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Construction of Invariant Manifold by Renormalization-group Method: reduction of dynamical systems and its applications

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We have first given a comprehensive review of the renormalization group(RG) method for global and asymp-totic analysis on the basis of the following articles[1, 2, 3, 4, 5, 6]: An emphasis is put on the relevance to the classical theory of envelopes and the existence of invariant manifolds of the dynamics under consideration. We clarify that an essential point of the method is to convert the problem from solving differential equations to obtaining suitable initial (or boundary) conditions: The RG equation determines the slow motion of the would-be integral constants in the unperturbative solution on the invariant manifold.

The RG method is applied to derive the relativistic Navier-Stokes equation from the Boltzmann equation[7, 8], as an example of the reduction of dynamics; the non-relativistc case was already treated successfully in [1, 6]. It turns out that the derived equation in the particle frame is a stable dissipative relativistic hydrodynamic equation [8]. We indicate that the usual constraint on the dissipative part of the energy-momentum tensor $\delta T^{\mu\nu}$ after Eckart in the particle frame is not compatible with the underlying relativistic Boltzmann equation. We demonstrate that the solution around the thermal equilibrium state obtained in the new equations in the particle frame is stable, performing the linear stability analysis with the use of the equation of state and the transport coefficients for a rarefied gas[8]. It is worth emphasizing that the establishment of a stable relativistic hydrodynamic equation in the particle frame is significant since the so called causal equations such as Israel-Stwert are usually constructed in the particle frame with the constraint of Eckart.

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