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An approach to robust minimax receding horizon control problems (Robust minimax receding horizon 制御問題の一解法)

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ABSTRACT

In this paper, an approach to robust finite RHC (receding horizon control) problem of constrained systems with structured uncertainties and bounded disturbances is developed. The problem is formulated as a minimax optimization problem of quadratic cost function with bounded constraint conditions. The proposed approach can be expected to solve such problems effectively.

KEY WORDS

Finite RHC (receding horizon control) problem, Robustness, *S*-procedure, Minimax optimization, Constrained system

1. Introduction

In last few decades, receding horizon control (RHC) based on the quadratic cost criterion has been widely accepted in the process industry. In the standard RHC formulation, the current control action is obtained by solving a finite or infinite horizon quadratic cost problem at every sample time using the current state of the plant as the initial state [GAR, 89].

One of the significant merits of RHC is easy handling of constraints during the design and implementation of the controller.

On the other hand, a drawback of RHC is its explicit lack of robust property with respect to model uncertanties or disturbances since the on-line minimized cost function is defined in terms of the nominal systems. Although many method of robust control synthesis for linear systems have been proposed, the number of available work of robust RHC with constrained systems is limited. The issue of robust RHC therefore still deserves further attention[BEM, 99, MAY, 00].

A possible strategy for robust RHC is solving the so-called minimax problem, namely minimization problem over the control input of the robust performance measure maximized by plant uncertainties or disturbances. One of the early works on robust RHC was proposed by Campo and Morari [CAM, 87], and further developed by Zheng and Morari [ZHE, 93] for SISO FIR plants.

Kothare *et al.* solve minimax RHC problems with state-space uncertainties through LMIs [KOT, 96]. Cuzzola *et al.* improve the Kothare's method [KOT, 96] to reduce conservativeness in [CUZ, 01]. Furthermore other methods of minimax RHC for systems with model uncertainty can be found in [ALL, 92, LEE, 97]. There has been some works of minimax RHC for systems with external disturbances in [BEM, 98, BEM, 00, SCO, 98]. Most these methods are, however, based on infinite horizon quadratic cost functions, since it is rather hard to solve the minimax finite quadratic cost problems.

In this paper, therefore, we propose an approach to minimax finite RHC of constrained systems with structured uncertainties and disturbance. The proposed approach using S-procedure can solve finite horizon quadratic cost problem efficiently. Using this approach, we can expect to reduce the conservativeness of control performance. Moreover, this approach is one of the general framework of the minimax robust finite RHC problem of bounded constrained systems.

2. Problem formulation

Consider the following discrete-time system with disturbances

$$x(k+1) = (A + L\triangle R_A)x(k) + (B + L\triangle R_B)u(k) + \eta(k) \tag{2.1}$$
$$y(k) = Cx(k) \tag{2.2}$$

where $x(k)$, $u(k)$, $y(k)$ and $\eta(k)$ denote the state, input, measured output and disturbance vector respectively, and where $\triangle$ is a diagonal structured uncertainties parameters matrix satisfied $\Delta^T\Delta \leq I$. L, R_A and R_B are constant matrices. All these vectors and matrices have appropriate dimensions. Then, we can transform this system as

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Lw(k) + \eta(k) && (2.3)\\ z(k) &= R_A x(k) + R_B u(k) && (2.4)\\ y(k) &= Cx(k) && (2.5) \end{aligned}$$

where $w(k)(= \Delta z(k))$. We assumed that the system is constrained with following conditions;

$$\begin{aligned} w^T(k+j)P_w w(k+j) &\leq 1\\ \eta^T(k+j)P_\eta \eta(k+j) &\leq 1\\ u^T(k+j)P_u u(k+j) &\leq 1\\ z^T(k+J)P_z z(k+j) &\leq 1\\ (j = 0, \cdots, N-1)& \end{aligned} \tag{2.6}$$

where P_w, P_η, P_u ($P_w, P_u, P_\eta \succ 0$) are positive symmetric matrices for weights of constraints. For this systems, the quadratic performance measure with finite horizon with positive weighting constant matrices Q and R ($Q, R \succ 0$) as :

$$J(k) = \sum_{j=0}^{N-1} \| x(k+j+1|k) \|_Q^2 + \| u(k+j|k) \|_R^2 \tag{2.7}$$

is used. $x(k+j|k)$, $y(k+j|k)$ and $u(k+j|k)$ are the predicted state of the plant, the predicted output of the plant and the future control input at time $k+j$ respectively. Then, the design problem is formulated as the following minimax optimization problem.

