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λ Credibility

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Abstract

This paper extends credibility measure in uncertainty theory to lambda credibility. Lambda credibility measure is a convex combination of possibility measure and necessity measure. We investigate differences between credibility measure and lambda credibility measure. Finally, we introduce lambda credibility expectation for a triangular fuzzy variable to have an insight on the differences between credibility and lambda credibility.

Keywords: unceratain theory, credibility measure,possibility measure, necessity measure, fuzzy variable, expectation operator

1 Introduction

Fuzzy set was first introduced by Zadeh [16] in 1965. This notion has been very useful in human decision making under uncertainty. We can see lots of papers which use this fuzzy set theory in K.Iwamura and B.Liu [1] [2], B.Liu and K.Iwamura [9] [10] [11], X.Ji and K.Iwamura [4], X.Gao and K.Iwamura [5], G.Wang and K.Iwamura [14], M.Wen and K.Iwamura [15] and others. We also have some books on fuzzy decision making under fuzzy environments such as D.Dubois and H.Prade [17], H-J.Zimmermann [18], M.Sakawa [20], J.Kacprzyk [19], B.Liu and A.O.Esogbue [8].

Recently B.Liu has founded a frequentionist fuzzy theory with huge amount of applications in fuzzy mathematical programming. We see it in books such as B.Liu [3] [7] [6]. In his book [6] published in 2004, B.Liu [6] has succeeded in establishing an axiomatic foundation for uncertainty theory, where they have created a notion of credibility measure, which is a mean of possibility measure and necessity measure.

In this paper, we further define λ credibility measure which is a convex combination of possibility measure and necessity measure. Then we investigate differences between credibility measure and λ credibility measure. We will see that credibility measure is much better than λ credibility measure because the former can deal with many applications in mathematical programming in fuzzy environments. Yet, our λ credibility is also a special fuzzy measure in T.Murofushi and M.Sugeno [21]. Hence there could come out some useful application of λ-credibility measure based on either Sugeno integral [13] or Choquet integral in the future, which is left unanswered in this paper.

The rest of the paper is organized as follows. The next section provides a brief review on the results of possibility, necessity and credibility. Section 3 presents the definition of λ-credibility and investigate its characteristics to recognize the importance of credibility measure. In section 4, we introduce λ-credibility expectation for a triangular fuzzy variable (a, b, c) to see that its formula differs depending on cases $0 \le a < b < c$, $a < 0 \le b < c$, $a < b < 0 \le c$, while they all reduces to the same expression $(a + 2b + c)/4$ if we set $\lambda = 1/2$. Finally in Section 5, we give a final conclusion on λ credibility.

2 Possibility, Necessity and Credibility

We start with the axiomatic definition of possibility given by B. Liu [6] in 2004. Let Θ be an arbitrary nonempty set, and let $\mathcal{P}(\Theta)$ be the power set of Θ.
The four axioms are listed as follows:

Axiom 1. $\text{Pos}\{\Theta\} = 1$.

Axiom 2. $\text{Pos}\{\emptyset\} = 0$.

Axiom 3. $\text{Pos}\{\cup_i A_i\} = \sup_i \text{Pos}\{A_i\}$ *for any collection* $\{A_i\}$ *in* $\mathcal{P}(\Theta)$.

Axiom 4. *Let* Θ_i *be nonempty sets on which* $\text{Pos}_i\{\cdot\}$ *satisfy the first three axioms,* $i = 1, 2, \cdots, n$, *respectively, and* $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$. *Then*

$$\text{Pos}\{A\} = \sup_{(\theta_1,\theta_2,\cdots,\theta_n)\in A} \text{Pos}_1\{\theta_1\} \wedge \text{Pos}_2\{\theta_2\} \wedge \cdots \wedge \text{Pos}_n\{\theta_n\} \quad (1)$$

for each $A \in \mathcal{P}(\Theta)$. *In that case we write* $\text{Pos} = \text{Pos}_1 \wedge \text{Pos}_2 \wedge \cdots \wedge \text{Pos}_n$.

We call Pos a possibility measure if it satisfies the first three axioms. We call the triplet $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ a possibility space.

