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d-primitive words and D(1)-concatenated words

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Abstract

In this paper, we study d-primitive words and D(1)-concatenated words. It is shown that neither $D(1)$, the set of all primitive words, nor $D(1)D(1)$, the set of all $D(1)$-concatenated words, is regular. We also show that every d-primitive word, with the length of two or more, is $D(1)$-concatenated.

1 Introduction

The notion of primitivity of words plays a central role in algebraic coding theory and combinatorial theory of words (See [3] [4], and [6]).

Recently attention has been drawn to $D(1)$, the set of all d-primitive words, which is a proper subset of Q, the set of all primitive words. ([1], [7]) In this paper, we study languages $D(1)$ and $D(1)D(1)$, the set of all $D(1)$-concatenated words. We consider regularity of $D(1)$ and $D(1)D(1)$, and a relation between $D(1)$ and $D(1)D(1)$.

In section 2 some basic definitions and results are presented. In section 3, the following results are proved. (1) Neither $D(1)$ nor $D(1)D(1)$ is regular. (2) For $u, v, w \in \Sigma^+$ with $|u| = |v|$, $uvw \in D(1)$ if and only if $uv^+w \subseteq D(1)$. In section 4, we consider a relation between $D(1)$ and $D(1)D(1)$. It is proved that for a word w in $D(1)$, with the length of two or more, w is in $D(1)D(1)$.

2 Preliminaries

Let Σ be an alphabet. Σ^* denotes the free moniod generated by Σ, that is, the set of all finite words over Σ, including the empty word ϵ, and $\Sigma^+ = \Sigma^* - \epsilon$. For w in Σ^* , $|w|$ denotes the length of w. Any subset of Σ^* is called a *language* over Σ.

For a word $u \in \Sigma^+$, if $u = vw$ for some $v, w \in \Sigma^*$, then v (w) is called a *prefix (suffix)* of u, denoted by $v \leq_p u$ $(w \leq_s u$, *resp.*). If $v \leq_p u$ $(w \leq_s u)$ and $u \neq v(w \neq u)$, then v (w) is called a *proper prefix (proper suffix)* of u, denoted by $v <_p u$ $(w <_s u$, resp.). For a word w, let $Pref(w)$ $(Suff(w))$ be the set of all prefixes (suffixes, resp.) of w.

A nonempty word u is called a *primitive word* if $u = f^n$, for some $f \in \Sigma^+$, and some $n \geq 1$ always implies that $n = 1$. Let Q be the set of all primitive words over Σ. A nonempty word u is a *non-overlapping word* if $u = vx = yv$ for some $x, y \in \Sigma^+$

always implies that $v = \epsilon$. Let $D(1)$ be the set of all non-overlapping words over Σ. A words in $D(1)$ is also called a *d-primitive word.* For $u \in \Sigma^+$, u is said to be $D(1)$-concatenated if there exist $x, y \in D(1)$ such that $xy = u$, i.e., $u \in D(1)D(1)$. (See [1] and [5]).

For $w \in \Sigma^+$ with $|w| \geq 2$, $Hlvs(w)$ is defined as follows. If $w = xy$ for $x, y \in \Sigma^*$, with $|x| = |y|$, then $Hlvs(w) = (x, y)$. If $w = xcy$, for $x, y \in \Sigma^*$, $c \in \Sigma$, with $|x| = |y|$, then $Hlvs(w) = (x, y)$. For $x, y \in \Sigma^+$, if $(Pref(x) - \{\epsilon\}) \cap (Suff(y) - \{\epsilon\}) = \phi$, then (x, y) is said to be a non-overlapping pair (n-o. pair).

Lemma 1 *([2]) Let $u \in \Sigma^+$. Then $u \notin D(1)$ iff there exists a unique word $v \in D(1)$ with $|v| \leq (1/2)|u|$ such that $u = vwv$ for some $w \in \Sigma^*$.*

Remark 1 *Let $u, v \in \Sigma^+$. Obviously $uv \in$ D(1) implies that (u, v) is a n-o. pair. The converse does not hold; for $u =$ abbbba, and $v =$ bb, (u, v) is a n-o. pair but uv is not in D(1). However, in the next Proposition, we show the above two are equivalent on the condition that u and v are in D(1).*

Proposition 2 *For $u \in \Sigma^+$, the following two are equivalent.*

(1) u, v, uv, and vu are in D(1).

(2) u, v are in D(1), and (u, v), (v, u) are n-o. pairs.

The next lemma is immediate by Lemma 1

Lemma 3 *(1) For a n-o. pair (x, y) and $c \in \Sigma$, with $|x| = |y|$, both xy and xcy are in $D(1)$. (2) Let $w \in D(1)$. For every $x \in Pref(w) - \{\epsilon\}$ and $y \in Suff(w) - \{\epsilon\}$, (x, y) is a n-o.pair.*

3 Regularity of D(1) and D(1)-concatenated words

Proposition 4 *$D(1)$ is not regular.*

Proposition 5 *$D(1)D(1)$ is not regular.*

Proposition 6 *Let $|u| = |w|$ for $u, v, w \in \Sigma^+$. Then $uvw \in D(1)$ if and only if $uv^+w \subseteq D(1)$.*

Remark 2 *Unfortunately, the previous proposition does not hold without the condition $|u| = |w|$. For example, let $u = babaa$, $v = ba$, and $w = a$. Then $uvw = bavaabaa \in D(1)$, but $uv^2w = (babaa)^2 \notin D(1)$.*

4 d-primitive words and D(1)-concatenated words

In this section we consider a relation between primitive words and D(1)-concatenated words.

Lemma 7 *Let $zxyx$ be in $D(1)$ for $z, x \in \Sigma^+$, $y \in \Sigma^*$. If z is in $D(1)$, then zx is also in $D(1)$.*

Proposition 8 *Let $|w| \geq 2$ for $w \in \Sigma^+$. If $w \in D(1)$, then w is a D(1)-concatenated word. In other words, for a word w in $D(1)$, with the length of two or more, w is in $D(1)D(1)$.*

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