$$\min_{u(k+j|k)} \max_{w(k+j|k),\eta(k+j|k)} J(k) \tag{2.8}$$

$$\begin{aligned} \textbf{subject to} \quad w^T(k+j)P_w w(k+j) &\leq 1\\ u^T(k+j)P_u u(k+j) &\leq 1\\ \eta^T(k+j)P_\eta \eta(k+j) &\leq 1\\ (j = 0, \cdots, N-1)& \end{aligned}$$

Since the saddle point may not exist in general, it is difficult to solve this problem. Hence, the objective in this paper is to elimenate the maximization procedure and transform this problem to simple minimaization problem which can be solved easily.

3. Transformation of minimax finite RHC problem

At each step k the following state feedback is employed;

$$u(k+j|k) = \begin{cases} 0 & (j=0) \\ -F_0 x(k+j|k) & (j=1,2,\cdots N-1) \end{cases} \tag{3.1}$$

where F_0 is a constant feedback matrix. Then, introducing the following vectors

$$\begin{aligned} X &:= \begin{bmatrix} x(k+1|k) & x(k+2|k) & \cdots & x(k+N|k) \end{bmatrix}^T \\ Z &:= \begin{bmatrix} z(k+1|k) & z(k+2|k) & \cdots & z(k+N|k) \end{bmatrix}^T \\ W &:= \begin{bmatrix} w(k|k) & w(k+1|k) & \cdots & w(k+N-1|k) \end{bmatrix}^T \\ \Lambda &:= \begin{bmatrix} \eta(k|k) & \eta(k+1|k) & \cdots & \eta(k+N-1|k) \end{bmatrix}^T \end{aligned}$$

and using state space equation, eqs. (2.3) $\sim$ (2.5), recursively, we can derive

$$X = \tilde{A}x(k) + \tilde{L}W + \Lambda \tag{3.2}$$

$$Z = \tilde{R}_F\tilde{A}x(k) + \tilde{R}_F\tilde{L}W + \tilde{R}_F\Lambda \tag{3.3}$$

where

$$\begin{aligned} \tilde{R}_F &:= R_A - R_B F \\ F &:= \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ -F_0 & 0 & 0 & \dots & 0 \\ 0 & -F_0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & -F_0 & 0 \end{bmatrix} \\ \tilde{A} &:= \begin{bmatrix} A \\ (A-BF_0)A \\ \vdots \\ (A-BF_0)^{N-2}A \end{bmatrix} \\ \tilde{L} &:= \begin{bmatrix} L & 0 & \dots & 0 \\ (A-BF_0)L & L & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ (A-BF_0)^{N-2}L & (A-BF_0)^{N-3}L & \dots & L \end{bmatrix} \end{aligned}$$

Hence, we can transform the minimax problem (2.8) to

$$\min_{F_0} \gamma \tag{3.4}$$

$$\begin{array}{llcrcl}
\textbf{subject to} & & \max_{W,\Lambda} \Pi & & \leq & \gamma \\
& w^T(k+j) & P_w & w(k+j) & \leq & 1 \\
& u^T(k+j) & P_u & u(k+j) & \leq & 1 \\
& \eta^T(k+j) & P_\eta & \eta(k+j) & \leq & 1 \\
& & (j=0,\cdots,N-1) & & &
\end{array}$$

where $\gamma > 0$ (scalar parameter) and where;

$$\Pi := \left\{ \| \tilde{A}x(k) + \tilde{L}W + \Lambda \|_{\hat{Q}}^2 + \| FX \|_{\hat{R}}^2 \right\},$$

$$\hat{Q} := \begin{bmatrix} Q & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & Q \end{bmatrix}, \qquad \hat{R} := \begin{bmatrix} R & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & R \end{bmatrix}$$

To eliminate the maximaization procedure, we have to remove W and Λ terms in the first constraint. For this, in the first place, following basis for all variables and transformation matrices are defined.