Note 2.1 : *We have* Θ_1, Pos_1 *and* Θ_2, Pos_2 *which satisfy the first three axioms and* $\Theta = \Theta_1 \times \Theta_2$, $\text{Pos} = \text{Pos}_1 \wedge \text{Pos}_2$ *which satisfy the four axioms.*

Theorem 2.1 *(B. Liu, 2004). Let* $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ *be a possibility space. Then we have*
(a) 0-1 boundedness: $0 \le \text{Pos}\{A\} \le 1$ *for any* $A \in \mathcal{P}(\Theta)$;
(b) monotonicity: $\text{Pos}\{A\} \le \text{Pos}\{B\}$ *whenever* $A \subset B$;
(c) subadditivity: $\text{Pos}\{A \cup B\} \le \text{Pos}\{A\} + \text{Pos}\{B\}$ *for any* $A, B \in \mathcal{P}(\Theta)$.
(d) $\text{Pos}\{A \cup B\} = \max(\text{Pos}\{A\}, \text{Pos}\{B\})$ *for any* $A, B \in \mathcal{P}(\Theta)$.

Note 2.2 : *We see that*

$$\mathrm{Pos}\{A \cup B\} = \max(\mathrm{Pos}\{A\}, \mathrm{Pos}\{B\}) \tag{2}$$

naturally brings subadditivity in Theorem 2.1(c).

Theorem 2.2 *(B. Liu, 2004). Let* $(\Theta_i, \mathcal{P}(\Theta_i), \mathrm{Pos}_i), i = 1, 2, \cdots, n$ *be possibility spaces,* $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$ *and* $\mathrm{Pos} = \mathrm{Pos}_1 \wedge \mathrm{Pos}_2 \wedge \cdots \wedge \mathrm{Pos}_n$. *Then* $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ *is a possibility space, which is called n-fold product possibility space of* $(\Theta_i, \mathcal{P}(\Theta_i), \mathrm{Pos}_i)$ $(i = 1, 2, \cdots, n)$.

Theorem 2.3 *(B. Liu, 2004). Let* $(\Theta_i, \mathcal{P}(\Theta_i), \mathrm{Pos}_i), i = 1, 2, \cdots$ *be possibility spaces. If* $\Theta = \Theta_1 \times \Theta_2 \times \cdots$ *and* $\mathrm{Pos} = \mathrm{Pos}_1 \wedge \mathrm{Pos}_2 \wedge \cdots$, *then the set function* Pos *is a possibility measure on* $\mathcal{P}(\Theta)$, *and* $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ *is a possibility space, which is called infinite product possibility space of* $(\Theta_i, \mathcal{P}(\Theta_i), \mathrm{Pos}_i)$ $(i = 1, 2, \cdots)$.

Let $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ be a possibility space, and A a set in $\mathcal{P}(\Theta)$. Then the necessity measure of A is defined by

$$\mathrm{Nec}\{A\} = 1 - \mathrm{Pos}\{A^c\}. \tag{3}$$

We have

Theorem 2.4 *(B. Liu, 2004). Let* $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ *be a possibility space. Then we have*
(a) $\mathrm{Nec}\{\Theta\} = 1$, $\mathrm{Nec}\{\emptyset\} = 0$;
(b) 0-1 boundedness: $0 \leq \mathrm{Nec}\{A\} \leq 1$ *for any* $A \in \mathcal{P}(\Theta)$;
(c) monotonicity: $\mathrm{Nec}\{A\} \leq \mathrm{Nec}\{B\}$ *whenever* $A \subset B$;
(d) $\mathrm{Nec}\{A\} + \mathrm{Pos}\{A^c\} = 1$ *for any* $A \in \mathcal{P}(\Theta)$;
(e) $\mathrm{Nec}\{A \cap B\} = \min(\mathrm{Nec}\{A\}, \mathrm{Nec}\{B\})$ *for any* $A, B \in \mathcal{P}(\Theta)$;
(f) $\mathrm{Nec}\{A\} = 0$ *whenever* $\mathrm{Pos}\{A\} < 1$;.

