 2$).

(2) x and y are in some gap subgroup of G, or the orders $|x|$ and $|y|$ are even and the involutions of $\langle x\rangle$ and $\langle y\rangle$ are conjugate in G.

Moreover, we say that (x, y) is an *H-pair* for a subgroup H of G, if $xH = yH$.

Theorem 4.1. *If an Oliver group G has a basic G^{nil}-pair, then $WLO(G) \neq 0$ and thus $LSm(G) \neq 0$.*

It is easy to see that G has a basic G^{nil}-pair in some assumptions. The next theorem is obtained by combining Theorem 5.1.

Theorem 4.2. *If an Oliver group G has an element of the center whose order is divisible by at least 3 distinct primes then G has a basic G^{nil}-pair.*

In the case when G has nontrivial center, if G has no basic G^{nil}-pair then the structure of G is almost determined. In this paper we omit it.

5. Outline of a proof of Theorem 1.1

We introduce outline of a proof of Theorem 1.1. The following result is one of keys.

Theorem 5.1. *Let G be an Oliver group with $a_G \geq 2$. If G/G^{nil} is isomorphic to none of the following groups then $WLO(G) \neq 0$.*

(1) *a p-group for a prime p*

(2) *$C_2 \times P$ for an odd prime p and a p-group P*

(3) *$P \times C_3$ for a 2-group P such that any element of P is self-conjugate*

Conversely, we obtain

Proposition 5.2. *Let N be a nilpotent group with $LO(N) = 0$. Then N is isomorphic to (1), (2) or (3) in Theorem 5.1.*

Let G be a nonsolvable group with $a_G \geq 2$. We point out again that Morimoto obtained $Sm(\mathrm{Aut}(A_6)) = 0$ and $Sm(P\Sigma L(2,27)) \neq 0$. So, suppose that $G \not\cong \mathrm{Aut}(A_6), P\Sigma L(2,27)$.

Pawalowski and Solomon obtained that $a_G > b_{G/G^{nil}}$ and $LO(G) \supset PO(G, G^{nil}) \neq 0$. Clearly the existence of a basic G^{nil}-pair yields $a_G > b_{G/G^{nil}}$.

If G is isomorphic to (1), (2) or (3) in Theorem 5.1, then we can show that there exists a basic G^{nil}-pair by the similar argument of the section 2 in [54] and then $LSm(G) \neq 0$ follows.

Remark 5.3. *For a nonsolvable group with $a_G \geq 2$, $LO(G) = 0$ implies that G is isomorphic to either $\mathrm{Aut}(A_6)$ or $P\Sigma L(2,27)$.*

6. Computation by GAP

We computed solvable Oliver groups G with $LO(G) = 0$ and $a_G \geq 2$ of order up to 2000 by using a software GAP [23] and found twelve groups of which ten groups are gap groups and the others are not.

Proposition 6.1. *If G is an Oliver group then the order of G is divisible by at least 3 distinct primes.*

We obtain 4 counterexamples to Laitinen's Conjecture:

Laitinen's Conjecture. *It might hold $LSm(G) \neq 0$ for an Oliver group G with $a_G \geq 2$.*

A counterexample is found first by Morimoto for $G = \mathrm{Aut}(A_6)$. The key point is next.

Lemma 6.2 ([42]). *If U and V are Smith equivalent representations then U^N and V^N are isomorphic for each subgroup N of G with index 1 or 2.*

This means that

$$Sm(G) \leq \bigcap_N \overline{PO}(G,N) \quad \text{and} \quad LSm(G) \leq \bigcap_N PO(G,N)$$

where N runs over subgroups of G with index 2.

Proposition 6.3. *If G/G^{nil} is an elementary abelian 2-group then $LSm(G) \subseteq LO(G)$. In addition if G is a gap Oliver group, it holds the equality $LSm(G) = LO(G)$.*

$SG(72,44)$, $SG(288,1025)$, $SG(432,734)$, $SG(576,8654)$ are our counterexamples. Here $SG(ord, type)$ is denoted the group SmallGroup($ord, type$) in the software GAP of order ord. Note that $SG(72,44)$ and $SG(288,1025)$ are gap groups and the others are not.

G	a_G	$LSm(G) = 0$	8 condition	$Sm(G) = 0$
$SG(72,44)$	2	True	Hold	True
$SG(288,1025)$	2	True	Hold	True
$SG(432,734)$	2	True	Not hold	True
$SG(576,8654)$	3	True	Hold	True

TABLE 1. Counterexamples to Laitinen's Conjecture

For a subset S of $RO(G)$, we define rank S by

$$\operatorname{rank} S = \max\{ \operatorname{rank} A \mid A \text{ is a subgroup and } A \subseteq S \}.$$

By definition it holds rank $WLO(G) \leq$ rank $LSm(G) \leq$ rank $PO(G, O^p(G))$ for each prime p.

Morimoto shows $LSm(G) \neq 0$ for $G = SG(864,2666), SG(864,4666)$ as well as $P\Sigma L(2,27)$ and then it is unknown whether $LSm(G) = 0$ or not for the following six gap groups G.

G	a_G	rank $LSm(G)$	G/G^{nil}	$LSm(G) = Sm(G)$
$SG(864,4663)$	3	0, 1, 2	C_8	False
$SG(864,4672)$	5	0, 1	$Q_8 \times C_3$	True
$SG(1176,220)$	2	0, 1	C_3	True
$SG(1176,221)$	2	0, 1	C_3	True
$SG(1152,155470)$	3	0, 1	C_6	True
$SG(1152,157859)$	3	0, 1	C_6	True

7. PROBLEM

In the section we post a problem with respect to an approach to show $LO(G) \subseteq LSm(G)$.

Problem 7.1. *Let G be an Oliver group which is not a gap group and let K be a subgroup of G with $K > O^2(G)$. Is either $C_K(x)$ or $C_K(y)$ a 2-group for involutions x and y of K outside of $O^2(K)$ which are not conjugate in G?*

The author confirmed that this problem is affirmative for all groups of order less than 2000.

Theorem 7.2. *Let G be an Oliver group which is not a gap group. Suppose that the problem is affirmative for each K. Then it holds $2LO(G) \subseteq WLO(G) \subseteq LO(G)$. In particular, it holds that* $\operatorname{rank} LO(G) \leq \operatorname{rank} LSm(G)$.

Note that $LO(G) = WLO(G)$ if G is a gap group.
Putting together with Proposition 6.3, we obtain

Corollary 7.3. *Let G be an Oliver group which is not a gap group. Suppose that the problem is affirmative for each K. If G/G^{nil} is an elementary abelian 2-group then it holds $WLO(G) = LO(G) = LSm(G)$. In particular, $LSm(G)$ is a group.*

Finally we point out that the problem is affirmative if and only if there exists $U - V \in LO(G)$ such that both two representations $U \oplus W$ and $V \oplus W$ satisfy (1) of the weak gap condition for any representation W. The author hope the problem will be solved affirmative.

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