$$\zeta := \begin{bmatrix} x(k) & W^T & \Lambda^T & 1 \end{bmatrix}^T \tag{3.5}$$

$$X = H_x\zeta \quad (H_x := \begin{bmatrix} \tilde{A} & \tilde{L} & I & 0 \end{bmatrix}) \tag{3.6}$$

$$FX = H_u\zeta \quad (H_u := \begin{bmatrix} F\tilde{A} & F\tilde{L} & F & 0 \end{bmatrix}) \tag{3.7}$$

$$Z = H_z\zeta \quad (H_z := \begin{bmatrix} \tilde{R}_F\tilde{A} & \tilde{R}_F\tilde{L} & \tilde{\Gamma} & 0 \end{bmatrix}) \tag{3.8}$$

$$\Lambda = H_\eta\zeta \quad (H_\eta := \begin{bmatrix} \mathbf{0} & \mathbf{0} & I & 0 \end{bmatrix}) \tag{3.9}$$

$$1 = (H_1\zeta)^T(H_1\zeta) \quad (H_1 := \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}) \tag{3.10}$$

By using these, we can express the first constraint condition of problem (3.4) ;

$$\max_{W,\Lambda} \left\{ \| H_x\zeta \|_{\hat{Q}}^2 + \| H_u\zeta \|_{\hat{R}}^2 \right\} \leq (H_1\zeta)^T\lambda(H_1\zeta) \tag{3.11}$$

Please take notice that both the left side and the right side of this inequality are expressed by the quadratic forms and they have positive scalar values. Hence, if the inequality is hold by maximum values of W and Λ in left side, this inequality must be hold by any other values of them. This fact means that we can eliminate the maximization procedure in the first constraint. We can only check the following condition instead of the first constraint of problem (3.4).

$$\left\{\| H_x\zeta \|_{\hat{Q}}^2 + \| H_u\zeta \|_{\hat{R}}^2\right\} \leq (H_1\zeta)^T\lambda(H_1\zeta) \tag{3.12}$$

In the second place, $H_w(j)$ is defined. This matrix pick out the suitable block from W and satisfy the relation of $w(k+j) = H_w^{(j)}\zeta$. Then, we can derive

$$\begin{array}{rcl} (H_w^{(j)}\zeta)^T P_w (H_w^{(j)}\zeta) & \leq & (H_1\zeta)^T(H_1\zeta) \\ & (j = 0, \cdots, N-1). & \end{array} \tag{3.13}$$

For the constraints of η, u and z, we can derive the following relations in the same way.

$$\begin{array}{rcl} (H_\eta^{(j)}\zeta)^T P_\eta (H_\eta^{(j)}\zeta) & \leq & (H_1\zeta)^T(H_1\zeta) \\ (H_u^{(j)}\zeta)^T P_u (H_u^{(j)}\zeta) & \leq & (H_1\zeta)^T(H_1\zeta) \\ & (j = 0, \cdots, N-1). & \end{array} \tag{3.14}$$

Furthermore, by using (3.5) $\sim$ (3.10), all constraints in minimax problem (3.4) can be transformed into

$$\forall\zeta \neq 0 \; ; \; \zeta^T \left(H_1^T\lambda H_1 - H_x^T\hat{Q}H_x - H_u^T\hat{R}H_u\right)\zeta \geq 0 \tag{3.15}$$

$$\textbf{subject to} \quad \begin{array}{rcl} \zeta^T(H_1^TH_1 - (H_w^{(j)})^T P_w H_w^{(j)})\zeta & \geq & 0 \\ \zeta^T(H_1^TH_1 - (H_u^{(j)})^T P_u H_u^{(j)})\zeta & \geq & 0 \\ \zeta^T(H_1^TH_1 - (H_\eta^{(j)})^T P_\eta H_\eta^{(j)})\zeta & \geq & 0 \\ & (j = 0, \cdots, N-1)\,. & \end{array} \tag{3.16}$$

Then, we can transform the original minimax problem (2.8) to the following one by using $\mathcal{S}$-procedure [BOY, 91].

