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On the Free Boundary Problem for the Navier-Stokes Equations in the L_p Framework and Related Topics

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Abstract. In this note, we consider a free boundary problem for the Navier-Stokes equation in several domains in $\mathbb{R}^n$ $(n \geq 2)$ with surface tension. We will state a local in time unique existence theorem in the space $W_{q,p}^{2,1}$ $(2 < p < \infty$ and $n < q < \infty)$, which is proved by using the maximal regularity theorem of the corresponding linearized problem. Also, we state the resolvent estimate, the generation of analytic semigroup and the maximal regularity theorem of the corresponding linearized problem.

1 Introduction and Results

1.1 Problem. In this note, we consider the motion of a viscous, incompressible fluid with free surface. The effect of surface tension on free surface is taken into account. Our problem considered in this note is to find a time dependent domain Ω_t for $t > 0$ occupied by a viscous incompressible fluid, a velocity vector field $v(x,t)$ and a scalor pressure $\theta(x,t)$, $x \in \Omega_t$ which satisfy the Navier-Stokes equations:

$$
\begin{aligned}
&\partial_t v + (v \cdot \nabla)v - \operatorname{Div} S(v,\theta) = f(x,t) && \text{in } \Omega_t,\ t > 0,\\
&\operatorname{div} v = 0 && \text{in } \Omega_t,\ t > 0,\\
&S(v,\theta)\nu_t = \sigma \mathcal{H} \nu_t - g_a x_n \nu_t && \text{on } \Gamma_t,\ t > 0,\\
&V_n = v \cdot \nu_t && \text{on } \Gamma_t,\ t > 0,\\
&v = 0 && \text{on } \Gamma_b,\ t > 0,\\
&v|_{t=0} = v_0 && \text{in } \Omega.
\end{aligned}
\tag{1.1}
$$

Here, $\Omega_0 = \Omega$ is an initial domain which is given, Γ_t and Γ_b denote the boundary of Ω_t, ν_t is the unit outward normal to Γ_t, $S(v,\theta) = \mu D(v) - \theta I$ is the stress tensor, $D(v) = (D(v))_{ij} = \partial v_i/\partial x_j + \partial v_j/\partial x_i$ is a deformention tensor, $\mathcal{H}$ is a mean curvature which is given by $\mathcal{H}\nu_t = \Delta_{\Gamma(t)} x$, $\Delta_{\Gamma(t)}$ is the Laplace-Beltrami operator on Γ_t, $\mu > 0$ is a vicsous coefficient, $\sigma > 0$ is a coefficient of surface tension, and $g_a > 0$ is the acceleration of gravity. V_n is the velocity of the evolution of Γ_t in the direction of outward normal ν_t.

1.2 Domains and their Boundaries. Throughout this note, we assume that Ω is a one of the following domains in $\mathbb{R}^n$: a bounded domain, an exteriord domain, a lower perturned half-space, a perturned layer, and a tube. Here, Ω is called an exterior domain if the complement of Ω is a bounded domain; a lower(upper) perturbed half space if there exist positive constans R and ω_0, and a function $\omega(x')$, $x' = (x_1, \dots, x_{n-1})$, defined on $\mathbb{R}^{n-1}$ such that $\Omega \cap B^R$ * $= \{x = (x', x_n) \in \mathbb{R}^n \mid x_n < \omega(x') + \omega_0 \ (x_n > -\omega(x') - \omega_0)\} \cap B^R$; a perturbed layer if there exist a lower perturbed half-space H_- and a upper perturbed half-space H_+ such that $\Omega = H_- \cap H_+$; a tube domain if there exists a bounded domain D in $\mathbb{R}^{n-1}$ such that $\Omega = \mathbb{R} \times D' = \{(x_1, x'') \in \mathbb{R}^n \mid -\infty < x_1 < \infty, \ x'' = (x_2, \dots, x_n) \in D\}$.

When Ω is a perturbed layer, denoting the boundary of H_- by Γ and H_+ by Γ_b we assume that $\Gamma \subset \{x = (x', x_n) \in \mathbb{R}^n \mid x_n > \omega_1\}$ and $\Gamma_b \subset \{x = (x', x_n) \in \mathbb{R}^n \mid x_n < -\omega_1\}$ with some positive constant ω_1. Let us denote the boundary of Ω by Γ when Ω is one of a bounded domain, an exterior domain, an perturbed half-space, and a tube domain. In this case, formally Γ_b is defined by the empty set. When Ω is a bounded domain or an exterior domain, we say that the boundary Γ of Ω belongs to the class W^m_q if the bounary Γ is locally represented by the graph of a W^m_q function. When Ω is a perturbed half-space, we say that the boundary Γ of Ω belongs to the class W^m_q if the bounded part $\Gamma \cap B_R$ belongs to the class W^m_q and $\omega_0(x')$ is a function in $W^m_q(\mathbb{R}^{n-1})$. When Ω is a tube domain, we say that the boundary Γ of Ω belongs to the class W^m_q if the section D belongs to the class W^m_q. Problem (1.1) contains the following special cases:

- If Ω is a bounded domain and $g_a = 0$, then (1.1) is a drop problem.
- If Ω is a perturbed layer, then (1.1) is an ocean problem.
- If Ω is a lower perturbed half-space, then (1.1) is an ocean problem without bottom.

1.3 Some History. Concerning the drop problem, Solonnikov proved a local in time unique existence theorem of (1.1) in the Sobolev-Slobodetskii space $W_2^{2+\alpha, 1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$ when $f = 0$ or $f = \kappa \nabla U$ (κ is the gravitational constant and U is the Newtonian potential), and arbitrary initial data in [28, 29, 35, 32]. In [29], Solonnikov proved a global in time unique existence theorem of (1.1) in $W_2^{2+\alpha, 1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$ for $f = 0$ provided that initial data are sufficiently small and the initial domain Ω is sufficiently close to a ball. Moglilevskiĭ and Solonnikov [11] proved a local in time unique existence theorem in Hölder spaces. Schweizer [21] proved a local in time unique existence theorem for small initial data by using the semigroup approach. Padula and Solonnikov [19] proved a global in time unique existence theorem in Hölder spaces by using the mapping of Ω_t on a ball instead of Lagrangean coordinates.

