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The characterization of the sampling set for Bloch-type spaces

小樽商科大学商学部 米田 力生 (Rikio Yoneda)

Otaru University of Commerce

§1. Introduction

Let D be the open unit disk in complex plane C. For $z,w\in D$, $0<r<1$, let $\varphi_w(z)=\dfrac{w-z}{1-\overline{w}z}$ and let $\rho(z,w)=\left|\dfrac{w-z}{1-\overline{w}z}\right|$ and $D(w,r)=\{z\in D,\rho(w,z)<r\}$. Let $H(D)$ be the space of all analytic functions on D.

For $\alpha>0$, the space L_α^∞ is defined to be the Banach space of Lebesgue measurable functions f on the open unit disk D with

$$\| f \|_{L_\alpha^\infty}=\sup_{z\in D}(1-|z|^2)^\alpha |f(z)|<+\infty.$$

Note that $L_0^\infty=L^\infty$ is the classical Banach space of Lebesgue measurable functions f on the open unit disk D with

$$\| f \|_{L^\infty}=\sup_{z\in D}|f(z)|<+\infty.$$

The space of bounded analytic functions on D will be denoted by H^∞.

The space $\mathcal{B}_\alpha$ of D is defined to be the space of analytic functions f on D such that

$$\| f \|_{\beta_\alpha}=|f(0)|+\| f \|_{\mathcal{B}_\alpha}<+\infty ,$$

where $\| f \|_{\mathcal{B}_\alpha}=\sup_{z\in D}(1-|z|^2)^\alpha |f'(z)|$. Note that $\mathcal{B}_1=\mathcal{B}$ is the Bloch space .

For $\alpha>0$, the Bloch-type space $\mathcal{B}^\alpha$ of D is defined by

$$\mathcal{B}^\alpha=H(D)\cap L_\alpha^\infty.$$

For $p > 0$, the space $L^2\left((1-|z|^2)^p dA(z)\right)$ is defined to be the space of Lebesgue measurable functions f on D such that

$$\| f \|_{L^2((1-|z|^2)^p dA(z))} = \left\{ \int_D |f(z)|^2 (p+1)(1-|z|^2)^p dA(z) \right\}^{\frac{1}{2}} < +\infty,$$

where $dA(z)$ denote the area measure on D.

The weighted Bergman space $L^2_a\left((1-|z|^2)^p dA(z)\right)$ is defined by

$$L^2_a\left((1-|z|^2)^p dA(z)\right) = H(D) \cap L^2\left((1-|z|^2)^p dA(z)\right).$$

For $\alpha > -1$, the weighted Dirichret space D^α_p is defined to be the space of analytic functions f on D such that

$$\int_D (1-|z|^2)^\alpha |f'(z)|^p (\alpha+1) dA(z) < +\infty.$$

In the case of $\alpha = 1$ and $p = 2$, then $D^1_2 = H^2$ is the Hardy space. In the case of $\alpha = 2$ and $p = 2$, then $D^2_2 = L^2_a$ is the Bergman space.

For g analytic on D, the operators I_g, J_g, M_g are defined by the following:

$$I_g(f)(z) = \int_0^z g(\zeta) f'(\zeta) d\zeta \ , \ J_g(f)(z) = \int_0^z f(\zeta) g'(\zeta) d\zeta \ , \ M_g(f)(z) = g(z) f(z).$$

If $g(z) = z$, then J_g is the integration operator. If $g(z) = \log \frac{1}{1-z}$, then J_g is the Cesáro operator.

Let X, Y be Banach spaces and let T be a linear operator from X into Y. Then T is called to be bounded below from X to Y if there exists a positive constant $C > 0$ such that $\| Tf \|_Y \geq C \| f \|_X$ for all $f \in X$, where $\| * \|_X$, $\| * \|_Y$ be the norm of X, Y, respectively.

