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Social Optimization in the Emission Trading of Greenhouse Gases

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Abstract. This paper considers the simultaneous optimization of the amount of discharge of greenhouse gases and the amount of removing its harms in both international and domestic markets. In the domestic market we introduce surcharges and bounties in order that the individual optimization accomplishes the total (social) optimization.

Key Words. greenhouse gas, emission trading, international market, domestic market, individual optimization, social optimization, surcharge, bounty

1. Introduction

The global warming proceeds and its various influences, for example, the ascent of sea-level and desertification are indicated. Many international conferences have held toward the regulation of greenhouse gases. The Kyoto protocol, which was agreed at 1997, has established numerical targets of the reduction as legal liability of each nation. Furthermore as a measure for its effective accomplishment the international emission trading has been introduced. The emission trading is a system in which each nation receives the quota of emission credits for greenhouse gases and the nation with insufficient quota buys the emission credits from the nation with surplus. This method aims to conquer the "market failure" in the external diseconomy by creating a new market toward the internalization of externality.

There are many literatures in the field of environmental economics. C.D.Kolstad [2] is a good introduction and a survey for this field. A.Malik [3] considers a method of enforcing tradable emission permits when there are noncompliant firms. A.Malik [4] shows that the condition for minimizing the cost of enforcing an environmental policy differs from that for minimizing the cost for firms to comply it. G.Amacher and A.S.Malik [1] compares the regulation by penalty with a taxation method in the case that regulations and firms bargain. With respect to an emission trading many literatures consider effective methods of enforcing it, but there are a few literatures that considers the difference between an individual and social optimization in the emission trading market.

This paper has four focuses:

(1) As decision variables we consider not only the amount of discharge but also the amount of removing harms of greenhouse gases.

(2) We consider both international and domestic markets.

(3) In both international and domestic markets we consider the individual optimization for each agent and the total (social) optimization.

(4) In the domestic markets we introduce surcharges and bounties in order that both individual and total optimizations are accomplished simultaneously.

2. International Market

We consider the international market in which there are n agents (nations). The total emission credits of greenhouse gases available for the world (in other words, for human beings) is $A(>0)$. Let $a_i(>0)$ be the quota of emission credits for agent i ; $\sum_{i=1}^{n} a_i = A$. Let p be the price of emission credit which is determined from the balance between supply and demand in the international market. Let $R_i(z)$ be the profit function of agent i which denotes the profit of agent i by discharging the amount z of greenhouse gases. We suppose that the function $R_i(z)$ is strictly concave increasing and that the derivative $R_i'(z)$ converges to zero as z approaches to infinity. Let $C_i(z)$ be the cost function of agent i which denotes the cost of removing harms for the amount z of greenhouse gases. We suppose that the function $C_i(z)$ is strictly convex increasing and that the derivative $C_i'(z)$ diverges as z approaches to infinity. The policy of agent i can be denoted by (x_i, y_i) where x_i is the discharge amount of greenhouse gases and y_i is the amount of greenhouse gases which harms are removed. If $x_i - y_i > (<) a_i$, then agent i buys (sells) emission credits $x_i - y_i - a_i (a_i - x_i + y_i)$ in the international market. When agent i uses a policy (x_i, y_i), his net profit $G_i(x_i, y_i)$ is given by

$$G_i(x_i, y_i) = R_i(x_i) - C_i(y_i) - p(x_i - y_i - a_i). \tag{1}$$

We put $(x,y)=\{(x_1,y_1),\cdots,(x_n,y_n)\}$ which is called a social policy. When a social policy (x,y) is used, let $T(x,y)$ be the total net profit of the world (the society of human beings), namely,

$$T(x,y)=\sum_{i=1}^{n} G_i(x_i,y_i). \tag{2}$$

2.1 International Total Optimization

In this subsection we consider the total optimization in the international market. The problem is as follows:

$$(\mathrm{P}_1)\begin{cases} T(x,y)=\sum_{i=1}^{n}[R_i(x_i)-C_i(y_i)] \quad \rightarrow \quad \underset{(x,y)}{\max} & (3)\\ \textit{subject to} & \\ \sum_{i=1}^{n}(x_i-y_i)=A & (4)\\ x_i,y_i\geq 0 \qquad (i=1,2,\cdots,n). & \end{cases}$$

The constraint (4) means the balance between supply and demand in the international market.