Concerning the ocean problem, Beale [4] proved a local in time unique existence theorem when $\sigma = 0$ and $n = 3$ in the Bessel potential spaces $H_2^{\ell, \frac{\ell}{2}}$ $(3 < \ell < \frac{7}{2})$. In [5], Beale proved a global in time unique solvability in $H_2^{\ell, \frac{\ell}{2}}$ $(3 < \ell < \frac{7}{2})$ when $\sigma > 0$, $n = 3$ and $f = 0$ provided that the initial data η_0 and u_0 are sufficiently small. Beale and Nishida [6] obtained an asymptotic power-like in time decay of global solutions. A local in time existence theorem for $\sigma > 0$ and $n = 2$ was established by Allain [2]. Tani [41] proved a local in time unique existence theorem in $W_2^{2+\alpha, 1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$ when $\sigma > 0$ and $n = 3$. When $\sigma = 0$ and $n = 3$, using Beale's method, Sylvester [39] showed a global in time unique exsitence theorem in $H_2^{\ell, \frac{\ell}{2}}$ $(\frac{9}{2} < \ell < 5)$ provided that initial data are sufficiently small. When $\sigma \geqq 0$ and $n = 3$, using Solonnikov's

* $B^R = \{x \in \mathbb{R}^n \mid |x| \geq R\} = \mathbb{R}^n \setminus B_R$ with $B_R = \{x \in \mathbb{R}^n \mid |x| < R\}$.

method, Tani and Tanaka [42] proved a global in time unique exsitence theorem in $W_2^{2+\alpha}$ with $\alpha \in (\frac{1}{2}, 1)$ provided that initial data are sufficiently small. Nishida, Teramoto and Yoshihara [15] considered the same problem as in Tani and Tanaka [42] under the assumption that the motion of fluid is horizontally periodic and that spatial mean of the motion of unknown free surface over the space period is equal to zero. They proved a global in time unique solvability and exponential stability in $H_2^{\ell, \frac{\ell}{2}}$ $(3 < \ell < \frac{7}{2})$ for sufficiently small initial data.

We make some remarks in case $\sigma = 0$, namely the surface tension is not taken into account. When Ω is a bounded domain, Solonnikov [27] and Shibata and Shimizu [23], [24] proved a local in time unique existence theorem for any initial data and external force f, and a global in time unique existence theorem for small initial data in $W_p^{2,1}$ $(n < p < \infty)$ and $W_{q,p}^{2,1}$ $(2 < p < \infty$ and $n < q < \infty)$, respectively. Mucha and Zajączkowski [12, 13] proved a local in time unique existence theorem for any initial data in $W_p^{2,1}$ $(n < p < \infty)$. Abels [1] proved a local in time unique existence theorem.

Roughly speaking, a free boundary problem for the Navier-Stokes equation becomes a parabolic system completely in case $\sigma = 0$, while some hyperbolic character appears in case $\sigma > 0$. These facts reflect the asymptotic behaviour of global in time solutions. In fact, to obtain a global in time existence theorem with exponential stability in the bounded domain case we need an assumption that the domain is close to a ball initially in case $\sigma > 0$ while we do not need any geometrical assumption on the domain in case $\sigma = 0$.

Finally, we mention the work due to Prüss and Simonett [20] where they treated two phase free boundary problem and they proved a local in time wellposedness under the assumptions that the initial interface with surface tension is close to a half-plane and that the first derivative of initial data of a height function is small enough. The reason why we mentione the Prüss and Simonett work is that they used the Dirichlet to Neumann map approach which seems to be new in the study of free boundary problem for the Navier-Stokes equations.

1.4 Formulation in the Lagrangean Coordinate. Aside from the dynamical boundary condition, a further kinematic condition for Γ_t is satisfied which gives Γ_t as a set of points $x = x(\xi, t)$, $\xi \in \Gamma$, where $x(\xi, t)$ is the solution of the Cauchy problem:

$$\frac{dx}{dt} = v(x,t), \quad x|_{t=0} = \xi. \tag{1.2}$$

This expresses the fact that the free surface Γ_t consists for all $t > 0$ of the same fluid particles, which do not leave it and are not incident on it from Ω_t.

The problem (1.1) can therefore be written as an initial boundary value problem in the given region Ω if we go over the Eulerean coordinates $x \in \Omega_t$ to the Lagrangean coordinates $\xi \in \Omega$ connected with x by (1.2). If a velocity vector field $u(\xi, t) = (u_1, \ldots, u_n)^*$ is known as a function of the Lagrangean coordinates ξ, then this connection can be written in the form:

$$x = \xi + \int_0^t u(\xi, \tau)\, d\tau \equiv X_u(\xi, t). \tag{1.3}$$

Passing to the Lagrangean coordinates in (1.1) and setting $\theta(X_u(\xi,t),t) = \pi(\xi,t)$, as the same procedure as $\sigma = 0$ case (cf. Appendix in [23]), we obtain

$$\begin{aligned}
&\partial_t u - \operatorname{Div} S(u,\pi) = \operatorname{Div} Q(u) + R(\pi) + f(X_u(\xi,t),t) && \text{in } \Omega,\ t > 0,\\
&\operatorname{div} u = E(u) = \operatorname{div}\tilde{E}(u) && \text{in } \Omega,\ t > 0,\\
&(S(u,\pi) + Q(u))\nu_{tu} - \sigma \mathcal{H}\nu_{tu} - g_a X_{u,n}\nu_{tu} = 0 && \text{on } \Gamma,\ t > 0,\\
&u = 0 && \text{on } \Gamma_b,\ t > 0,\\
&u|_{t=0} = u_0(\xi) && \text{in } \Omega,
\end{aligned} \tag{1.4}$$

where $u_0(\xi) = v_0(x)$. Here ν_{tu} is the outer normal to Γ_t given by $\nu_{tu} = {}^tA^{-1}\nu_0/|{}^tA^{-1}\nu_0|$, where A is the matrix whose element $\{a_{jk}\}$ is the Jacobian of (1.3):

$$a_{jk} = \frac{\partial x_j}{\partial \xi_k} = \delta_{jk} + \int_0^t \frac{\partial u_j}{\partial \xi_k}\, d\tau.$$

$Q(u)$, $R(\pi)$, $E(u)$ and $\tilde{E}(u)$ are nonlinear terms of the following forms:

$$\begin{aligned} Q(u) &= \mu V_1\Big(\int_0^t \nabla u\, d\tau\Big)\nabla u, \quad & R(\pi) &= V_2\Big(\int_0^t \nabla u\, d\tau\Big)\nabla\pi, \\ E(u) &= V_3\Big(\int_0^t \nabla u\, d\tau\Big)\nabla u, \quad & \tilde{E}(u) &= V_4\Big(\int_0^t \nabla u\, d\tau\Big)u \end{aligned} \tag{1.5}$$

with some polynomials $V_j(\cdot)$ of $\int_0^t \nabla u\, d\tau$, $j = 1, 2, 3, 4$, such as $V_j(0) = 0$.