And the operator norm is defined by the following:

$$\| T \|_{\mathrm{sup}, X \to Y} = \sup_{\|x\|_X = 1} \| Tx \|_Y .$$

And we also define the operator infimum norm by the following:

$$\| T \|_{\mathrm{inf}, X \to Y} = \inf_{\|x\|_X = 1} \| Tx \|_Y .$$

In [7] G.McDonald and C.Sundberg proved the following result:

Theorem (G.McDonald and C.Sundberg).([7]) *For any inner function φ, the following are equivalent:*

(i) *The operator $M_\varphi : L_a^2 \to L_a^2$ is bounded below.*

(ii) *φ is a finite product of interpolating Blaschke products.*

In [1] Paul S.Bourdon proved the following result :

Theorem (Paul S.Bourdon).([1]) *Let $h \in H^\infty$. The operator $M_h : L_a^2 \to L_a^2$ is bounded below if and only if $h = \varphi F$, where $F, 1/F \in H^\infty$ and where φ is a finite product of interpolating Blaschke products.*

In [2] J.Bonet, P.Domanski and M.Lindstrom proved the following result :

Theorem (J.Bonet, P.Domanski and M.Lindstrom). ([2]) *Let $h \in H^\infty$. If v is a positive radial function in the unit disc D, and $-\Delta \log v(z) \asymp (1-|z|^2)^{-2}$, then the operator M_h is bounded below on $H_v^\infty = \{f \in H(D) : \sup_{z\in D}|f(z)|v(z) < \infty\}$ if and only if $h = \varphi F$, where $F, 1/F \in H^\infty$ and where φ is a finite product of interpolating Blaschke products.*

The above results do only hold for the multiplication operator from the Bergman space L_a^2 to the same Bergman space L_a^2 (from the weighted space H_v^∞ to the same weighted space H_v^∞). In [12] we study when the operator M_h is bounded below from the weighted Bergman spaces $L_a^2\left((1-|z|^2)^\alpha dA(z)\right)$ to the another weighted Bergman spaces $L_a^2\left((1-|z|^2)^\beta dA(z)\right)$using sampling set for the Bloch-type space. Moreover we make a sampling set for $\mathcal{B}^\alpha$ using a finite (product of interpolating) Blaschke products.

§2. Statement of main results.

Definition 2.1. *Let $\alpha > 0$. A set Γ of the open unit disk D is called a sampling set for $\mathcal{B}^\alpha$ if there exists a positive constant $C > 0$ such that*

$$\sup_{z\in D}(1-|z|^2)^\alpha|f(z)| \le C \sup_{z\in\Gamma}(1-|z|^2)^\alpha|f(z)|,$$

for all $f \in \mathcal{B}^\alpha$.

Definition 2.2. *Let $\alpha > 0$. A set Γ of the open unit disk D is called a sampling set for $\mathcal{B}_\alpha$ if there exists a positive constant $C > 0$ such that*

$$\sup_{z\in D}(1-|z|^2)^\alpha|f'(z)| \le C\sup_{z\in\Gamma}(1-|z|^2)^\alpha|f'(z)|,$$

for all $f \in \mathcal{B}_\alpha$.

By using a sampling set for $\mathcal{B}_\alpha$, we can prove the following result with respect to the operator I_g:

Theorem 2.3.([13]) *Let $\beta \ge \alpha > 0$ and $g \in H(D)$. The operator $I_g : \mathcal{B}_\alpha \to \mathcal{B}_\beta$ is bounded. Then the operator $I_g : \mathcal{B}_\alpha \to \mathcal{B}_\beta$ is bounded below if and only if there exists a positive constant $(1 >)\epsilon > 0$ such that $\{z \in D, (1-|z|^2)^{\beta-\alpha}|g(z)| \ge \epsilon\}$ is a sampling set for $\mathcal{B}_\alpha$.*

Corollary 2.4. ([13]) *Let $\alpha > 0$ and $g \in H(D)$. The operator I_g is bounded on $\mathcal{B}_\alpha$. Then the operator I_g is bounded below on $\mathcal{B}_\alpha$ if and only if there exists a positive constant $\epsilon > 0$ such that $\{z \in D, |g(z)| \ge \epsilon\}$ is a sampling set for $\mathcal{B}_\alpha$.*

By using a sampling set for $\mathcal{B}^\alpha$, we can prove the following result with respect to the operator J_g:

Theorem 2.5.([13]) *Let $\beta \ge \alpha > 1$ and $g \in H(D)$. The operator $J_g : \mathcal{B}_\alpha \to \mathcal{B}_\beta$ is bounded. Then the operator $J_g : \mathcal{B}_\alpha \to \mathcal{B}_\beta$ is bounded below if and only if there exists a positive constant $\epsilon > 0$ such that $\{z \in D, (1-|z|^2)^{\beta-\alpha+1}|g'(z)| \ge \epsilon\}$ is a sampling set for $\mathcal{B}^{\alpha-1}$.*

Corollary 2.6.([13]) *Let $\alpha > 1$ and $g \in H(D)$. The operator J_g is bounded on $\mathcal{B}_\alpha$. Then the operator J_g is bounded below on $\mathcal{B}_\alpha$ if and only if* *there exists a positive constant* $\epsilon > 0$ *such that* $\{z \in D, (1-|z|^2)|g'(z)| \geq \epsilon\}$ *is a sampling set for* $\mathcal{B}^{\alpha-1}$.