For a possibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ and A a set in $\mathcal{P}(\Theta)$, B. Liu and Y-K. Liu [12] [6] have introduced the credibility measure of A by

$$\mathrm{Cr}\{A\} = \frac{1}{2}\left(\mathrm{Pos}\{A\} + \mathrm{Nec}\{A\}\right). \tag{4}$$

We easily have

Theorem 2.5 *(B. Liu, 2004).*

$$\mathrm{Pos}\{A\} \geq \mathrm{Cr}\{A\} \geq \mathrm{Nec}\{A\} \tag{5}$$

for any $A \in \mathcal{P}(\Theta)$ *of a possibility space* $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$.

Theorem 2.6 *(B. Liu, 2004). Let* $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ *be a possibility space. Then we have*
(a) $\mathrm{Cr}\{\Theta\} = 1$, $\mathrm{Cr}\{\emptyset\} = 0$;
(b) 0-1 boundedness: $0 \leq \mathrm{Cr}\{A\} \leq 1$ *for any* $A \in \mathcal{P}(\Theta)$;
(c) monotonicity: $\mathrm{Cr}\{A\} \leq \mathrm{Cr}\{B\}$ *whenever* $A \subset B$;
(d) self duality: $\mathrm{Cr}\{A\} + \mathrm{Cr}\{A^c\} = 1$ *for any* $A \in \mathcal{P}(\Theta)$;
(e) subadditivity: $\mathrm{Cr}\{A \cup B\} \leq \mathrm{Cr}\{A\} + \mathrm{Cr}\{B\}$ *for any* $A, B \in \mathcal{P}(\Theta)$.

We further have

Theorem 2.7 *Let* $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ *be a possibility space. Let* $\mathrm{Cr}\{\ \}$ *be the credibility measure on* $\mathcal{P}(\Theta)$*. Then*
(a) $\mathrm{Pos}\{A\} < 1$ *implies* $\mathrm{Cr}\{A\} = \frac{1}{2}\mathrm{Pos}\{A\}$*;*
(b) $\mathrm{Cr}\{A\} \geq \frac{1}{2}$ *implies* $\mathrm{Pos}\{A\} = 1$.

A fuzzy variable is defined as a function from Θ of a possibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ to the set of reals. Let ξ be a fuzzy variable defined on the possibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$. Then the set

$$\xi_\alpha = \{\xi(\theta) \mid \theta \in \Theta, \mathrm{Pos}\{\theta\} \geq \alpha\}$$

is called the α-level set of ξ. The set

$$\Theta^+ = \{\theta \in \Theta | \mathrm{Pos}\{\theta\} > 0\}$$

is called the kernel of the possibility space and the set

$$\{\xi(\theta) \mid \theta \in \Theta, \mathrm{Pos}\{\theta\} > 0\} = \{\xi(\theta) \mid \theta \in \Theta^+\}$$

is called the support of ξ.

Let ξ be a fuzzy variable defined on the possibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$. Then its membership function is derived through the possibility measure Pos by

$$\mu(x) = \mathrm{Pos}\{\theta \in \Theta \mid \xi(\theta) = x\}, \quad x \in \Re. \tag{6}$$

Theorem 2.8 *(B. Liu, 2004). Let* $\mu : \Re \to [0, 1]$ *be a function with* $\sup \mu(x) = 1$*. Then there is a fuzzy variable whose membership function is* μ*.*

3 λ Credibility

Let $\mathrm{Pos}\{\ \}$ and $\mathrm{Nec}\{\ \}$ be a possibility measure and the necessity measure on $\mathcal{P}(\Theta)$. Let λ be a real number between 0 and 1, i.e., $\lambda \in [0, 1]$. Define a set function $\lambda\mathrm{Cr}\{\ \}$ on $\mathcal{P}(\Theta)$ by

$$\lambda\mathrm{Cr}\{A\} = \lambda\mathrm{Pos}\{A\} + (1 - \lambda)\mathrm{Nec}\{A\}. \tag{7}$$

For $\lambda = 1$, we have $\lambda\mathrm{Cr}\{A\} = \mathrm{Pos}\{A\}$ and for $\lambda = 0$, we have $\lambda\mathrm{Cr}\{A\} = \mathrm{Nec}\{A\}$ and we see that $0.5\mathrm{Cr}\{A\} = \mathrm{Cr}\{A\}$, for any $A \in \mathcal{P}(\Theta)$. $\lambda\mathrm{Cr}\{\ \}$ is not $\mathrm{Cr}\{\ \}$ multiplied by λ!