$$\min_{F_0} \gamma \tag{3.17}$$

$$\textbf{subject to} \quad \begin{array}{l} H_1^T\gamma H_1 - H_x^T\hat{Q}H_x - H_u^T\hat{R}H_u \\ \quad - \displaystyle\sum_{j=0}^{N-1} [\tau_j^w S_j^w + \tau_j^u S_j^u + \tau_j^\eta S_j^\eta] \succeq 0 \\ \\ (j = 0, \cdots, N-1) \end{array}$$

where

$$\begin{aligned} S_j^w &= (H_1^T H_1 - (H_w^{(j)})^T P_w H_w^{(j)}) , \\ S_j^u &= (H_1^T H_1 - (H_u^{(j)})^T P_u H_u^{(j)}) , \\ S_j^\eta &= (H_1^T H_1 - (H_\eta^{(j)})^T P_\eta H_\eta^{(j)}) , \end{aligned}$$

and where τ_j^w, τ_j^u, τ_j^η and τ_j^z are positive semi-definite scalars. It must be noted that this transformation satisfies only a sufficent condition of S-procedure, since S-procedure is not the so-called "*lossless*" in this case. We can not therefore avoid that the design results are slightly conservative. Nevertheless, we can expect the reduction of conservativeness in design result by this technique in contrast with the results by preexisiting methods. Because the conservativeness caused by S-procedure is too small to put a matter for practical purposes.

Finally, using "*Schur-complement*" [ZHO, 96], we can transformed the minimization problem (3.17) into the following problem which can be solved easily.

$$\min_{F_0,\tau} \gamma \tag{3.18}$$

subject to

$$\begin{bmatrix} H_1^T \gamma H_1 - \Sigma & H_x^T & H_u^T \\ H_x & \hat{Q}^{-1} & 0 \\ H_u & 0 & \hat{R}^{-1} \end{bmatrix} \succeq 0$$

$$\tau_j \geq 0 \quad (j = 0, \cdots, N-1)$$

where

$$\Sigma := \sum_{j=0}^{N-1} [\tau_j^w S_j^w + \tau_j^u S_j^u + \tau_j^\eta S_j^\eta].$$

4. Conclusion

A new approach to minimax finite RHC of constrained systems with structured uncertainties has been proposed. The proposed approach can be expected to solve the control design problem with the finite horizon quadratic cost function efficiently.

The proposed approach is easily extended the systems with other constraints which are specified by ellipsoidal bounds, for example, state estimation errors and so on as follows.

In the case that $x(k)$ is not full measured and we need to estimate $x(k)$, where the bound of estimation error $e(k) = x(k) - \hat{x}(k)$ is guranteed an ellipsoidal set as:

$$e^T(k)P_e e(k) \leq 1 \quad (P_e : \text{positive symmetric matrix for weight}) \ . \tag{4.1}$$

This specification of estimation error is standard one. Now we introduce H_e as:

$$H_e := [1\,0\cdots0 \ -\hat{x}(k)] \ , \tag{4.2}$$

then the relation of $e(k) = H_e\zeta$ is hold. And the conditicn below is also hold.

$$\zeta^T(H_1^T H_1 - H_e^T P_e H_e)\zeta \geq 0 \ . \tag{4.3}$$

Since this condition has same form as other constraints (3.16), we can include this condition into the condition of problem (3.17) by using a new variable τ_e. Furthermore, in this case, a new output equation with measurement noise $\psi(k)$ is needed as follows in stead of eq. (2.2).

$$y(k) = Cx(k) + \psi(k) \quad (\,\psi^T(k)P_\psi\psi(k) \leq 1\,) \ . \tag{4.4}$$

We can also include this constraint into the condition of problem (3.17) by using a new variable τ_ψ.

Although every constraint used in this paper has been specified by the ellipsoidal bound which has one single center, it can be extended to the intersection of ellipsoidal bounds, for example:

$$z(k) \in \cup_{l=1\cdots N_1}\{z : \begin{bmatrix} z \\ 1 \end{bmatrix} P_{z,l} \begin{bmatrix} z \\ 1 \end{bmatrix} \leq 1\} \ .$$

However, it should be noted that this extension cause the rise of computaional complexity due to the increase of the number of variables (τ_*) of S-procedure.

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