1.5 A Local in Time Unique Existence Theorem. Let $L_q(D)$ and $W_q^m(D)$ denote the usual Lebesgue space and Sobolev space on a domain D, respectively. The space $\hat{W}_q^1(\Omega)$ for the pressure term is defined by the formula:

$$\hat{W}_q^1(\Omega) = \{\theta \in L_{q,\mathrm{loc}}(\overline{\Omega}) \mid D_j\theta \in L_q(\Omega) \ \ (j = 1, \dots, n)\}$$

where $D_j\theta = \partial\theta/\partial x_j$. The space $B_{q,p}^{2(1-1/p)}(\Omega)$ for the initial data is defined by the real interpolation:

$$B_{q,p}^{2(1-1/p)}(\Omega) = [L_q(\Omega), W_q^2(\Omega)]_{1-1/p,p}.$$

Given Banach space X, $L_p((a,b),X)$ and $W_p^1((a,b),X)$ denote the sets of all $L_p(a,b)$ and $W_p^m(a,b)$ functions with values in X, respectively, and set

$$W_{q,p}^{2,1}(\Omega \times (0,T)) = L_p((0,T), W_q^2(\Omega)) \cap W_p^1((0,T), L_q(\Omega)).$$

Given Banach space X, X^n denotes the n-product space of X, that is $X^n = \{u = (u_1, \dots, u_n) \mid u_i \in X \ \ (i = 1, \dots, n)\}$. If $\|\cdot\|_X$ stands for the norm of X, then the norm of X^n is also denoted by $\|\cdot\|_X$ which is defined by the formula: $\|u\|_X = \sum_{j=1}^n \|u_j\|_X$.

1.6 THEOREM *Let $\Omega \subset \mathbb{R}^n$ ($n \geqq 2$) be one of the following domains: a bounded domain, an exterior domain, a lower perturbed half-space, a perturbed layer, and a tube. Assume that $\Gamma \in W_q^3$ and $\Gamma_b \in W_q^2$. Let $2 < p < \infty$, $n < q < \infty$ and $2(1 - 1/p) > 1 + 1/q$. Then for any $u_0 \in B_{q,p}^{2(1-1/p)}(\Omega)^n$ satisfying the compatibility conditions:*

$$\operatorname{div} u_0 = 0 \ \ \text{in } \Omega, \quad D(u_0) - (D(u_0)\nu_0, \nu_0)\nu_0 = 0 \ \ \text{on } \Gamma, \quad u_0 = 0 \ \ \text{on } \Gamma_b, \tag{1.6}$$

and $f \in L_p((0,\infty), L_q(\mathbb{R}^n))^n$ such that $D_j f \in L_\infty(\mathbb{R}^n \times (0,\infty))^n$ for $j = 1, \dots, n$, where $D_j = \partial/\partial x_j$, there exists a $T > 0$ such that the problem (1.4) admits a unique solution

$$(u, \pi) \in W_{q,p}^{2,1}(\Omega \times (0,T))^n \times L_p((0,T), \hat{W}_q^1(\Omega)).$$

1.7 REMARK If $2(1 - 1/p) > 1 + 1/q$, then the regularity of the first dirivative of u_0 is greater than $1/q$, and therefore the trace of $D(u_0) - (D(u_0)\nu_0, \nu_0)\nu_0$ on Γ exists.

2 Reduction to Linearized Problems

We consider the bondary condition of (1.4):

$$(S(u,\pi)+Q(u))\nu_{tu}-\sigma\mathcal{H}\nu_{tu}+g_a X_{u,n}\nu_{tu}=0. \tag{2.1}$$

Let $\mathbf{\Pi}_t$ and $\mathbf{\Pi}_0$ be projections to tangent hyperplanes of Γ_t and Γ_0 which are defined by the formulas:

$$\mathbf{\Pi}_t d = d-(d,\nu_{tu})\nu_{tu}, \quad \mathbf{\Pi}_0 d = d-(d,\nu_0)\nu_0. \tag{2.2}$$

for an arbitrary vector field d defined on Γ_t and Γ_0, respectively. Applying $\mathbf{\Pi}_t$ to (2.1), we obtain

$$\mathbf{\Pi}_t\big((S(u,\pi)+Q(u))\nu_{tu}-\sigma\mathcal{H}\nu_{tu}+g_a X_{u,n}\nu_{tu}\big)=\mathbf{\Pi}_t(\mu D(u)+Q(u))\nu_{tu}=0, \tag{2.3}$$

which implies that

$$\mathbf{\Pi}_0\mu D(u)\nu_0=\mathbf{\Pi}_0\mu D(u)\nu_0-\mathbf{\Pi}_t(\mu D(u)+Q(u))\nu_{tu}. \tag{2.4}$$

By using the fact that $\mathcal{H}\nu_{tu}=\Delta_{\Gamma(t)}X_u$, and taking the innerproduct of (2.1) with ν_{tu}, we obtain

$$\nu_{tu}\cdot(S(u,\pi)+Q(u))\nu_{tu}-\sigma\nu_{tu}\cdot\Delta_{\Gamma_t}X_u+g_a X_{u,n}=0. \tag{2.5}$$