Definition 2.7.([13]) *The space BMOA is defined to be the space of analytic functions f on D such that* $\sup_{a\in D}\int_D (1-|\varphi_a(z)|^2)|f'(z)|^2 dA(z) < +\infty$.
In the case of $0 < \alpha < 1$, *The space* Q_α *is defined to be the space of analytic functions f on D such that* $\sup_{a\in D}\int_D (1-|\varphi_a(z)|^2)^\alpha |f'(z)|^2 dA(z) < +\infty$.

Proposition 2.8. ([13]) *Let* $g \in H^\infty$. *If the operator* $I_g : H^2 \to H^2$ *is bounded below, then* $I_g : BMOA \to BMOA$ *is bounded below. If the operator* I_g $: L^2_a \to L^2_a$ *is bounded below, then* $I_g : \mathcal{B} \to \mathcal{B}$ *is bounded below. For* $0 < p < 1$, *if the operator* $I_g : D^\alpha_2 \to D^\alpha_2$ *is bounded below, then* $I_g : Q_\alpha \to Q_\alpha$ *is bounded below.*

We determined the integration operators I_g on the Bergman spaces that have a closed range using sampling set for $\mathcal{B}$.

Theorem 2.9. ([13]) *Suppose that* $g \in H^\infty$. *Then there is a constant* $k > 0$ *such that*

$$\int_D |f'(z)|^2|g(z)|^2(1-|z|^2)^2 dA(z) \geq k\int_D |f'(z)|^2(1-|z|^2)^2 dA(z)$$

for all $f \in L^2_a$ *if and only if there exists a positive constant* $\epsilon > 0$ *such that* $\{z \in D, |g(z)| \geq \epsilon\}$ *is a sampling set for* $\mathcal{B}$.

Remark 2.10. ([13]) Carefully examining the proof of the above theorem, we see the following are also the equivalent conditions respectively:

$\sup_{z\in D}(1-|z|^2)|g(z)\varphi'_w(z)| \geq C$ for all $w \in D$.

For any $\epsilon < C$, $\rho(\Gamma, w) \leq R < 1$ for all $w \in D$, R depending only on ϵ, where $\Gamma = \{z \in D, |g(z)| \geq \epsilon\}$.

Proposition 2.11. ([13]) *Let $g \in \mathcal{B}$. If $J_g : L_a^2\left((1-|z|^2)^2\, dA(z)\right) \to L_a^2\left((1-|z|^2)^2\, dA(z)\right)$ is bounded below, then $J_g : \mathcal{B}_2 \to \mathcal{B}_2$ is bounded below.*

We determined the integration operators J_g on the weighted Bergman spaces that have a closed range using sampling set for $\mathcal{B}^1$.

Theorem 2.12. ([13]) *Suppose that $g \in \mathcal{B}$. Then there is a constant $k > 0$ such that*

$$\int_D |f(z)|^2|g'(z)|^2(1-|z|^2)^4 dA(z) \geq k \int_D |f'(z)|^2(1-|z|^2)^4 dA(z)$$

for all $f \in L_a^2\left((1-|z|^2)^2\, dA(z)\right)$ if and only if there exists a positive constant $\epsilon > 0$ such that $\{z \in D, (1-|z|^2)|g'(z)| \geq \epsilon\}$ is a sampling set for $\mathcal{B}^1$.

Remark 2.13.([13]) Carefully examining the proof of the above theorem, we see the following are also the equivalent conditions respectively:

$\sup_{z\in D}(1-|z|^2)^2|g'(z)\varphi_w'(z)| \geq C$ for all $w \in D$.

For any $\epsilon < C$, $\rho(\Gamma, w) \leq R < 1$ for all $w \in D$, R depending only on ϵ, where $\Gamma = \{z \in D, (1-|z|^2)|g'(z)| \geq \epsilon\}$.

