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Fuzzy Subjective Conflict Analysis

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Abstract

For an analysis of a noncooperative game we propose a fuzzy subjective conflict analysis which applies a conflict analysis to a fuzzy subjective game considering a subjective sense of value of a player and a fuzziness and nonadditivity of human judgments. Using this method we can explain the variety in selecting a strategy by a player and dispel the gap between the individual rationality and the social one.

Key words : Noncooperative game, conflict analysis, subjective game, fuzzy integral, subjective stable equilibrium

1 Introduction

As a concept of solution for a noncooperative game, the Nash equilibrium point (NEP : Nash[6]) is well-known. It is a point at which all players maintain the balance by aiming to maximize his own expected payoff, in other words, no player has a motive of changing alone his own strategy. The NEP is used widely and deeply as an important analysis instrument in the micro-economics. But it has the following three problems.

(1) There are many noncooperative games with many NEPs. In these case a player can't distinguish the NEP which he should follow, therefore the NEP can't become a suitable guide for selecting a strategy.

(2) In many actual game experiments it is reported that even if the game has a unique NEP, many players don't follow their Nash equilibrium strategies (NES). Furthermore strategies which players select in practice have a great variety (Minas et al [3]).

(3) The NEP is based on the individual rationality, but doesn't necessarily satisfy the Parato optimality (the social rationality).

As a method for dispelling the gap between the NEP and social efficiency, the metagame analysis is known, which considers the selection of strategy to be a result of multistage thinking by a player (Howard [2]). The metagame analysis aims to find a new equilibrium (which is called a metagame equilibrium point) satisfying the Parato optimality. The conflict analysis improves the metagame analysis at the following two points:

(i) In the case of considering the deviation from a NEP, the metagame analysis considers the deviation of only one player, but the conflict analysis allows the deviation of a group of players.

(ii) In the case of considering a punishment for a deviating player, the metagame analysis doesn't require its credibility and accept a punishment by which the payoff of the punisher becomes smaller than the non-punishment case. The conflict analysis accepts only a punishment with credibility.

By the conflict analysis we can find the point satisfying both individual and social rationality.

Nakai[4] proposes a subjective game. The selection of a strategy by a player is likely to depend on something expect original payoffs. For example, a strategy of a player may vary according to feelings (goodwill or ill will) for his opponent. While there is a player aiming to maximize his own payoff, a more offensive player devotes himself to minimizing the payoff of his opponent. Moreover there is a player who is going to take a self-sacrificing action. It is almost impossible for a player to know perfectly the motive for selecting a strategy by his opponent even if it is a complete information game. Each player has no method for knowing motives of other players except expectations. His expectation is expressed by a motive distribution. On the other hand, the payoff matrix under each motive for each player is constructed from the original payoff matrix of all players. For a certain player P we define his subjective game as a noncooperative game constructed by the expected payoff matrix concerning the motive distribution which player P estimates in a place of each player. We consider that each player should take his Nash equilibrium strategy for his subjective game. We call this strategy his subjective Nash equilibrium strategy. After all, each player has no choice but to play his own subjective game. Then if all players have the same understanding for their own motive distributions, their strategies are reasonable and accepted, but if there are large discrepancies among their understandings, they surprise that the others take absurd actions. By introducing the concept of subjective game, we can take in subjective judgments of a player and represent the variety of strategy selection. Then the above-mentioned problem (2) is solved. Moreover even if an original game has many NEPs, it occurs that the subjective game has only an NEP, in this case the above-mentioned problem (1) is solved. By the way, we use expectation values with respect to a motive distribution. This means that we assume the additivity of human judgments. But human judgment has many ambiguous parts and does not always maintain the additivity. There are many nonadditive judgments, for example, synergy effort or offset effort. As a method of representing ambiguities and nonadditivities of human judgments a fuzzy measure is well-known. Furthermore the fuzzy integral is used for a comprehensive evaluation of policy. Furuyama and Nakai[1] proposes a fuzzy subjective game by introducing fuzzy evaluation in a subjective game.

Appling a conflict analysis to a fuzzy subjective game, we develop a fuzzy subjective conflict analysis considering subjective feelings of a player and ambiguities and nonadditivities of human judgments. Then we can clarify more deeply real behaviors of players and solve the problem of the gap between the individual rationality and the social one, that is, the above-mentioned problem (2) and (3) are resolved and problem (1) is resolved partially.

2 Fuzzy subjective game

We represent an original n-person noncooperative game as follows:

$N = \{1, 2, \cdots, n\}$: the set of players

$A_i = \{\alpha_{i1}, \cdots, \alpha_{im_i}\}$: the set of actions of player $i(= 1, \cdots, n)$ where the integer m_i is the number of possible actions for player i.