Substituting (1.3) for (2.5), we obtain

$$\nu_{tu}\cdot(S(u,\pi)+Q(u))\nu_{tu}-\sigma\nu_{tu}\cdot\Delta_{\Gamma_t}\left(\xi+\int_0^t u(\xi,\tau)\,d\tau\right)+g_a\left(\xi_n+\int_0^t u_n(\xi,\tau)\,d\tau\right)=0,$$

which is equivalent to

$$\begin{aligned}
&\nu_0\cdot S(u,\pi)\nu_0+(m-\sigma\nu_0\Delta_\Gamma)\int_0^t \nu_0\cdot u\,d\tau\\
&=m\int_0^t\nu_0\cdot u\,d\tau-\sigma\Big\{\nu_0\cdot\Big(\Delta_\Gamma\int_0^t u\,d\tau\Big)-\nu_{tu}\cdot\Big(\Delta_{\Gamma_t}\int_0^t u\,d\tau\Big)\Big\}\\
&\quad+\nu_0\cdot S(u,\pi)\nu_0-\nu_{tu}\cdot S(u,\pi)\nu_{tu}\\
&\quad-\nu_{tu}\cdot Q(u)\nu_{tu}+\sigma\Big\{\nu_0\cdot\Big(\Delta_\Gamma\int_0^t u\,d\tau\Big)-\Delta_\Gamma\Big(\int_0^t\nu_0\cdot u\,d\tau\Big)\Big\}\\
&\quad+\sigma(\nu_{tu}\cdot\Delta_{\Gamma_t}\xi-\nu_0\cdot\Delta_\Gamma\xi)+\sigma\nu_0\Delta_\Gamma\xi-g_a\xi_n-g_a\int_0^t u_n\,d\tau.
\end{aligned} \tag{2.6}$$

where $\Delta_\Gamma=\Delta_{\Gamma_0}$. For the notational simplicity, we set

$$\begin{aligned}
F(u)&=\sigma\Big\{\nu_0\cdot\Big(\Delta_\Gamma\int_0^t u\,d\tau\Big)-\nu_{tu}\cdot\Big(\Delta_{\Gamma_t}\int_0^t u\,d\tau\Big)\Big\}\\
&\quad+\sigma\Big\{\Delta_\Gamma\Big(\int_0^t\nu_0\cdot u\,d\tau\Big)-\nu_0\cdot\Big(\int_0^t u\,d\tau\Big)\Big\}+\sigma(\nu_0\cdot\Delta_\Gamma\xi-\nu_{tu}\cdot\Delta_{\Gamma_t}\xi)\\
H_n(u)&=m\int_0^t\nu_0\cdot u\,d\tau+\nu_0\cdot S(u,\pi)\nu_0-\nu_{tu}\cdot S(u,\pi)\nu_{tu}-\nu_{tu}\cdot Q(u)\nu_{tu}-g_q\int_0^t u_n\,d\tau\\
h_n(\xi)&=\sigma\nu_0\Delta_\Gamma\xi-g_a\xi_n
\end{aligned} \tag{2.7}$$

Then, finally we arrive at the equation:

$$\nu_0 \cdot S(u,\pi)\nu_0 + (m - \sigma\nu_0\Delta_\Gamma)\int_0^t \nu_0 \cdot u\,d\tau + F(u) = H_n(u) + h_n(\xi) \quad \text{on } \Gamma.$$

In (2.7), since Δ_{Γ_t} contains the second derivative with respect to variables on Γ, in order to avoid the loss of regularity we apply the inverse operator $(m - \sigma\Delta_\Gamma)^{-1}$ with sufficiently large number m to $F(u)$. We proceed that

$$\nu_0 \cdot S(u,\pi)\nu_0 + (m - \sigma\Delta_\Gamma)\left(\nu_0 \cdot \int_0^t u\,d\tau + (m - \sigma\Delta_\Gamma)^{-1}F(u)\right) = H_n(u) + h_n(\xi) \quad \text{on } \Gamma. \quad (2.8)$$

We define a new function η by the formula:

$$\eta = \nu_0 \cdot \int_0^t u\,d\tau + (m - \sigma\Delta_\Gamma)^{-1}F(u) \quad \text{on } \Gamma. \quad (2.9)$$

From (2.8) and (2.9), we obtain the system of two equations on Γ as follows:

$$\begin{aligned} \nu_0 \cdot S(u,\pi)\nu_0 + (m - \sigma\Delta_\Gamma)\eta &= H_n(u) + h_n(\xi) \\ \partial_t\eta - \nu_0 \cdot u &= (m - \sigma\Delta_\Gamma)^{-1}\dot{F}(u), \end{aligned} \quad (2.10)$$

where $\dot{F}(u)$ denotes the time derivative of $F(u)$. We conclude that (1.4) is reduced to the equations

$$\begin{aligned} &\partial_t u - \operatorname{Div} S(u,\pi) = \operatorname{Div} Q(u) + R(\pi) + f(X_u(\xi,t),t) && \text{in } \Omega, \\ &\operatorname{div} u = E(u) = \operatorname{div}\tilde{E}(u) && \text{in } \Omega, \\ &\partial_t\eta - \nu_0 \cdot u = G(u) && \text{on } \Gamma, \\ &\mathbf{\Pi}_0 D(u)\nu_0 = H'(u) && \text{on } \Gamma, \\ &\nu_0 \cdot S(u,\pi)\nu_0 + (m - \sigma\Delta_\Gamma)\eta = H_n(u) + h_n(\xi) && \text{on } \Gamma, \\ &u = 0 && \text{on } \Gamma_b, \\ &u|_{t=0} = u_0(\xi) \text{ in } \Omega, \quad \eta|_{t=0} = 0 \text{ on } \Gamma, \end{aligned} \quad (2.11)$$

