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LBOs and Debt Ratio in a Growing Industry

北海道大学・経済学研究科　後藤 允* (Makoto Goto)
Graduate School of Economics and Business Administration,
Hokkaido University

1 Introduction

M&A is one of the most important topics for all public firms. Especially, leveraged buyouts (LBOs) is increasing in recent years rapidly. LBOs need small capital due to debt financing and are used in management buyouts (MBOs). Goto et al. (2009) investigated the mechanism of LBO by using takeover frameworks of Lambrecht and Myers (2007) in a declining industry. They focus the bidder's option to acquire the target company, and find the optimal timing and expending capital, moreover the optimal capital structure of the new company.

Due to Lambrecht and Myers (2007), M&A can be divided into the following two types:

1. a type of seeking a synagy effect and a growth opportunity,

2. a type of seeking effectiveness by dismissal, integration and disinvestment.

Goto et al. (2009) is categorize to the second type. Our focus is a growing industry where the target company has a growth option, so our type is the first one.

M&A is widely investigated from the view point of both practical and theoretical aspects. We orient a theoretical analysis, especialay using a real options approach. Existing literature using a real options approach has the following studies. Shleifer and Vishny (2003) claim misvalue in stock markets causes takeover. Rhodes-Kropf and Viswanathan (2004) show market bias leads to correlation between takeover action and market estimation. Lambrecht (2004) provides an M%A model motivated scale ecomony in a growing industry. Morellec and Zhdanov (2005) analyze the roll of multi-bidders and inperfect imformation on takeover action. Lambrecht and Myers (2007) provides a real options model of takeover and disinvestment in a declining industry. Leland (2007), Lambrecht and Myers (2008) and Tian et al. (2008) investigate M&A using debts.

Goto et al. (2009) forcused LBO in a declining industry and show that uncertainty leads to delay in LBO, and that LBO leads to junk bonds, usual leverage and low risk. Key points in this work are following. We forcus LBO in a growing industry where the target company has a growth option. And we investigate the impact of growth options on LBO. As our main results, we show that a growth option leads to delay in LBO, high leverage and low risk, and that default risk has the opposite sensitivity before/after growth.

The rest of the paper is organized as follows. Section 2 describes the model settings. Section 3 derive the value functions. Next, we present numerical illustrations in section 4. Lastly, section 5 concludes the paper.

*Kita 9, Nishi 7, Kita-ku, Sapporo 060-0809, Japan; E-mail: goto@econ.hokudai.ac.jp

2 The model

We consider the target company whose capital is all equity. X_t is the EBIT of target company:

$$\mathrm{d}X_t = \alpha X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t, \tag{1}$$

where α is the instantaneous expected growth rate of X_t, σ (> 0) is the instantaneous volatility of X_t, and W_t is a standard Brownian moption. The target company is assumed to have an opportunity to expand its business to γ times by paying out the cost K. In other words, the target company has a growth option. We have the value of the target company:

$$V_T(x) = \sup_{t_T \in \mathcal{T}} \mathbb{E}\left[\int_t^{t_T} \mathrm{e}^{-r(s-t)}(1-\tau)X_s \mathrm{d}s + \int_{t_T}^{\infty} \mathrm{e}^{-r(s-t)}(1-\tau)\gamma X_s \mathrm{d}s - \mathrm{e}^{-r(t_T-t)}K \middle| X_t = x\right], \tag{2}$$

where r is the discount rate, τ is the tax rate, $\mathcal{T}$ denotes the collection of admissible stopping times in $[t, \infty)$. The growth time is

$$t_T = \inf\{t > 0 : X_t \geq X_T\}, \tag{3}$$

where X_T is the growth threshold.