We easily have

Theorem 3.1 *Let* λ *be a real number such that* $0 \leq \lambda \leq 1$*. Then*
(a) $\lambda\mathrm{Cr}\{\Theta\} = 1, \lambda\mathrm{Cr}\{\emptyset\} = 0$*;*
(b) $\mathrm{Nec}\{A\} \leq \lambda\mathrm{Cr}\{A\} \leq \mathrm{Pos}\{A\}$ *for any* $A \in \mathcal{P}(\Theta)$ *and so 0-1 boundedness holds for* $\lambda\mathrm{Cr}\{\ \}$*;*

(c) monotonicity: $\lambda\mathrm{Cr}\{A\} \le \lambda\mathrm{Cr}\{B\}$ *whenever* $A \subset B$;

(d) $2 \ge \lambda\mathrm{Cr}\{A\} + \lambda\mathrm{Cr}\{A^c\} \ge 1$ *whenever* $\lambda \ge \frac{1}{2}$

while $0 \le \lambda\mathrm{Cr}\{A\} + \lambda\mathrm{Cr}\{A^c\} \le 1$ *whenever* $\lambda \le \frac{1}{2}$ *for any* $A \in \mathcal{P}(\Theta)$;

(e) restricted subadditivity: $\lambda\mathrm{Cr}\{A \cup B\} \le \lambda\mathrm{Cr}\{A\} + \lambda\mathrm{Cr}\{B\}$ *for any* $A, B \in \mathcal{P}(\Theta)$ *provided that* $\lambda \ge \frac{1}{2}$.

Corollary 3.1 *Let* λ *be such that* $0 < \lambda < 1$. *Then we get*

(a) $\lambda\mathrm{Cr}\{A\} = 1$ *if and only if* $\mathrm{Pos}\{A\} = \mathrm{Nec}\{A\} = 1$,

(b) $\lambda\mathrm{Cr}\{A\} = 0$ *if and only if* $\mathrm{Pos}\{A\} = \mathrm{Nec}\{A\} = 0$.

Theorem 3.2 *Let* $0 \le \lambda \le 1$. *Suppose that there exists a set* $A \in \mathcal{P}(\Theta)$ *such that* $\mathrm{Pos}\{A\} > \mathrm{Nec}\{A\}$ *with* $\lambda\mathrm{Cr}\{A\} + \lambda\mathrm{Cr}\{A^c\} = 1$. *Then we get* $\lambda = \frac{1}{2}$.

Corollary 3.2 *Suppose that there exist* s, t *with* $0 \le s < t \le 1$ *such that for any* $\lambda \in [s, t]$, *any* $A \in \mathcal{P}(\Theta)$, $\lambda\mathrm{Cr}\{A\} + \lambda\{A^c\} = 1$. *Then we have* $\mathrm{Pos}\{A\} = \mathrm{Nec}\{A\}$ *for any* $A \in \mathcal{P}(\Theta)$.

Example 3.1. Let $\Theta = \{\theta_1, \theta_2\}$ with $\mathrm{Pos}\{\theta_1\} = 1.0, \mathrm{Pos}\{\theta_2\} = 0.8$. Then $\mathrm{Nec}\{\theta_1\} = 1 - \mathrm{Pos}\{\theta_2\} = 0.2$, $\mathrm{Nec}\{\theta_2\} = 1 - 1 = 0$. $\lambda\mathrm{Cr}\{\Theta\} = 1$, $\lambda\mathrm{Cr}\{\emptyset\} = 0$, $\lambda\mathrm{Cr}\{\theta_1\} = 0.8\lambda + 0.2$, $\lambda\mathrm{Cr}\{\theta_2\} = 0.8\lambda$, $\lambda\mathrm{Cr}\{\theta_1\} + \lambda\mathrm{Cr}\{\theta_2\} = 1.6\lambda + 0.2$ for $0 \le \lambda \le 1$. Therefore

$$\lambda\mathrm{Cr}\{\theta_1\} + \lambda\mathrm{Cr}\{\theta_2\} = 1 \quad \text{if and only if} \quad \lambda = \frac{1}{2}. \tag{8}$$

For $\lambda = 0.4$, $\lambda\mathrm{Cr}\{\theta_1\} + \lambda\mathrm{Cr}\{\theta_2\} = 0.84 < 1$.