where

$$\begin{aligned} F(u) &= \sigma\Big\{\nu_0 \cdot \Big(\Delta_\Gamma \int_0^t u\,d\tau\Big) - \nu_{tu} \cdot \Big(\Delta_{\Gamma_t} \int_0^t u\,d\tau\Big)\Big\} \\ &\quad + \nu_0 \cdot \Delta_\Gamma \int_0^t u\,d\tau - \nu_{tu} \cdot \Delta_{\Gamma(t)} \int_0^t u\,d\tau + \nu_0 \cdot \Delta_\Gamma\xi - \nu_{tu} \cdot \Delta_{\Gamma(t)}\xi, \\ G(u) &= (m - \sigma\Delta_\Gamma)^{-1}\dot{F}(u), \\ H'(u) &= \mu(\mathbf{\Pi}_0 D(u)\nu_0 - \mathbf{\Pi}_t D(u)\nu_{tu}) - \mu\mathbf{\Pi}_t Q(u)\nu_{tu}, \\ H_n(u) &= m\int_0^t \nu_0 \cdot u\,d\tau + \nu_0 \cdot S(u,\pi)\nu_0 - \nu_{tu} \cdot S(u,\pi)\nu_{tu} - \nu_{tu} \cdot Q(u)\nu_{tu} - g_a \int_0^t u_n\,d\tau, \\ h_n(\xi) &= \sigma\nu_0\Delta_\Gamma\xi - g_a\xi_n, \end{aligned}$$

and $Q(u)$, $R(\pi)$, $E(u)$ and $\tilde{E}(u)$ are nonlinear terms defined by (1.5).

3 Stokes Problem Arising in the Study of the Free Boundary Problem for the Navier-Stokes Equation with Surface Tension

3.1 Stokes Problem. In view of (2.11), now we consider the following time dependent inear problem:

$$\begin{aligned}
&\partial_t u - \operatorname{Div} S(u,\pi) = f, && x \in \Omega,\ t>0,\\
&\operatorname{div} u = f_d = \operatorname{div} \tilde f_d && x \in \Omega,\ t>0,\\
&\partial_t \eta - \nu_0 \cdot u = d && x \in \Gamma,\ t>0,\\
&S(u,\pi)\nu_0 + (m - \sigma\Delta_\Gamma)\eta\,\nu_0 = h && x \in \Gamma,\ t>0,\\
&u = 0 && x \in \Gamma_b,\ t>0,\\
&u|_{t=0} = u_0,\ \eta|_{t=0} = \eta_0.
\end{aligned} \tag{3.1}$$

Also, we consider the following resolvent problem correspondint to (3.1):

$$\begin{aligned}
&\lambda u - \operatorname{Div} S(u,\pi) = f, && x \in \Omega,\\
&\operatorname{div} u = g && x \in \Omega,\\
&\lambda \eta - \nu_0 \cdot u = d && x \in \Gamma,\\
&S(u,\pi)\nu_0 + (m - \sigma\Delta_\Gamma)\eta\,\nu_0 = h && x \in \Gamma,\\
&u = 0 && x \in \Gamma_b.
\end{aligned} \tag{3.2}$$

3.2 Some Spaces of Bessel Potentials. For the boundary data h in (3.1)we shall introduce some spaces of Bessel potentials. Given $\alpha \geq 0$, we set

$$\begin{aligned}
&< D_t >^\alpha u(t) = \mathcal{F}^{-1}[(1+s^2)^{\alpha/2}\mathcal{F}u(s)](t),\\
&H_p^\alpha(\mathbb{R}, X) = \{u \in L_p(\mathbb{R}, X) : < D_t >^\alpha u \in L_p(\mathbb{R}, X)\},\\
&\|u\|_{H_p^\alpha(\mathbb{R},X)} = \| < D_t >^\alpha u\|_{L_p(\mathbb{R},X)} + \|u\|_{L_p(\mathbb{R},X)}.
\end{aligned}$$

Here and hereafter, $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and its inverse formula, respectively. Set

$$\begin{aligned}
&H_{q,p}^{1,1/2}(D \times \mathbb{R}) = H_p^{1/2}(\mathbb{R}, L_q(D)) \cap L_p(\mathbb{R}, W_q^1(D)),\\
&H_{q,p}^{1,1/2}(D \times (0,T)) = \{u \mid \text{there exists a } v \in H_{q,p}^{1,1/2}(D\times\mathbb{R}) \text{ such that } u = v \text{ on } D \times (0,T)\},\\
&\|u\|_{H_{q,p,0}^{1,1/2}(D\times(0,T))} = \inf\{\|v\|_{H_{q,p}^{1,1/2}(D\times\mathbb{R})} \mid v \in S(u)\}
\end{aligned}$$

where $S(u) = \{v \in H_{q,p}^{1,1/2}(D \times \mathbb{R}) \mid v = u \text{ on } D \times (0,T)\}$.

3.3 Maximal Regularity Theorem with Zero Initial Data. Instead of (3.1), we consider the following linear problem with zero initial data:

$$\begin{aligned}
&\partial_t u - \operatorname{Div} S(u,\pi) = f, && x \in \Omega,\ t>0,\\
&\operatorname{div} u = f_d = \operatorname{div} \tilde f_d && x \in \Omega,\ t>0,\\
&\partial_t \eta - \nu_0 \cdot u = d && x \in \Gamma,\ t>0,\\
&S(u,\pi)\nu_0 + (m - \sigma\Delta_\Gamma)\eta\,\nu_0 = h && x \in \Gamma,\ t>0,\\
&u = 0 && x \in \Gamma_b,\ t>0,\\
&u|_{t=0} = 0,\ \eta|_{t=0} = 0.
\end{aligned} \tag{3.3}$$

To solve (2.11) locally in time by the usual contraction mapping principle, the following maximal regularity theorem for (3.3) plays an essential role.

3.4 THEOREM *Let $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ be one of domains: a bounded domain, an exterior domain, a upper perturbed half-space, a perturbed layer, and a tube domain. Let $1 < p, q < \infty$, $\max(q, 2) \leq r < \infty$ and $r > n-1$. Assume that $\Gamma \in W_r^3$ and $\Gamma_b \in W_r^2$. If right members f, f_d, $\tilde{f}_d$, d, h in* (3.3) *belong to the following spaces:*

$$f \in L_p((0,T), L_q(\Omega))^n, \quad f_d \in L_p((0,T), W_q^1(\Omega)), \quad \tilde{f}_d \in W_p^1((0,T), L_q(\Omega))^n,$$
$$d \in L_p((0,T), W_q^{2-1/q}(\Gamma)), \quad h \in H_{q,p}^{1,1/2}(\Omega \times (0,T))^n$$

and satisfy the compatibility conditions: $\tilde{f}_d|_{t=0} = 0$, $h|_{t=0} = 0$, and $\tilde{f}_d \cdot \nu_b|_{\Gamma_b} = 0$, then (3.3) *admits a unique solution (u, π, η) which belong to the following spaces:*

$$u \in W_{q,p}^{2,1}(\Omega \times (0,T)), \quad \pi \in L_p((0,T), \hat{W}_q^1(\Omega)),$$
$$\eta \in W_p^1((0,T), W_q^{2-1/q}(\Gamma)) \cap L_p((0,T), W_q^{3-1/q}(\Gamma)).$$