THEOREM1. For a parameter λ, we put

$$\begin{aligned} I_1 &= \{\ i\,|\,R_i'(0)>\lambda\ ,\quad C_i'(0)<\lambda\ \}\\ I_2 &= \{\ i\,|\,R_i'(0)>\lambda\ ,\quad C_i'(0)\geq\lambda\ \}\\ I_3 &= \{\ i\,|\,R_i'(0)\leq\lambda\ ,\quad C_i'(0)\geq\lambda\ \}\\ I_4 &= \{\ i\,|\,R_i'(0)\leq\lambda\ ,\quad C_i'(0)<\lambda\ \} \end{aligned} \tag{5}$$

and

$$f(\lambda)=\sum_{i\in I_1\cup I_2} R_i'^{-1}(\lambda)-\sum_{i\in I_1\cup I_4} C_i'^{-1}(\lambda). \tag{6}$$

Let λ^* be the unique root of the equation $f(\lambda)=A$. The optimal solution of the total optimization problem (P1) is given as follows:

$$x_i^*=\left[R_i'^{-1}(\lambda^*)\right]^+ \qquad i=1,2,\cdots,n \tag{7}$$

$$y_i^*=\left[C_i'^{-1}(\lambda^*)\right]^+ \qquad i=1,2,\cdots,n \tag{8}$$

where $\alpha^+=\max\{\alpha,0\}$.

Proof. Since the problem (P_1) is a concave programming problem, by the Karush-Kuhn-Tucker theorem, the necessary and sufficient conditions for a policy $(x^*, y^*) = \{(x_1^*, y_1^*), \cdots, (x_n^*, y_n^*)\}$ to be optimal are as follows: There is a constant (the Lagrangian multiplier) λ such that

$$R_i'(x_i^*) - \lambda \begin{Bmatrix} = \\ \le \end{Bmatrix} 0 \qquad if \quad x_i^* \begin{Bmatrix} > \\ = \end{Bmatrix} 0 \tag{9}$$

$$-C_i'(y_i^*) + \lambda \begin{Bmatrix} = \\ \le \end{Bmatrix} 0 \qquad if \quad y_i^* \begin{Bmatrix} > \\ = \end{Bmatrix} 0 \tag{10}$$

$$A - \sum_{i=1}^{n} (x_i^* - y_i^*) = 0 \tag{11}$$

$$x_i^*, y_i^* \ge 0 \qquad (i = 1, 2, \cdots, n).$$

Using the relation (9) and the decreasing of $R_i'(z)$, we obtain

$$x_i^* = \begin{cases} 0 \\ R_i'^{-1}(\lambda) > 0 \end{cases} \qquad if \quad R_i'(0) \begin{Bmatrix} \le \\ > \end{Bmatrix} \lambda. \tag{12}$$

Similarly using the relation (10) and the increasing of $C_i'(z)$, we obtain

$$y_i^* = \begin{cases} C_i'^{-1}(\lambda) > 0 \\ 0 \end{cases} \qquad if \quad C_i'(0) \begin{Bmatrix} < \\ \ge \end{Bmatrix} \lambda. \tag{13}$$

Substituting these results (12) and (13) into the equation (11), we obtain $f(\lambda) = A$ where the function $f(\lambda)$ is defined by (6). Since the function $f(\lambda)$ is strictly decreasing in λ, the equation $f(\lambda) = A$ has a unique solution $\lambda = \lambda^*$. Then we can obtain the result from the relation (12) and (13) for $\lambda = \lambda^*$. (q.e.d.)

Theorem 1 shows the following properties:

(1) If the initial marginal profit $R_i'(0)$ is larger (smaller) than the constant level λ^*, then the optimal amount of discharge is positive (zero). If it is positive, the optimal amount satisfies that the posterior marginal profit equals to λ^*.

(2) If the initial marginal cost $C_i'(0)$ is larger (smaller) than λ^*, then the optimal

amount of removing harms is zero (positive). If positive, the optimal amount satisfies that the posterior marginal cost equals to λ^*.

(3) The smaller the total emission credit A becomes, the larger the constant level λ^* becomes, and then the number of agents with positive amount of discharge (removing harms) decreases (increases).

(4) In general the price is determined such that the marginal profit equals to the marginal cost. In the problem (P1), since $R_i'(x_i^*) = \lambda^* = C_i'(y_i^*)$, the international price of emission credits is λ^*.