Then the bidder establishes a SPC to acquire the target by capitalizing $I > 0$. The SPC issues a corporate bond whose value is D and coupon payment is c. The bond is a nonrecourse loan. The SPC acquires the target's stock and they merge into a new subsidiary company of the bidder at time t_B. And we need the following capital constraint:

$$V_T(X_{t_B}) = I + D. \tag{4}$$

3 The Value Functions

We have the value of the target company analytically:

$$V_T(x) = \begin{cases} A_T x^{\beta_1} + \dfrac{(1-\tau)x}{r-\alpha}, & \text{for } x < X_T, \\ \dfrac{(1-\tau)\gamma x}{r-\alpha}, & \text{for } x \geq X_T, \end{cases} \tag{5}$$

$$A_T = \left(\frac{(1-\tau)(\gamma-1)X_T}{r-\alpha} - K\right) X_T^{-\beta_1}, \tag{6}$$

$$X_T = \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{(1-\tau)(\gamma-1)}K, \tag{7}$$

where β_1 is the positive root of the following characteristic equation:

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0. \tag{8}$$

Next the value of the new subsidiary company consists of the equity and debt values:

$$V_N(x) = E_N(x) + D_N(x). \tag{9}$$

$E_N(x)$ is the equity value of the new company which has to choose the growth time t_G and the default time before and after growth t_D^1 and t_G^2:

$$\begin{aligned} E_N(x) = \sup_{t_G, t_D^1, t_D^2 \in \mathcal{T}} \mathbb{E}\Bigg[& \mathbf{1}_{\{t_D^1 < t_G\}} \int_t^{t_D^1} \mathrm{e}^{-r(s-t)}(1-\tau)(X_s - c)\mathrm{d}s \\ & + \mathbf{1}_{\{t_D^1 \geq t_G\}} \bigg(\int_t^{t_G} \mathrm{e}^{-r(s-t)}(1-\tau)(X_s - c)\mathrm{d}s + \int_{t_G}^{t_D^2} \mathrm{e}^{-r(s-t)}(1-\tau)(\gamma X_s - c)\mathrm{d}s \\ & - \mathrm{e}^{-r(t_G - t)} K \bigg) \bigg| X_t = x \Bigg]. \end{aligned} \tag{10}$$

Coupon payment c is determined by the capital constraint:

$$V_T = I + D. \tag{11}$$

We can solve equation (10) by dividing into two parts at growth time:

$$E_N(x) = \begin{cases} 0, & \text{for } x \leq X_D^1, \\ A_G x^{\beta_1} + A_D^1 x^{\beta_2} + \dfrac{(1-\tau)x}{r-\alpha} - \dfrac{(1-\tau)c}{r}, & \text{for } X_D^1 < x < X_G, \\ E_N^2(x) - K, & \text{for } x \geq X_G, \end{cases} \tag{12}$$

$$E_N^2(x) = \begin{cases} 0, & \text{for } x \leq X_D^2, \\ A_D^2 x^{\beta_2} + \dfrac{(1-\tau)\gamma x}{r-\alpha} - \dfrac{(1-\tau)c}{r}, & \text{for } x > X_D^2, \end{cases} \tag{13}$$

where β_2 is the negative root of the characteristic equation (8), X_G is the growth threshold and X_D^1 and X_D^2 are the default thresholds before and after growth, respectively:

$$t_G = \inf\{t > 0 : X_t \geq X_G\}, \tag{14}$$

$$t_D^1 = \inf\{t > 0 : X_t \leq X_D^1\}, \tag{15}$$

$$t_D^2 = \inf\{t > t_G : X_t \leq X_D^2\}. \tag{16}$$

Although A_D^2 and X_D^2 are found analytically:

$$A_D^2 = \left(\frac{(1-\tau)c}{r} - \frac{(1-\tau)\gamma X_D^2}{r-\alpha} \right) (X_D^2)^{-\beta_2}, \tag{17}$$

$$X_D^2 = \frac{\beta_2}{\beta_2 - 1} \frac{r-\alpha}{\gamma r} c, \tag{18}$$

A_G, A_D^1, X_G and X_D^1 are found numerically.

Given the growth strategy X_G and the default strategy X_D^1 and X_D^2, the value of the debt issued to acquire the target, i.e., the debt passed to the new company $D_N(x)$ $(= D)$ is calculated. Here we assume investment cost to growth K is expended by equity. Therefore, debt holders have no concern with the investment, so that coupon payment c never change before/after the investment. We have