Moreover, there exists $\tilde{\pi}|_\Gamma = \pi|_\Gamma$ such that $\tilde{\pi} \in H_{q,p}^{1,1/2}(\Omega \times (0,T))$. Also, there holds the estimate:

$$\begin{aligned}
&\|u\|_{L_p((0,T),W_q^2(\Omega))} + \|u\|_{W_p^1((0,T),L_q(\Omega))} + \|\nabla\pi\|_{L_p((0,T),L_q(\Omega))} + \|\tilde{\pi}\|_{H_{q,p}^{1,1/2}(\Omega\times(0,T))} \\
&\qquad + \|\eta\|_{W_p^1((0,T),W_q^{2-(1/q)}(\Gamma))} + \|\eta\|_{L_p((0,T),W_q^{3-(1/q)}(\Gamma))} \\
&\quad \leq C\Big(\|f\|_{L_p((0,T),L_q(\Omega))} + \|d\|_{L_p((0,T),W_q^{2-1/q}(\Gamma))} \\
&\qquad + \|f_d\|_{L_p((0,T),W_q^1(\Omega))} + \|\tilde{f}_d\|_{W_p^1((0,T),L_q(\Omega))} + \|h\|_{H_{q,p}^{1,1/2}(\Omega\times(0,T))}\Big).
\end{aligned}$$

3.5 2nd Helmholtz Decomposition and Resolvent Estimates. To state our resolvent estimate concerning the problem (3.2), at this point we shall introduce the 2nd Helmholtz decomposition. Let $1 < q < \infty$ and set

$$J_q(\Omega) = \{u \in L_q(\Omega)^n \mid \operatorname{div} u = 0 \quad \text{in } \Omega, \quad \nu_b \cdot u|_{\Gamma_b} = 0\},$$
$$G_q(\Omega) = \{\nabla\pi \mid \pi \in \hat{W}_q^1(\Omega), \quad \pi|_\Gamma = 0\},$$

where ν_b is the unit outward normal to Γ_b. Given $f \in L_q(\Omega)^n$, let $\pi \in \hat{W}_q^1(\Omega)$ be a unique weak solution to the Dirichlet-Neumann problem for the Laplace operator:

$$\Delta\pi = \operatorname{div} f \ \text{ in } \Omega, \quad \pi = 0 \ \text{ on } \Gamma, \quad \frac{\partial\pi}{\partial\nu_b} = \nu_b \cdot f \ \text{ on } \Gamma_b,$$

where $\partial\pi/\partial\nu_n = \nu_n \cdot \nabla\pi$ and $\nabla\pi = (D_1\pi, \ldots, D_n\pi)$. When Ω is one of the domains: a bounded domain, an exterior domain, a upper perturbed half-space, a perturbed layer, and a tube, the unique existence of such π follows, which will be discussed elsewhere. If we define the operators P_q and Q_q by the formulas: $P_q f = f - \nabla\pi$ and $Q_q f = \pi$, then P_q and Q_q are bounded linear operators from $L_q(\Omega)^n$ into $J_q(\Omega)$ and $\hat{W}_q^1(\Omega)$, respectively. Moreover, we have $f = P_q f + \nabla Q_q f$ and this decomposition is unique. Therefore, we have

$$L_q(\Omega)^n = J_q(\Omega) \oplus G_q(\Omega)$$

where $\oplus$ means the direct sum, which is called the second Helmholtz decomposition.

3.6 THEOREM *Let Ω be one of the domains: a bounded domain, an exterior domain, a upper perturbed half-space, a perturbed infinite layer, and a tube. Let $1 < q < \infty$, $\max(q, 2) \le r < \infty$ and $r > n - 1$. Assume that $\Gamma \in W^3_r$ and $\Gamma_b \in W^2_r$. Set $\Sigma_{\epsilon,\lambda_0} = \{\lambda \in \mathbb{C} \mid |\arg\lambda| \le \pi - \epsilon,\ |\lambda| \ge \lambda_0\}$. Then, for any $\epsilon \in (0, \pi/2)$ there exists a $\lambda_0 > 0$ such that for any $\lambda \in \Sigma_{\epsilon,\lambda_0}$, $f \in L_q(\Omega)^n$, $g \in \hat{W}^1_{0,q'}(\Omega)^* \cap W^1_q(\Omega)$, $d \in W^{2-1/q}_q(\Gamma)$ and $h \in W^1_q(\Omega)^n$, (3.2) admits a unique solution $(u, p, \eta) \in W^2_q(\Omega)^n \times \hat{W}^1_q(\Omega) \times W^{3-1/q}_q(\Gamma)$ such that*

$$
\begin{aligned}
&\|(|\lambda|u, |\lambda|^{\frac{1}{2}}\nabla u, \nabla^2 u, \nabla p)\|_{L_q(\Omega)} + |\lambda|\|\eta\|_{W^{2-1/q}_q(\Gamma)} + \|\eta\|_{W^{3-1/q}_q(\Gamma)} \\
&\le C[\|f\|_{L_q(\Omega)} + \|d\|_{W^{2-1/q}_q(\Gamma)} + |\lambda|\|g\|_{\hat{W}^1_{0,q'}(\Omega)^*} + |\lambda|^{\frac{1}{2}}\|(g,h)\|_{L_q(\Omega)} + \|\nabla(g,h)\|_{L_q(\Omega)}]. \\
&\|p\|_{L_q(\Omega^R)} \le C[\|h\cdot\nu_0\|_{L_q(\Gamma)} + \|Q_q f\|_{L_q(\Omega)} \\
&+ |\lambda|^{-\delta}(\|f\|_{L_q(\Omega)} + \|d\|_{W^{2-1/q}_q(\Gamma)} + |\lambda|\|g\|_{\hat{W}^1_{0,q'}(\Omega)^*} + |\lambda|^{\frac{1}{2}}\|(g,h)\|_{L_q(\Omega)} + \|\nabla(g,h)\|_{L_q(\Omega)})].
\end{aligned}
$$