$a_i(\alpha_1, \cdots, \alpha_n)$: the payoff of player i when player $1, \cdots, n$ select actions $\alpha_1, \cdots, \alpha_n$ respectively ($\alpha_j \in A_j$).

$x_i =< x_{i1}, \cdots, x_{im_i} >$: a mixed strategy of player i where x_{ij} is a probability that player i uses an action α_{ij}.

$a_i(x_1, \cdots, x_n)$: the expected payoff of player i when player $1, \cdots, n$ use mixed strategies $x_1, \cdots, x_n$ respectively.

Then

$$a_i(x_1, \cdots, x_n) = \sum_{j_1=1}^{m_1} \cdots \sum_{j_n=1}^{m_n} a_i(\alpha_{1j_1}, \cdots, \alpha_{nj_n}) x_{1j_1} \cdots x_{nj_n} \tag{1}$$

Next we construct a fuzzy subjective game for a certain player P (one of players $1, \cdots, n$). We consider l motives $m_1, \cdots, m_l$ for selecting a strategy. We put $M = \{m_1, \cdots, m_l\}$. It is assumed that player P thinks that player i follows the motive selected by his subjective motive distribution $\theta^i =< \theta^i_1, \cdots, \theta^i_l >$ where θ^i_k is a probability that player P thinks that player i follows motive m_k. Of course $0 \leq \theta^i_k \leq 1$ $(k = 1, \cdots, l)$ and $\sum_{k=1}^{l} \theta^i_k = 1$.

We translate a motive distribution θ^i into a fuzzy measure, especially we use a λ-fuzzy measure which is defined as follows:

Definition 1. For a constant λ (> -1), a set function $g_\lambda(\cdot)$ defined on the family of subsets of the motive set M is a λ-fuzzy measure if and only if it satisfies the following four conditions:

(1) $g_\lambda(\phi) = 0 \qquad (\phi \text{ is null set})$

(2) $g_\lambda(M) = 1$

(3) $A \subseteq B(\subseteq M) \Rightarrow g_\lambda(A) \leq g_\lambda(B) \qquad$ (simplicity)

(4) $A \cap B = \phi \Rightarrow g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B) \qquad (2)$

To transform the motive distribution θ^i into a λ-fuzzy measure we put

$$g^i_\lambda(\{m_k\}) = \beta\theta^i_k \qquad (k = 1, \cdots, l) \tag{3}$$

where β is a nonnegative constant. Using the relation (2), we can obtain

$$g^i_\lambda(\{m_{k(1)}, \cdots, m_{k(q)}\}) = \sum_{j=1}^{q} \lambda^{j-1}\beta^j \sum_{(p_1,\cdots,p_j)} \theta^i_{p_1}\theta^i_{p_2}\cdots\theta^i_{p_j} \tag{4}$$

for any subset $\{m_{k(1)}, \cdots, m_{k(q)}\}$ of the motive set M where $\sum_{(p_1,\cdots,p_j)}$ denotes the sum of all cases in sampling j elements from the set $\{k(1), \cdots, k(q)\}$. From the relation $g^i_\lambda(M) = 1$, we can decide the value of β. Substituting it into the equation (4), we can obtain the λ-fuzzy measure $g^i_\lambda(\cdot)$. To decide the value of the parameter λ we need more additional information and should ask player P his evaluation $g^i_\lambda(S)$ for any subset S of M.

Next we decide payoff of each player under each motive. We put

$a^k_i(\alpha_1, \cdots, \alpha_n)$: the payoff of player i in the case that player i follows the motive m_k and that player $1, \cdots, n$ use actions $\alpha_1, \cdots, \alpha_n$ respectively

which can be constructed from the original game. Using a fuzzy integral, specially, the Choquet integral, we evaluate comprehensively payoffs of players.

Definition 2. Let $g(\cdot)$ be a fuzzy measure defined on the family of subsets of the set M. Suppose that a real-valued function $h(\cdot)$ defined on the set M satisfies the following relation (if necessary, change the number of each motive)

$$h(m_1) \geq h(m_2) \geq \cdots \geq h(m_l).$$

If the right side of the following equation (5) is called the Choquet integral of the function $h(\cdot)$ by the fuzzy measure $g(\cdot)$ and is denoted by the left side.

$$(C)\int h\, dg = \sum_{k=1}^{l}[h(m_k) - h(m_{k+1})]g(M_k) \tag{5}$$

where $M_k = \{m_1, \cdots, m_k\}$ and $h(m_{l+1}) = 0$.