$$\begin{aligned} D_N(x) = \mathbb{E}\Bigg[& \int_t^{t_D^1 \wedge t_D^2} \mathrm{e}^{-r(s-t)} c\mathrm{d}s + \mathbf{1}_{\{t_D^1 < t_G\}} \mathrm{e}^{-(t_D^1 - t)}(1-\theta) \frac{(1-\tau)X_{t_D^1}}{r-\alpha} \\ & + \mathbf{1}_{\{t_D^1 \geq t_G\}} \mathrm{e}^{-(t_D^2 - t)}(1-\theta) \frac{(1-\tau)\gamma X_{t_D^2}}{r-\alpha} \bigg| X_t = x \Bigg], \end{aligned} \tag{19}$$

where θ is the default cost (LGD). Dividing into two parts at growth time again, we have

$$D_N(x) = \begin{cases} (1-\theta)\dfrac{(1-\tau)x}{r-\alpha}, & \text{for } x \le X_D^1, \\ B_G x^{\beta_1} + B_D^1 x^{\beta_2} + \dfrac{c}{r}, & \text{for } X_D^1 < x < X_G, \\ D_N^2(x), & \text{for } x \ge X_G, \end{cases} \tag{20}$$

$$D_N^2(x) = \begin{cases} (1-\theta)\dfrac{(1-\tau)\gamma x}{r-\alpha}, & \text{for } x \le X_D^2, \\ B_D^2 x^{\beta_2} + \dfrac{c}{r}, & \text{for } x > X_D^2, \end{cases} \tag{21}$$

where

$$B_D^2 = \left((1-\theta)\frac{(1-\tau)\gamma X_D^2}{r-\alpha} - \frac{c}{r}\right)(X_D^2)^{-\beta_2}, \tag{22}$$

$$B_G = (B_D^2 - B_D^1) X_G^{\beta_2 - \beta_1}, \tag{23}$$

and B_D^1 is a complicated closed form.

Then total value of the new company is following:

$$V_N(x) = E_N(x) + D_N(x), \tag{24}$$

$$= \begin{cases} (1-\theta)\dfrac{(1-\tau)x}{r-\alpha}, & \text{for } x \le X_D^1, \\ (A_G + B_G)x^{\beta_1} + (A_D^1 + B_D^1)x^{\beta_2} + \dfrac{(1-\tau)x}{r-\alpha} + \dfrac{\tau c}{r}, & \text{for } X_D^1 < x < X_G, \\ V_N^2(x) - K, & \text{for } x \ge X_G, \end{cases} \tag{25}$$

and the value after growth is

$$V_N^2(x) = \begin{cases} (1-\theta)\dfrac{(1-\tau)\gamma x}{r-\alpha}, & \text{for } x \le X_D^2, \\ (A_D^2 + B_D^2)x^{\beta_2} + \dfrac{(1-\tau)\gamma x}{r-\alpha} + \dfrac{\tau c}{r}, & \text{for } x > X_D^2. \end{cases} \tag{26}$$

We can interpret four terms of equation (25) as the growth option, the default cost, the earning profit and the tax benefit, in order.

Finally, we consider the bidder's option to acquire the target company. After acquisition by expending the capital to establish SPC I, the bidder gets the equity of the new company:

$$F_B(x) = \sup_{t_B \in \mathcal{T}} \mathbb{E}\left[e^{-rt_B}(E_N(X_{t_B}) - I)\right], \tag{27}$$

$$= \begin{cases} A_B x^{\beta_1}, & \text{for } x < X_B, \\ A_G x^{\beta_1} + A_D^1 x^{\beta_2} + \dfrac{(1-\tau)x}{r-\alpha} - \dfrac{(1-\tau)c}{r} - I, & \text{for } x \ge X_B, \end{cases} \tag{28}$$

where t_B is the acquiring time and X_B is the acquiring threshold:

$$t_B = \inf\{t > 0 : X_t \ge X_B\}. \tag{29}$$

Although A_B has a closed form:

$$A_B = \left(A_G X_B^{\beta_1} + A_D^1 X_B^{\beta_2} + \frac{(1-\tau)X_B}{r-\alpha} - \frac{(1-\tau)c}{r} - I\right) X_B^{-\beta_1}, \tag{30}$$

Table 1: Parameter Values

parameter		value
volatility	σ	0.15
expected growth rate	α	0.02
discount rate	r	0.1
effective tax rate	τ	0.6
scale parameter	γ	1.5
investment cost	K	5
loss given default	θ	0.5
expended capital	I	2

X_B is found numerically. The capital constraint must hold at the acquiring time:

$$V_T(X_B) = I + D(X_B; c). \tag{31}$$

4 Numerical Illustrations

In this section, we use the basic parameter values in table 1 for the numerical calculation. Figures 1 and 2 are the value functions of the new company and the value of the bidder's option, respectively, for the basic parameter.