Here, $\delta = \min(1/2, 1 - 1/q)$, $q' = q/(q-1)$, $\hat{W}^1_{0,q'}(\Omega) = \{v \in \hat{W}^1_{q'}(\Omega) \mid v|_\Gamma = 0\}$, $\hat{W}^1_{0,q'}(\Omega)^*$ stands for the dual space of $\hat{W}^1_{0,q'}(\Omega)$ with norm $\|\cdot\|_{\hat{W}^1_{0,q'}(\Omega)^*}$. Also, Ω^R is defined as follows: $\Omega^R = \Omega$ when Ω is one of the domains: a bounded domain, a perturbed infinite layer and a tube domai, $\Omega^R = \Omega \cap B_R$ with sufficiently large $R > 0$ when Ω is an exterior domain , and $\Omega^R = \Omega \cap \mathbb{R}^{n-1} \times (-R, R)$ with sufficiently large $R > 0$ when Ω is a upper perturbed half-space.

3.7 Generation of Analytic Semigroup. Now, we shall discuss the unique solvability of the initial value problem:

$$
\begin{aligned}
&\partial_t u - \operatorname{Div} S(u, \pi) = 0, \quad \operatorname{div} u = 0 && x \in \Omega,\ t > 0, \\
&\partial_t \eta - \nu_0 \cdot u = 0 && x \in \Gamma,\ t > 0, \\
&S(u, \pi)\nu_0 + (m - \sigma\Delta_\Gamma)\eta\,\nu_0 = 0 && x \in \Gamma,\ t > 0, \\
&u = 0 && x \in \Gamma_b,\ t > 0, \\
&u|_{t=0} = u_0, \quad \eta|_{t=0} = \eta_0.
\end{aligned} \tag{3.4}
$$

We shall discuss an analytic semigroup approach to the initial-boundary value problem (3.4). Since the time derivative of π is missing in (3.4) we shall eliminate π from (3.4). For a while instead of (3.4) we shall consider the resolvent problem:

$$
\begin{aligned}
&\lambda u - \operatorname{Div} S(u, \pi) = f, \quad \operatorname{div} u = 0 && \text{in } \Omega, \\
&\lambda\eta - \nu_0 \cdot u = g, && \text{on } \Gamma, \\
&S(u, \pi)\nu_0 + (m - \sigma\Delta_\Gamma)\eta\nu_0 = 0 && \text{on } \Gamma, \\
&u = 0 && \text{on } \Gamma_b.
\end{aligned} \tag{3.5}
$$

and we shall discuss how to eliminate π from (3.5).

Substituting the 2nd Helmholtz decomposition $f = P_q f + \nabla Q_q f$ into (3.5) and using the fact that $Q_q f|_\Gamma = 0$, we have

$$
\begin{aligned}
&\lambda u - \operatorname{Div} S(u, \pi - Q_q f) = P_q f, \quad \operatorname{div} u = 0 && \text{in } \Omega, \\
&\lambda\eta - \nu_0 \cdot u = \eta_0, && \text{on } \Gamma, \\
&S(u, \pi - Q_q f)\nu_0 + (m - \sigma\Delta_\Gamma)\eta\nu_0 = 0 && \text{on } \Gamma, \\
&u = 0 && \text{on } \Gamma_b.
\end{aligned} \tag{3.6}
$$

We note that $\operatorname{Div} S(u,\pi) = \mu\Delta u - \nabla\pi$ when $\operatorname{div} u = 0$. Denoting $\pi - Q_q f$ by π again in (3.6), from now on we consider (3.5) under the condition that $\operatorname{div} f = 0$. Then, applying the divergence to the first equation of (3.5), taking the innerproduct of the boundary condition on Γ with ν_0 and taking the trace of the innerproduct of the first equation with ν_b to Γ_b, we have

$$\begin{aligned}&\Delta\pi = 0 \quad \text{in } \Omega,\\ &\pi|_\Gamma = \{\nu_0\cdot(\mu D(u)\nu) + \sigma(m-\Delta_\Gamma)\eta - \operatorname{div} u\}|_\Gamma, \ \frac{\partial\pi}{\partial\nu_b}\Big|_{\Gamma_b} = \mu(\nu_b\cdot\Delta u + \frac{\partial}{\partial\nu_b}\operatorname{div} u)|_{\Gamma_b},\end{aligned} \tag{3.7}$$

where we have used the facts that $\operatorname{div} u = 0$ in Ω and $\nu_0\cdot\nu_0 = 1$ on Γ. We decompose π into $\pi_1 + \pi_2$, where π_1 and π_2 satisfy the following equations:

$$\Delta\pi_1 = 0 \ \text{in } \Omega, \ \pi_1|_\Gamma = \nu_0\cdot\mu D(u)\nu_0 - \operatorname{div} u|_\Gamma, \ \frac{\partial\pi_1}{\partial\nu_b}\Big|_{\Gamma_b} = \mu[\nu_b\cdot\Delta u + \frac{\partial}{\partial\nu_b}\operatorname{div} u]|_{\Gamma_b}, \tag{3.8}$$

$$\Delta\pi_2 = 0 \ \text{in } \Omega, \ \pi_2|_\Gamma = \sigma(m-\Delta_\Gamma)\eta|_\Gamma, \qquad \frac{\partial\pi_2}{\partial\nu_b}\Big|_{\Gamma_b} = 0. \tag{3.9}$$