And for $\lambda = 0.6$, $\lambda\mathrm{Cr}\{\theta_1\} + \lambda\mathrm{Cr}\{\theta_2\} = 1.16 > 1$.

So, $\lambda\mathrm{Cr}\{\ \}$ is not self dual for any λ, $0 < \lambda < 1$ with $\lambda \ne \frac{1}{2}$.

Example 3.2. Let ξ be a triangular fuzzy variable with its membership function

$$\mu(x) = \begin{cases} x, & \text{if } 0 \le x \le 1 \\ -x + 2, & \text{if } 1 < x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Then we get

$$\mathrm{Pos}\{\xi \le x\} = \begin{cases} 0, & \text{if } x \le 0 \\ x, & \text{if } 0 < x \le 1 \\ 1, & \text{if } x > 1. \end{cases}$$

and

$$\mathrm{Pos}\{\xi > x\} = \begin{cases} 1, & \text{if } x \le 1 \\ -x + 2, & \text{if } 1 < x \le 2 \\ 0, & \text{if } x > 2. \end{cases}$$

For $0 < x < 1$, $\mathrm{Pos}\{\xi \le x\} + \mathrm{Pos}\{\xi > x\} = x + 1 > 1$. Therefore, we get $\lambda = \frac{1}{2}$ provided that $\lambda\mathrm{Cr}\{\ \}$ is self dual. This fact suggests the importance of $0.5\mathrm{Cr}\{\ \}$, i.e., credibility measure. Furthermore, we have

Theorem 3.3 *Let ξ be a triangular fuzzy variable with its membership function*

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } a \le x \le b \\ \frac{x-c}{b-c}, & \text{if } b < x \le c \\ 0, & \text{otherwise} \end{cases}$$

where $a \le b \le c$. Suppose that $\mathrm{Pos}\{\xi \le x\} + \mathrm{Pos}\{\xi > x\} = 1$ *holds for any real number x, i.e.,* $\lambda\mathrm{Cr}\{\ \}$ *is self dual for fuzzy events $\{\xi \le x\}$. Then we have $a = b = c$.*

Example 3.3: Let ξ be a fuzzy variable with its membership function $\mu(x) = \delta_a(x)$, where

$$\delta_a(x) = \begin{cases} 1, & x = a \\ 0, & x \ne a. \end{cases}$$

We call δ_a a Dirac measure. Then we get

$$\mathrm{Pos}\{\xi \le x\} = \begin{cases} 0, & x < a \\ 1, & x \ge a \end{cases} \text{ and } \mathrm{Pos}\{\xi > x\} = \begin{cases} 1, & x < a \\ 0, & x \ge a. \end{cases}$$

Hence we get $\mathrm{Pos}\{\xi \le x\}+\mathrm{Pos}\{\xi > x\} = 1$ for any number x. Therefore $\lambda\mathrm{Cr}\{\xi \le x\}+\lambda\mathrm{Cr}\{\xi > x\} = 1$ for any number x. $\lambda\mathrm{Cr}\{\ \}$ is self dual for fuzzy events $\{\xi \le x\}$, where x is a real number.