In [6] D.Leucking proved the following result:

Theorem (D.Leucking). ([6]) *Let $\alpha > -1$. There is a constant $C > 0$ such that*

$$\int_D |f(z)|^2(1-|z|^2)^\alpha dA(z) \leq C \int_G |f(z)|^2(1-|z|^2)^\alpha dA(z)$$

for all $f \in L_a^2((1-|z|^2)^\alpha dA(z))$ if and only if a subset G of D satisfy the condition that there exist $\delta > 0$ and $r > 0$ such that $\delta|D(a,r)| \leq |D(a,r) \cap G|$, where $|D(a,r)|$ is the (normalized) area of $D(a,r)$.

Using a sampling set for $\mathcal{B}^{\alpha}$, in [12], we proved the following result with respect to the operator M_h:

Theorem 3.1. ([12]) *Let* $0 < \alpha \leq \beta$ *and* $h \in H(D)$ *with* $\sup_{z\in D}(1-|z|^2)^{\beta-\alpha}|h(z)| < +\infty$. *Then* $M_h : \mathcal{B}^{\alpha} \to \mathcal{B}^{\beta}$ *is bounded below if and only if there exists a positive constant* $\epsilon > 0$ *such that* $\Gamma = \{z \in D, (1-|z|^2)^{\beta-\alpha}|h(z)| \geq \epsilon\}$ *is a sampling set for* $\mathcal{B}^{\alpha}$.

Next we study when the operator M_h is bounded below from $L^2_a\left((1-|z|^2)^{2\alpha}dA(z)\right)$ to $L^2_a\left((1-|z|^2)^{2\beta}dA(z)\right)$ using sampling set for the Bloch-type space.

In [14] R.Zhao proved the following :

Theorem (R.Zhao). *Let* $\alpha > 0$. *Let* $f \in H(D)$. *Then* $f \in \mathcal{B}^{\alpha}$ *if and only if*

$$\sup_{a\in D}\int_D (1-|z|^2)^{2\alpha-2}(1-|\varphi_a(z)|^2)^2|f(z)|^2 dA(z) < +\infty.$$

i.e. $\| f \|^2_{\mathcal{B}^{\alpha}}$ *is comparable to* $\sup_{a\in D}\int_D (1-|z|^2)^{2\alpha-2}(1-|\varphi_a(z)|^2)^2|f(z)|^2 dA(z)$.

To prove Theorem 3.3, in [12], we proved the following result with respect to the multiplication operator M_h for $h \in H(D)$ with $\sup_{z\in D}(1-|z|^2)^{\beta-\alpha}|h(z)| < +\infty$:

Theorem 3.2. ([12]) *Suppose* $\sup_{z\in D}(1-|z|^2)^{\beta-\alpha}|h(z)| < +\infty$ *and* $0 < \alpha \leq \beta$. *If* $M_h : L^2_a\left((1-|z|^2)^{2\alpha}dA(z)\right) \to L^2_a\left((1-|z|^2)^{2\beta}dA(z)\right)$ *is bounded below, then* $M_h : \mathcal{B}^{\alpha} \to \mathcal{B}^{\beta}$ *is bounded below.*

We determine the multiplication operator M_h on the weighted Bergman spaces that have a closed range using sampling set for $\mathcal{B}^{\alpha}$.

Theorem 3.3.([12]) *Let $\beta \geq \alpha > 0$. Suppose $h \in H(D)$ with $\sup_{z\in D}(1-|z|^2)^{\beta-\alpha}|h(z)| < +\infty$. Then the following are equivalent.*

(1) $M_h : L_a^2((1-|z|^2)^{2\alpha}dA(z)) \to L_a^2\left((1-|z|^2)^{2\beta}dA(z)\right)$ *is bounded below i.e. there is a constant $k > 0$ such that*

$$\left\{\int_D |M_h f(z)|^2(1-|z|^2)^{2\beta}dA(z)\right\}^{\frac{1}{2}} \geq k\left\{\int_D |f(z)|^2(1-|z|^2)^{2\alpha}dA(z)\right\}^{\frac{1}{2}}$$

for all $f \in L_a^2((1-|z|^2)^{2\alpha}dA(z))$.

(2) $M_h : \mathcal{B}^\alpha \to \mathcal{B}^\beta$ *is bounded below.*

(3) *There exists a positive constant $\epsilon > 0$ such that $\{z \in D, (1-|z|^2)^{\beta-\alpha}|h(z)| \geq \epsilon\}$ is a sampling set for $\mathcal{B}^\alpha$.*

Applying the inner function φ to the above theorem, and applying $\alpha = \beta$, the following result holds.