2.2 International Individual Optimization

In this subsection we consider the individual optimization problem for agent (nation) i in the international market. It is supposed that the price p of emission credit and the quota a_i of emission credit for agent i are constants. The problem is formulated as follows:

$$(\mathrm{P}_2) \quad \begin{cases} G_i(x_i, y_i) = R_i(x_i) - C_i(y_i) - p(x_i - y_i - a_i) \quad \rightarrow \quad \max\limits_{x_i, y_i} & (14) \\ \textit{subject to} \\ x_i, y_i \geq 0. \end{cases}$$

The following theorem is clear.

THEOREM2. The optimal solution of the problem (P2) is given by

$$\tilde{x}_i = \left[R_i'^{-1}(p)\right]^+ \qquad i = 1, \cdots, n \tag{15}$$

$$\tilde{y}_i = \left[C_i'^{-1}(p)\right]^+ \qquad i = 1, \cdots, n. \tag{16}$$

Since $p = \lambda^*$, we know that both individual and total (social) optimization in the international market are accomplished simultaneously.

3. Domestic Market

In this section we restrict our attention to agent k (nation k) in the international market and consider the domestic market in the nation k. Then in this section the word "agent" means a company or an individual person in the nation k. In this market the price of emission credit is $p=\lambda^*$ which is determined in the international market. Furthermore the optimal amount of discharge $x^*(=x_k^*)$, the optimal amount of removing harms $y^*(=y_k^*)$ and the quota of emission credits $a(=a_k)$ are considered to be constants. We suppose that there are m agents in nation k. Let $R_j(z)$ and $C_j(z)$ be a profit function and a cost function of agent $j(=1,\cdots,m)$ respectively which are assumed to satisfy the same conditions as in Section 2. A policy of agent j is denoted by (u_j,v_j) where u_j is the amount of discharge and v_j is the amount of removing harms of agent j. A social policy of nation k is denoted by $(u,v)=\{(u_1,v_1),\cdots,(u_m,v_m)\}$.

3.1 Domestic Total Optimization

The total optimization problem of nation k is formulated as follows:

$$
(\mathrm{P}_3)\quad \begin{cases} T(u,v)=\sum_{j=1}^{m}\left[R_j(u_j)-C_j(v_j)\right]-\lambda^*(x^*-y^*-a) \quad \rightarrow \quad \max\limits_{(u,v)} & (17) \\ subject\ \ to & \\ \quad \sum_{j=1}^{m} u_j = x^* & (18) \\ \quad \sum_{j=1}^{m} v_j = y^* & (19) \\ \quad u_j, v_j \geq 0 \qquad (j=1,2,\ldots m). & \end{cases}
$$

THEOREM3. For a parameter λ and ξ, we put

$$
\begin{aligned}
J_1 &= \{\ j \mid R_j'(0) > \mu \ ,\ C_j'(0) < \xi \ \} \\
J_2 &= \{\ j \mid R_j'(0) > \mu \ ,\ C_j'(0) \ge \xi \ \} \\
J_3 &= \{\ j \mid R_j'(0) \le \mu \ ,\ C_j'(0) \ge \xi \ \} \\
J_4 &= \{\ j \mid R_j'(0) \le \mu \ ,\ C_j'(0) < \xi \ \}
\end{aligned} \tag{20}
$$

and

$$g(\mu) = \sum_{j \in J_1 \cup J_2} R_j'^{-1}(\mu) \tag{21}$$

$$h(\xi) = \sum_{j \in J_1 \cup J_4} C_j'^{-1}(\xi). \tag{22}$$

Let μ^* and ξ^* be unique roots of equations $g(\mu) = x^*$ and $h(\xi) = y^*$ respectively. The optimal solution of the problem (P3) is given as follows:

$$u_j^* = \left[R_j'^{-1}(\mu^*)\right]^+ \qquad j = 1, \cdots, m \tag{23}$$

$$v_j^* = \left[C_j'^{-1}(\xi^*)\right]^+ \qquad j = 1, \cdots, m. \tag{24}$$

Proof. The necessary and sufficient conditions for a policy

$(u^*, v^*) = \{(u_1^*, v_1^*), \cdots, (u_m^*, v_m^*)\}$ to be optimal are as follows:

There are two Lagrangian multipliers μ and ξ such that

$$R_j'(u_j^*) - \mu \le 0 \qquad j = 1, \cdots, m \tag{25}$$

$$-C_j'(v_j^*) + \xi \le 0 \qquad j = 1, \cdots, m \tag{26}$$

$$\sum_{j=1}^m u_j^*\left[R_j'(u_j^*) - \mu\right] + \sum_{j=1}^m v_j^*\left[-C_j'(v_j^*) + \xi\right] = 0 \tag{27}$$

$$\sum_{j=1}^m u_j^* = x^* \tag{28}$$

$$\sum_{j=1}^m v_j^* = y^* \tag{29}$$

$$u_j^*, v_j^* \ge 0 \qquad j = 1, \cdots, m.$$

From the equations (25), (27) and the decreasing of $R_j'(z)$, we obtain

$$u_j^* = \begin{cases} 0 \\ R_j'^{-1}(\mu) > 0 \end{cases} \qquad if \quad R_j'(0)\begin{Bmatrix} \le \\ > \end{Bmatrix}\mu. \tag{30}$$

Similarly from the equations (26), (27) and the increasing of $C_j'(z)$, we obtain

$$v_j^* = \begin{cases} C_j'^{-1}(\xi) > 0 \\ 0 \end{cases} \qquad if \quad C_j'(0)\begin{Bmatrix} < \\ \ge \end{Bmatrix}\xi. \tag{31}$$

Substituting these results in (28) and (29), we obtain

$$g(\mu) = x^* \tag{32}$$

$$h(\xi) = y^*. \tag{33}$$

Since the function $g(\mu)$ ($h(\xi)$) is strictly decreasing (increasing) in μ (ξ), the equations (32) and (33) have unique roots $\mu = \mu^*$ and $\xi = \xi^*$ respectively. Then we obtain the result of Theorem 3. (q.e.d.)

3.2 Domestic Individual Optimization

The individual optimization problem for agent j in nation k is formulated as follows. Let b_j be the quota of emission credits for agent j $\left(\sum_{j=1}^{m} b_j = a\right)$.

$$(\mathrm{P}_4) \quad \begin{cases} G_j(u_j, v_j) = R_j(u_j) - C_j(v_j) - \lambda^*(u_j - v_j - b_j) \quad \to \quad \max\limits_{u_j, v_j} \\ subject \quad to \\ u_j, v_j \ge 0. \end{cases} \tag{34}$$

The problem (P_4) has the same for m as the problem (P_2) and therefore the following theorem is clear.

THEOREM4. The optimal solution of the problem (P_4) is as follows:

$$\tilde{u}_j = \left[R_j'^{-1}(\lambda^*)\right]^+ \qquad j = 1,2,\cdots,m \tag{35}$$

$$\tilde{v}_j = \left[C_j'^{-1}(\lambda^*)\right]^+ \qquad j = 1,2,\cdots,m. \tag{36}$$

3.3 Surcharge and Bounty

From Theorem 3 and 4, we know that in the domestic market the result of the individual optimization is different from one of the total (social) optimization. In order to accomplish both optimizations simultaneously, we introduce surcharge and bounty. Let $r(\geq 0)$ be the surcharge for discharging a unit of greenhouse gas. Let $s(\geq 0)$ be the bounty for removing a unit of harm. We want to obtain the optimal amounts of surcharge and bounty for the simultaneous accomplishment of both optimizations. Since the surcharge and bounty are the give and take between the government and a company, the total (social) net profit of nation k is invariant in spite of introducing surcharge and bounty. Then the total optimization problem with surcharge and bounty is the same as the problem (P_3). On the other hand, the individual optimization problem of agent j is as follows:

$$(P_4') \begin{cases} G_j(u_j,v_j)=R_j(u_j)-C_j(v_j)-\lambda^*(u_j-v_j-b_j)-ru_j+sv_j \quad \to \max_{u_j,v_j} & (37) \\ \text{subject to} \\ u_j,v_j \geq 0. \end{cases}$$

The following theorem is clear.

THEOREM5. The optimal solution of the problem (P_4') is as follows:

$$\tilde{u}_j = \tilde{u}_j(r,s) = \left[R_j'^{-1}(\lambda^*+r)\right]^+ \qquad j=1,2,\cdots,m \tag{38}$$

$$\tilde{v}_j = \tilde{v}_j(r,s) = \left[C_j'^{-1}(\lambda^*+s)\right]^+ \qquad j=1,2,\cdots,m. \tag{39}$$

Considering conditions that the result of Theorem 3 coincides with one of Theorem 5, the following theorem is clear.

THEOREM6. In the domestic market, in order that the individual optimization accomplishes the total (social) optimization, the optimal surcharge r^* and the optimal bounty s^* are as follows:

$$r^* = \mu^* - \lambda^* \tag{40}$$

$$s^* = \xi^* - \lambda^* \tag{41}$$

where μ^* and ξ^* are given by Theorem 3.

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