When Ω is one of the domains: a bounded domain, an exterior domain, a upper perturbed half-space, a perturbed layer, and a tube domain, we know that given $u \in W^2_q(\Omega)^n$ there exists a unique $\pi_1 \in \hat W^1_q(\Omega)$ which solves (3.8) and enjoys the estimate: $\|\pi_1\|_{\hat W^1_q(\Omega)} \le C\|u\|_{W^2_q(\Omega)}$. Also, we know that given $\eta \in W^{3-1/q}_q(\Gamma)$ there exists a unique $\pi_2 \in \hat W^1_q(\Omega)$ which solves (3.9) and enjoys the estimate: $\|\pi_2\|_{\hat W^1_q(\Omega)} \le C\|\eta\|_{W^{3-1/q}_q(\Gamma)}$. From these observations, let us define the maps

$$\begin{aligned}K_1 &: W^2_q(\Omega)^n \to \hat W^1_q(\Omega) \quad \text{by } \pi_1 = K_1 u \quad \text{for } u \in W^2_q(\Omega)^n,\\ K_2 &: W^{3-1/q}_q(\Gamma) \to \hat W^1_q(\Omega) \quad \text{by } \pi_2 = K_2\eta \quad \text{for } \eta \in W^{3-1/q}_q(\Gamma),\end{aligned}$$

respectively. We set $\pi = K_1 u + K_2\eta$. By using these symbols, the equation (3.5) is rewritten in the form:

$$\begin{aligned}&\lambda u - \mu\Delta u + \nabla(K_1 u + K_2\eta) = f && \text{in } \Omega\\ &\lambda\eta - \nu_0\cdot u = g && \text{on } \Gamma\\ &S(u, K_1 u + K_2\eta)\nu_0 + \sigma(m-\Delta_\Gamma)\eta\nu_0 = 0 && \text{on } \Gamma\\ &u = 0 && \text{on } \Gamma_b\end{aligned} \tag{3.10}$$

for $f \in J_q(\Omega)$. We set

$$\begin{aligned}A_q\begin{pmatrix}\eta\\ u\end{pmatrix} &= \begin{pmatrix}0 & -\nu_0\cdot R\\ \nabla K_2 & -\mu\Delta + \nabla K_1\end{pmatrix}\begin{pmatrix}\eta\\ u\end{pmatrix} \quad \text{for } (u,\eta) \in \mathcal{D}(A_q),\\ \mathcal{D}(A_q) &= \{(u,\eta) \in (W^2_q(\Omega)^n \cap J_q(\Omega)) \times W^{3-1/q}_q(\Gamma) \mid \\ &\qquad S(u, K_1 u + K_2\eta)\nu_0 + \sigma(m-\Delta_\Gamma)\eta\nu_0|_\Gamma = 0, \ \ u|_{\Gamma_b} = 0\},\\ X_q &= \{(f,g) \in J_q(\Omega) \times W^{2-1/q}_q(\Gamma)\}.\end{aligned}$$

Then (3.10) is formulated

$$(\lambda + A_q)\begin{pmatrix}\eta\\ u\end{pmatrix} = \begin{pmatrix}g\\ f\end{pmatrix}.$$

Applying THEOREM 3.6, we obtain the following theorem.

3.8 THEOREM *Let Ω be one of the domains: a bounded domain, an exterior domain, a upper perturbed half-space, a perturbed infinite layer, and a tube domain. Let $1 < q < \infty$ and* $\max(q,2) \le r < \infty$, $r > n-1$. *Assume that $\Gamma \in W^3_r$ and $\Gamma_b \in W^2_r$. Then, A_q generates an analytic semigroup $\{T(t)\}_{t\ge0}$ on $J_q(\Omega)$.*

By analytic semigroup theory and THEOREM 3.6, we have

$$\|T(t)(f,g)\|_{W^2_q(\Omega)^n\times W_q^{3-(1/q)}(\Gamma)} \le Ce^{ct}t^{-1}\|(f,g)\|_{L_q(\Omega)\cdot\times W_q^{2-(1/q)}(\Gamma)} \quad \text{for } (f,g)\in X_q,$$
$$\|T(t)(f,g)\|_{W^2_q(\Omega)^n\times W_q^{3-(1/q)}(\Gamma)} \le Ce^{ct}\|(f,g)\|_{W_q(\Omega)^2\times W_q^{3-(1/q)}(\Gamma)} \quad \text{for } (f,g)\in \mathcal{D}(A_q).$$

Therefore, by the real interpolation method, we have the maximal regularity theorem for the initial-boundary value problem (3.4), which was proved in [25].

3.9 THEOREM *Let Ω be one of the domains: a bounded domain, an exterior domain, a upper perturbed half-space, a perturbed infinite layer, and a tube domain. Let $1 < q < \infty$ and* $\max(q,2) \le r < \infty$, $r > n-1$. *Assume that $\Gamma \in W^3_r$ and $\Gamma_b \in W^2_r$. Set $\mathcal{D}_{p,q} = [X_q, \mathcal{D}(A_q)]_{1-1/p,p}$, where $[\cdot,\cdot]_{\theta,p}$ stands for the real interpolation functor. Let us $T(t)(f,g) = (u,\eta)$ for $(f,g)\in\mathcal{D}_{p,q}$. Then, we have*

$$u \in W^{2,1}_{q,p}(\Omega\times(0,\infty))^n, \quad \eta\in W^1_p((0,\infty),W_q^{2-(1/q)}(\Gamma))\cap L_p((0,\infty),W_q^{3-(1/q)}(\Gamma)).$$

Moreover, there exist positive constants C and γ such that

$$\|e^{-\gamma t}u\|_{L_p((0,\infty),W^2_q(\Omega))} + \|e^{-\gamma t}u\|_{W^1_p((0,\infty),L_q(\Omega))} + \|e^{-\gamma t}\eta\|_{L_p((0,\infty),W_q^{3-(1/q)}(\Gamma))}$$
$$+ \|e^{-\gamma t}\eta\|_{W^1_p((0,\infty),W_q^{2-(1/q)}(\Gamma))} \le C(\|f\|_{B^{2(1-(1/p))}_{q,p}(\Omega)} + \|g\|_{B^{3-(1/q)-(1/p)}_{q,p}(\Gamma)}).$$

Here, we have set

$$B^{2(1-(1/p))}_{q,p}(\Gamma) = [L_q(\Omega), W^2_q(\Omega)]_{1-(1/p),p}$$
$$B^{3-(1/q)-(1/p)}_{q,p}(\Gamma) = [W_q^{2-(1/q)}(\Gamma), W_q^{3-(1/q)}(\Gamma)]_{1-(1/p),p}.$$

3.10 REMARK The compatibility condition is hidden in the definition of the space $\mathcal{D}_{q,p}$.

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