Next we provide comparative statics with respect to some parameters. We consider the non-option model for comparison. Non-option means that the target company has no opportunity to expand its business. Setting $\gamma = 1$ and $K = 0$ in equations (2), (10) and (19) mathematically, we have

$$\bar{V}_T(x) = \frac{(1-\tau)x}{r-\alpha}, \tag{32}$$

$$\bar{E}_N(x) = \begin{cases} 0, & \text{for } x \leq \bar{X}_D, \\ \left(\frac{(1-\tau)c}{r} - \frac{(1-\tau)\bar{X}_D}{r-\alpha}\right) x^{\beta_2} + \frac{(1-\tau)x}{r-\alpha} - \frac{(1-\tau)c}{r}, & \text{for } x > \bar{X}_D, \end{cases} \tag{33}$$

$$\bar{D}_N(x) = \begin{cases} (1-\theta)\frac{(1-\tau)x}{r-\alpha}, & \text{for } x \leq \bar{X}_D, \\ \left((1-\theta)\frac{(1-\tau)\bar{X}_D}{r-\alpha} - \frac{c}{r}\right) x^{\beta_2} + \frac{c}{r}, & \text{for } x > \bar{X}_D, \end{cases} \tag{34}$$

$$\bar{V}_N(x) = \begin{cases} (1-\theta)\frac{(1-\tau)x}{r-\alpha}, & \text{for } x \leq \bar{X}_D, \\ -\left(\theta\frac{(1-\tau)\bar{X}_D}{r-\alpha} + \frac{\tau c}{r}\right) x^{\beta_2} + \frac{(1-\tau)x}{r-\alpha} + \frac{\tau c}{r}, & \text{for } x > \bar{X}_D, \end{cases} \tag{35}$$

$$\bar{F}_B(x) = \begin{cases} (\bar{E}_N(\bar{X}_B) - I)x^{\beta_1}, & \text{for } x < \bar{X}_B, \\ \bar{E}_N(x) - I, & \text{for } x \geq \bar{X}_B. \end{cases} \tag{36}$$

Although the default threshold $\bar{X}_D$ has a closed form:

$$\bar{X}_D = \frac{\beta_2}{\beta_2 - 1}\frac{r-\alpha}{r}c, \tag{37}$$

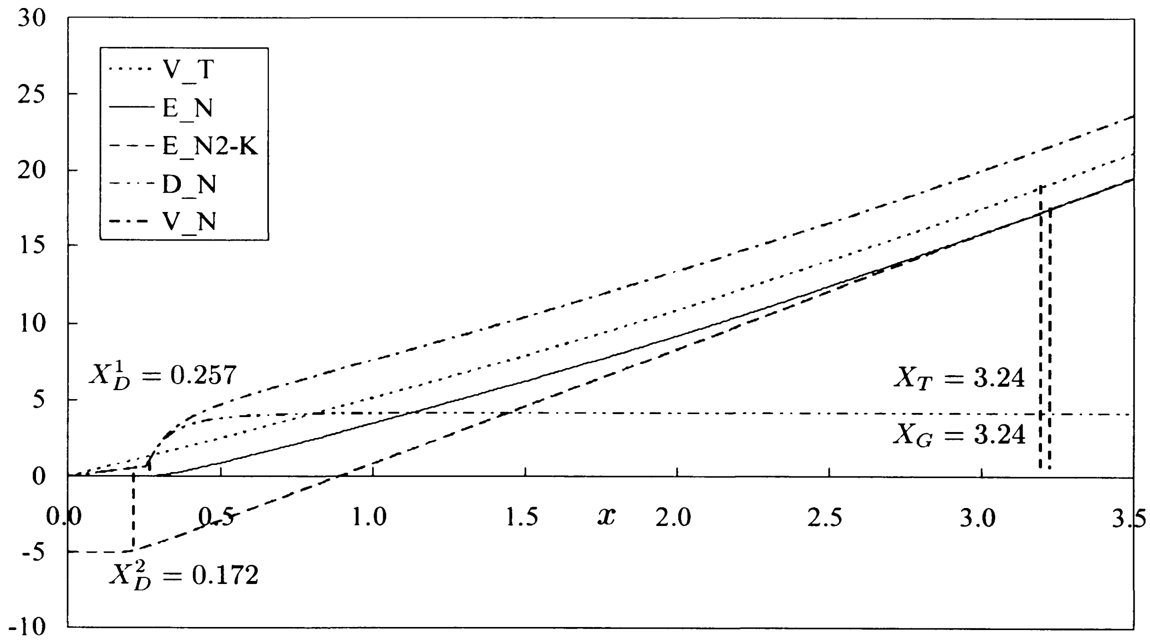

Figure 1: The value functions of the new company for basic parameter values

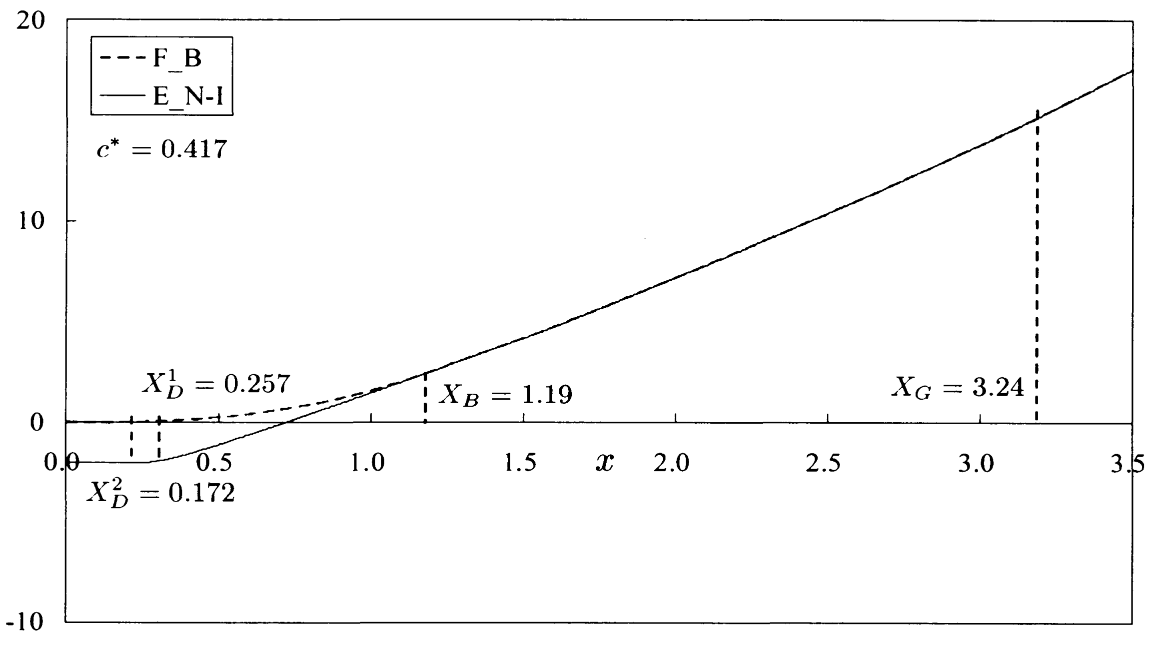

Figure 2: The value of the bidder's option for basic parameter values

Table 2: Leverage ratio w.r.t. I

	Growth option model		Non-option model	
I	c^*	ratio	c^*	ratio
1.0	0.19	0.47	0.18	0.46
1.5	0.29	0.47	0.26	0.46
2.0	0.42	0.48	0.35	0.46
2.5	0.57	0.49	0.44	0.46
3.0	0.77	0.50	0.53	0.46
3.5	1.08	0.52	0.62	0.46
	+	+	+	0

Table 3: Risk w.r.t. I

	Growth option model					Non-option model	
I	X_B	X_G	AtoD	AtoG	GtoD	X_B	AtoD
1.0	0.56	3.24	0.45	2.67	3.16	0.55	0.44
1.5	0.86	3.24	0.68	2.37	3.12	0.83	0.66
2.0	1.19	3.24	0.93	2.05	3.06	1.10	0.88
2.5	1.54	3.24	1.19	1.69	3.00	1.38	1.11
3.0	1.96	3.24	1.49	1.27	2.92	1.65	1.33
3.5	2.53	3.24	1.87	0.71	2.79	1.93	1.55
	+	0	+	−	−	+	+

the acquiring threshold $\bar{X}_B$ is found numerically.

