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Mapping theorems for C-spaces

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We assume that all spaces are normal unless otherwise stated. We refer the readers to [2] for dimension theory.

In this note we study mapping theorems for C-spaces.

A space X is a *C-space* (an *A-weakly infinite-dimensional space*) if for every countable collection $\{\mathcal{G}_i : i \in \mathbb{N}\}$ of open covers (two-element open covers, respectively) of X there exists a countable collection $\{\mathcal{H}_i : i \in \mathbb{N}\}$ of collections of pairwise disjoint open subsets of X such that $\mathcal{H}_i < \mathcal{G}_i$ for every $i \in \mathbb{N}$ and $\bigcup_{i=1}^{\infty} \mathcal{H}_i$ covers X (cf. [1]).

Evidently, every C-space is A-weakly infinite-dimensional. However, it is not known whether the converse is true.

Polkowski [5] proved the following theorem.

Theorem 1 (Polkowski [5]). *If $f : X \longrightarrow Y$ is a closed mapping of an A-weakly infinite-dimensional countably paracompact space X onto a space Y and there exists an integer $k \geq 1$ such that $|f^{-1}(y)| \leq k$ for every $y \in Y$, then Y is A-weakly infinite-dimensional.*

We proved that the following theorem, which is an analogous result for C-spaces.

Theorem 2. *If $f : X \longrightarrow Y$ is a closed mapping of a countably paracompact C-space X onto a paracompact space Y and there exists an integer $k \geq 1$ such that $|f^{-1}(y)| \leq k$ for every $y \in Y$, then Y is a C-space.*

Problem. Does theorem 1 (or theorem 2) hold for closed mappings with finite fibers?

In [4], Pol proved the following theorem.

Theorem 3 (Pol [4]). *If $f : X \longrightarrow Y$ is a continuous mapping of a compact metrizable space X onto a metrizable space Y such that $|f^{-1}(y)| \leq \aleph_0$ for every $y \in Y$, then X is an A-weakly infinite-dimensional space* (resp. a C-space) if and only if Y is an A-weakly infinite-dimensional space (resp. a C-space).

Does Theorem 3 remain true if we replace '$|f^{-1}(y)| \leq \aleph_0$' by '$|f^{-1}(y)| < \mathbf{c}$'? In [5], Polkowski proved the following theorem.

Theorem 4 (Polkowski [5]). *If $f : X \longrightarrow Y$ is a continuous mapping of a compact A-weakly infinite-dimensional space X onto a space Y such that $|f^{-1}(y)| < \mathbf{c}$ for every $y \in Y$, then Y is A-weakly infinite-dimensional.*

Similarly, the following theorem holds.

Theorem 5. *If $f : X \longrightarrow Y$ is a continuous mapping of a compact C-space X onto a space Y such that $|f^{-1}(y)| < \mathbf{c}$ for every $y \in Y$, then Y is a C-space.*

On the other hand, Hattori and Yamada proved that the following theorem.

Theorem 6 (Hattori and Yamada [3]).

(i) *If $f : X \longrightarrow Y$ is a closed mapping of a countably paracompact* (or hereditarily normal) space X onto a C-space Y such that $f^{-1}(y)$ is A-weakly infinite-dimensional for every $y \in Y$, then X is A-weakly infinite-dimensional.

(ii) If $f : X \longrightarrow Y$ is a closed mapping of a paracompact space X onto a C-space Y such that $f^{-1}(y)$ is a C-space for every $y \in Y$, then X is a C-space.

Problem. Does Theorem 6(i) remain true if we replace '$f^{-1}(y)$ is a C-space' by '$f^{-1}(y)$ is A-weakly infinite-dimensional'?

References

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