4 λ-Credibility Expectation

Let ξ be a triangular fuzzy variable (a, b, c) with its membership function $\mu(x)$ as follows;

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } a \le x \le b \\ \frac{x-c}{b-c}, & \text{if } b < x \le c \\ 0, & \text{otherwise.} \end{cases}$$

Then we get

$$\lambda\mathrm{Cr}\{\xi \le x\} = \begin{cases} 1, & \text{if } x \le a \\ \frac{b-\lambda a}{b-a} - \frac{1-\lambda}{b-a}x, & \text{if } a < x \le b \\ \frac{-\lambda c}{b-c} + \frac{\lambda}{b-c}x, & \text{if } b < x \le c \\ 0, & \text{if } c < x. \end{cases}$$

Define λ- credibility expectation $\lambda\mathrm{Cr}\{\ \}$of ξ by

$$\mathrm{E}_{\lambda\mathrm{Cr}}[\xi] = \int_0^{+\infty} \lambda\mathrm{Cr}\{\xi \ge x\}\mathrm{d}x - \int_{-\infty}^{0} \lambda\mathrm{Cr}\{\xi \le x\}\mathrm{d}x,$$

where integrals are defined through Lebesgue integral,i.e., well defined in case either of the two takes finite value.

Then we see the followings,
in case $0 \le a < b < c$, we get

$$E_{\lambda Cr}[\xi] = \int_0^{+\infty} \lambda Cr\{\xi \ge x\}dx \tag{9}$$

$$= \int_0^a 1dx + \int_a^b \{\frac{b-\lambda a}{b-a} - \frac{1-\lambda}{b-a}x\}dx$$

$$= \frac{b+a+(c-a)\lambda}{2},$$

in case $a < b < 0 \le c$, we get

$$E_{\lambda Cr}[\xi] = \left(\frac{-c^2}{2(b-c)} + \frac{a-b}{2} + \frac{b^2-2bc}{2(b-c)}\right)\lambda + \frac{b^2}{2(b-c)}. \tag{10}$$

If we set $\lambda = \frac{1}{2}$ in equation (9),(10) we get $E_{0.5Cr}[\xi] = \frac{a+2b+c}{4}$, which is identical to the credibility expectation for a triangular fuzzy variable ξ. This fact shows us the importance and adequateness of the credibility expectation operator. Finally let us see the case of $a = 0, b = 1, c = 3$. In this case we get

$$E_{\lambda Cr}[\xi] = \frac{1}{2} + \frac{3}{2}\lambda,$$

which coincides with the credibility expectation of ξ, $E[\xi] = \frac{0+2\cdot 1+3}{4} = \frac{5}{4}$, if we take $\lambda = \frac{1}{2}$.
We further see that $\frac{1}{2} \le E_{\lambda Cr}[\xi] \le 2$ and $\frac{1}{2}(\frac{1}{2}+2) = \frac{5}{4} = E[\xi]$, where $E_{Nec}[\xi] = \frac{1}{2}$ and $E_{Pos}[\xi] = 2$.

5 Conclusion

We have introduced λ credibility $\lambda Cr\{\ \}$ on a possibility space $(\Theta, \mathcal{P}(\Theta), Pos)$ and have shown that $\lambda Cr\{\ \}$ is 0-1 bounded, monotone for set inclusion, over self dual or under self dual according to $\lambda \ge \frac{1}{2}$ or $\lambda \le \frac{1}{2}$ and that it satisfies restricted subadditivity. We further have shown that for a non-trivial possibility space, where there exists a set $A \in \mathcal{P}(\Theta)$ such that $Pos\{A\} > Nec\{A\}$, self duality of $\lambda Cr\{\ \}$ naturally leads to $\lambda = \frac{1}{2}$. We have given some examples for which self duality naturally holds when $\lambda = \frac{1}{2}$, or a triangular fuzzy variable ξ reduces to a Dirac measure if we demand $\lambda Cr\{\ \}$ be self dual for fuzzy events $\{\xi \le x\}$. Finally we have defined $\lambda Cr\{\ \}$ expectation of a fuzzy variable ξ. For a triangular fuzzy variable (a, b, c), the $\lambda Cr\{\ \}$ expectation $E_{\lambda Cr}[\xi]$ has different formula in λ, dependent on $0 < a$ or $a < 0 < b$ or $a < b < 0 < c$. But if we let $\lambda = \frac{1}{2}$ then they all reduce to $(a+2b+c)/4$, the expectation of ξ in the sense of B.Liu [6] in 2004. So, we believe that $0.5\mathrm{Cr}\{\ \} = \mathrm{Cr}\{\ \}$ is the most useful one among $\lambda\mathrm{Cr}\{\ \}$, $0 < \lambda < 1$.

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