We analyze leverage ratio and risk. The former is defined by D_N/V_N and the latter by the difference between thresholds, such as acquisition and default: $X_B - X_D^1$ ($\bar{X}_B - \bar{X}_D$ for non-option model), acquisition and growth: $X_G - X_B$ and growth and default: $X_G - X_D^2$. Tables 2, 4 and 6 illustrate comparative statics of leverage ratio with respect to I, σ and α, respectively. Tables 3, 5 and 7 illustrate comparative statics of risk with respect to I, σ and α, respectively. Tables 8 and 9 illustrate comparative statics of both leverage ratio and risk with respect to γ and K, respectively.

5 Conclusion

In this paper, we have investigated the bidder's option to acquire the target company in LBO. While Goto et al. (2009) considered a declining industry, we assume that target has a growth option. An important setting is that the optimal timing is determined under the capital constraint. As our main result, we find that a growth option leads to delay in LBO, high leverage and low risk. And when the growth option is easily to exercised, the default risk is low before growth and high after growth except for the impact of uncertainty. For future works, we will

Table 4: Leverage ratio w.r.t. σ

	Growth option model		Non-option model	
σ	c^*	ratio	c^*	ratio
0.05	0.13	0.32	0.13	0.32
0.10	0.23	0.40	0.22	0.40
0.15	0.42	0.48	0.35	0.46
0.20	1.52	0.58	0.61	0.52
	+	+	+	+

Table 5: Risk w.r.t. σ

	Growth option model					Non-option model	
σ	X_B	X_G	AtoD	AtoG	GtoD	X_B	AtoD
0.05	0.67	2.63	0.57	1.96	2.56	0.67	0.56
0.10	0.84	2.90	0.69	2.06	2.80	0.83	0.68
0.15	1.19	3.24	0.93	2.05	3.06	1.10	0.88
0.20	2.84	3.67	2.04	0.83	3.11	1.59	1.26
	+	+	+	−	+	+	+

Table 6: Leverage ratio w.r.t. α

	Growth option model		Non-option model	
α	c^*	ratio	c^*	ratio
0.00	0.19	0.37	0.18	0.37
0.01	0.26	0.42	0.25	0.41
0.02	0.42	0.48	0.35	0.46
0.03	1.02	0.56	0.56	0.51
	+	+	+	+

Table 7: Risk w.r.t. α

	Growth option model					Non-option model	
α	X_B	X_G	AtoD	AtoG	GtoD	X_B	AtoD
0.00	0.96	3.49	0.82	2.53	3.40	0.95	0.82
0.01	1.02	3.35	0.85	2.33	3.24	1.00	0.83
0.02	1.19	3.24	0.93	2.05	3.06	1.10	0.88
0.03	1.91	3.14	1.35	1.23	2.76	1.33	1.02
	+	−	+	−	−	+	+

Table 8: Leverage ratio and Risk w.r.t. γ

γ	c^*	ratio	X_B	X_G	AtoD	AtoG	GtoD
1.1	0.35	0.46	1.10	16.18	0.89	15.08	15.98
1.3	0.37	0.47	1.12	5.39	0.89	4.27	5.22
1.5	0.42	0.48	1.19	3.24	0.93	2.05	3.06
1.7	0.61	0.52	1.43	2.31	1.06	0.89	2.09
	+	+	+	−	+	−	−

Table 9: Leverage ratio and Risk w.r.t. K

K	c^*	ratio	X_B	X_G	AtoD	AtoG	GtoD
5	0.42	0.48	1.19	3.24	0.93	2.05	3.06
10	0.37	0.47	1.13	6.47	0.90	5.35	6.32
15	0.36	0.46	1.11	9.71	0.89	8.59	9.56
20	0.36	0.46	1.11	12.95	0.89	11.84	12.80
	−	−	−	+	−	+	+

compare with optimal debt issuing as Goto et al. (2009) investigated. Another topic is analysis of levered target.

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