Corollary 3.4.([12]) *Let $\alpha > 0$. For any inner function φ, the following are equivalent.*

(1) M_φ *is bounded below on* $L_a^2((1-|z|^2)^{2\alpha}dA(z))$

(2) *There exists a positive constant $\epsilon > 0$ such that $\{z \in D, |\varphi(z)| \geq \epsilon\}$ is a sampling set for $\mathcal{B}^\alpha$.*

(3) $\inf_{w\in D}\left\{\sup_{z\in D}(1-|\varphi_w(z)|^2)^\alpha|\varphi(z)|\right\} > 0.$

(4) *φ is a finite product of interpolating Blaschke products.*

(5) *For some $\epsilon > 0$ and $0 < r < 1$ the area of the subset of the disc $D_{z,r} = \{w : \rho(z,w) < r\}$ where $|\varphi(w)| > \epsilon$ is comparable to the area of the whole disc $D_{z,r}$, $z \in D$.*

Remark 3.5.([12]) *Let $\beta \geq \alpha > 0$. Suppose $h \in H(D)$ with $\sup_{z\in D}(1-|z|^2)^{\beta-\alpha}|h(z)| < +\infty$.* Then we see that

$$\| M_h \|_{\inf, L_a^2((1-|z|^2)^{2\alpha}dA(z)) \to L_a^2((1-|z|^2)^{2\beta}dA(z))} \approx \| M_h \|_{\inf, \mathcal{B}^\alpha \to \mathcal{B}^\beta} \approx \inf_{w\in D}\left\{\sup_{z\in D}(1-|\varphi_w(z)|^2)^\alpha(1-|z|^2)^{\beta-\alpha}|\varphi(z)|\right\}. \tag{3.5}$$

K.Seip gives a complete description of interpolating sequences and sampling set (sequences) in terms of (Seip's) densities. Without using (Seip's) densities, we now generate a sampling set for $\mathcal{B}^\alpha$ using a finite (product of interpolating)Blaschke products.

Theorem 3.6. ([12]) *Let $0 < k < 1$ be some suitable constant (to satisfy the equivalence of (3.5)) and put*

$$\epsilon = k \inf_{w\in D}\Big\{\sup_{z\in D}(1-|\varphi_w(z)|^2)^\alpha|\varphi(z)|\Big\}.$$

Suppose φ is a finite product of interpolating Blaschke products and then $\{z_n\}_n$ in the open unit disk D is a finite union of interpolating Blaschke sequences. Then $D\setminus\bigcup_{n=1}^{\infty}\Big\{z\in D : \Big|\frac{z_n-z}{1-\overline{z_n}z}\Big| < \epsilon\Big\}$ is a sampling set for $\mathcal{B}^\alpha$.

Remark 3.7. Carefully examining the proof of Theorem 3.6, in fact, the sampling set for $\mathcal{B}^\alpha$ in Theorem 3.6 can be generated more smaller than it. So we reserch it for finite Blaschke products.

The following result is the case of $\varphi(z) = \varphi_{z_n}(z) = \frac{z_n-z}{1-\overline{z_n}z}$ for arbitrary $z_n \in D$.

Theorem 3.8.([12]) *Let $\alpha > 0$. For arbitrary point $z_n \in D$, let $\varphi(z) = \varphi_{z_n}(z) = \frac{z_n-z}{1-\overline{z_n}z}$. Then $\inf_{w\in D}\big\{\sup_{z\in D}(1-|\varphi_w(z)|^2)^\alpha|\varphi(z)|\big\} = \big(\frac{2\alpha}{2\alpha+1}\big)^\alpha\sqrt{\frac{1}{2\alpha+1}}$ and for some suitable constant $0 < k < 1$(to satisfy the equivalence of (3.5)), the set $D\setminus\big\{z\in D : \rho(z_n,z) < k\big(\frac{2\alpha}{2\alpha+1}\big)^\alpha\sqrt{\frac{1}{2\alpha+1}}\big\}$ is a sampling set for $\mathcal{B}^\alpha$.*

The following result is the case of $\varphi(z) = \frac{z_{n_1}-z}{1-\overline{z_{n_1}}z}\frac{z_{n_2}-z}{1-\overline{z_{n_2}}z}$ for arbitrary $z_{n_1}, z_{n_2} \in D$.

Theorem 3.9.([12]) *Let $\alpha > 0$. For some points $z_{n_1}, z_{n_2} \in D$, let $\varphi(z) = \frac{z_{n_1}-z}{1-\overline{z_{n_1}}z}\frac{z_{n_2}-z}{1-\overline{z_{n_2}}z}$.*

Then $\Gamma = \inf_{w\in D}\left\{\sup_{z\in D}(1-|\varphi_w(z)|^2)^\alpha|\varphi(z)|\right\}$ *depends on* $\varphi_{z_{n_1}}(z_{n_2})$ *and for some suitable constant* $0 < t < 1$(*to satisfy the equivalence of* (3.5)), *the set* $D\setminus\left(\left\{z\in D:\rho(z_{n_1},z)<T\right\}\cup\left\{z\in D:\rho(z_{n_2},z)<T\right\}\right)$ *is a sampling set for* $\mathcal{B}^\alpha$, *where* $T = \frac{-\rho(z_{n_1},z_{n_2})(1-\Gamma)+\sqrt{\rho(z_{n_1},z_{n_2})^2\cdot(1-\Gamma)^2+4\Gamma}}{2}$

The following result is well-known([3]):

Theorem.([3 , p.287-293]) *For a sequence* $\{z_n\}$ *in the open unit disk* D, *the following are equivalent:*
(1) $\{z_n\}$ *is an interpolating Blaschke sequence,*
(2) $\{z_n\}$ *satisfies the condition*

$$\inf_n \prod_{j\neq n} \rho(z_j, z_n) > 0,$$

(3) $\{z_n\}$ *satisfies the condition*

$$\inf_n (1-|z_n|^2)\,|B'(z_n)| > 0,$$

where $B(z)$ *is a Blaschke product with* $\{z_n\}$ *as its zero sequence.*

Theorem 3.10.([12]) *Let* $\alpha > 0$ *and* $k \geq 3$. *For some points* $z_{n_1}, z_{n_2}, z_{n_3}, \cdots z_{n_k} \in D$, *let* $\varphi(z) = \frac{z_{n_1}-z}{1-\overline{z_{n_1}}z}\frac{z_{n_2}-z}{1-\overline{z_{n_2}}z}\frac{z_{n_3}-z}{1-\overline{z_{n_3}}z}\cdots\frac{z_{n_k}-z}{1-\overline{z_{n_k}}z}$. *Then* $\Gamma = \inf_{w\in D}\left\{\sup_{z\in D}(1-|\varphi_w(z)|^2)^\alpha|\varphi(z)|\right\}$ *depends on* $\varphi_{z_{n_i}}(z_{n_1}), \varphi_{z_{n_i}}(z_{n_2}), \cdots \varphi_{z_{n_i}}(z_{n_k})$. *For some suitable constant* $0 < t < 1$(*to satisfy the equivalence of* (3.5)), *the set* $D\setminus\bigcup_{i=1}^{k}\left\{z\in D:\rho(z_{n_i},z)<T_i\right\}$ *is a sampling set for* $\mathcal{B}^\alpha$, *where the constant* T_i *depends on* $\varphi_{z_{n_i}}(z_{n_1}), \varphi_{z_{n_i}}(z_{n_2}), \cdots \varphi_{z_{n_i}}(z_{n_k})$, t *and* Γ.

Remark 3.11.([12]) In Theorem 3.8, the radius of the disk to be removed is a constant $\left(k\left(\frac{2\alpha}{2\alpha+1}\right)^\alpha\sqrt{\frac{1}{2\alpha+1}}\right)$ (independent of the center of the disk to be removed). In Theorem 3.9, the radius T of the disk to be removed is a constant (independent of the center of the disk to be removed, only dependent on $\varphi_{z_{n_1}}(z_{n_2})$). But, in Theorem 3.10, the radius T_i of the disk to be removed is a non-constant that depend on the center $\{z_{n_i}\}$ of the disk. With respect to the Bloch space and the (nonweighted) Bergman spaces, we also see that the results corresponding to Theorem 3.6, Theorem 3.8, Theorem 3.9,Theorem 3.10 hold by using Theorem 2.3, corollary 2.4, and Theorem 2.9.

The constant $\Gamma = \inf_{w\in D}\left\{\sup_{z\in D}(1-|\varphi_w(z)|^2)^\alpha|\varphi(z)|\right\}$ resemble in

$$\Gamma' := \inf_n \prod_{j\neq n} \rho(z_j, z_n)$$

and

$$\Gamma'' := \inf_n (1-|z_n|^2)\,|B'(z_n)|\,,$$

where $B(z)$ is a Blaschke product with $\{z_n\}$ as its zero sequence. If we can compare this Γ to these constants Γ' Γ'', we will know the relationship between the sampling set and interpolation sequences moreover.

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