Let $\tilde{a}_i(\alpha_1,\cdots,\alpha_n)$ be the Choquet integral of the payoff $a_i^k(\alpha_1,\cdots,\alpha_n)$ $(k=1,\cdots,l)$ by the λ-fuzzy measure $g_\lambda^i(\cdot)$ which is obtained by translating the motive distribution. We call the following game $\tilde{G}_P$ the fuzzy subjective game (FSG) of player P.

$$\tilde{G}_P=[\tilde{a}_i(\alpha_1,\cdots,\alpha_n)\ (i=1,\cdots,n)|\alpha_j\in A_j\ (j=1,\cdots,n)] \tag{6}$$

The NEP of the FSG $\tilde{G}_P$ is called the fuzzy subjective NEP (FSNEP) of player P. Moreover the strategy of player P at the FSNEP is called the fuzzy subjective Nash equilibrium strategy (FSNES) of player P. Therefore it is desirable that player P uses his FSNES.

3 Conflict analysis

In this section we explain the conflict analysis and a stable equilibrium point (SEP).

In the original game G, if players $1,\cdots,n$ select actions $u_1,\cdots,u_n$ $(u_i\in A_i\ :\ i=1,\cdots,n)$ respectively, we call $u=(u_1,\cdots,u_n)$ a result for the situation of decision makings. We denote the set of results by U. Let $p_i(u)$ be the payoff which the result u brings to player i, that is, $p_i(a)=a_i(u_1,\cdots,u_n)$. Paying attention to a specific subset $S(\subseteq N)$, we may write $u=\{u_S,u_{-S}\}$ where $u_S=\{u_i\}_{i\in S}$ and $u_{-S}=\{u_i\}_{i\in N\backslash S}$. Let $U_s\,(U_{-S})$ be the set of $u_s\,(u_{-S})$.

We consider any group $S(\subseteq N)$ and any result $u=\{u_S,u_{-S}\}\in U$. For any $u'_S\in U_S$, we call $\{u'_S,u_{-S}\}$ the movement from the result u by the group S. Let $M_S(u)$ be the set of movements. Furthermore we put

$$m_S^+(u)=\{u'\in U|p_i(u)<p_i(u')\quad for\ \forall i\in S\} \tag{7}$$

$$m_S^-(u)=\{u'\in U|p_i(u')\leq p_i(u)\quad for\ \text{some}\ i\in S\} \tag{8}$$

If a movement from a result $u(\in U)$ by a group $S(\subseteq N)$ guarantees more payoff than the result u to all players in S, then the movement is called an improvement from the result u by the group S. Let $M_S^+(u)$ be the set of improvements from u by S, that is,

$$M_S^+(u)=M_S(u)\cap m_S^+(u). \tag{9}$$

When an improvement $u'(\in M_S^+(u))$ is given, if there exists an improvement $u''(\in M_{S'}^+(u'))$ from u' by another group $S'(\neq S)$ which gives at least one player in S a payoff less than or equal to a payoff from the result u, we call u'' a punishment for u' by S'. Let $\bar{M}_{S,S'}(u,u')$ be the set of punishments, that is,

$$\bar{M}_{S,S'}(u,u')=M_{S'}^+(u')\cap m_S^-(u). \tag{10}$$

Definition 3. A result $u(\in U)$ is a stable equilibrium point (SEP) if and only if either of the following two conditions is satisfied:

(1) There is no improvement from the result u by any group $S(\subseteq N)$, that is,

$$M_S^+(u) = \phi \qquad for\ any\ S(\subseteq N). \tag{11}$$

(2) Even if there is an improvement u' from the result u by the group $S(\subseteq N)$, there exists a punishment for u' by some group $S'(\neq S)$,that is,

$$\bar{M}_{S,S'}(u, u') \neq \phi \qquad for\ some\ S'(\subseteq N). \tag{12}$$

All players are balanced at a SEP since any group of players has no incentive to deviate from the SEP.

4 Fuzzy subjective conflict analysis

In this section we propose the fuzzy subjective conflict analysis (FSCA). In the following we show the procedure in case of applying the FSCA to the decision making by a certain player P in a noncooperative n-person game $G = (N, \{A_i\}_{i\in N}, \{a_i\}_{i\in N})$.

step 1 : The player P enumerates motives (evaluation criteria) for selecting a strategy from a standpoint of player i.

step 2 : From a standpoint of player i the player P carries out paired comparisons among motives and decides the degree of importance of each motive by a method of the Analytic Hierarchy Processes (AHP), that is, the eigenvector method or the geometric average method. Then the motive distribution θ^i of player i by the player P is obtained.

step 3 : We translate the motive distribution θ^i into a λ-fuzzy measure. To decide the value of the parameter λ, we have the player P evaluate the effect (measure) of the set $S(\subseteq N)$ containing at least two motives from a standpoint of player i.

step 4 : The player P evaluates all results $u = (u_1, \cdots, u_n) \in U$ under each evaluation criterion (motive) from a standpoint of player i.

step 5 : We calculate the fuzzy integral (the Choquet integral) of evaluation points for each result obtained in step 4 with respect to the λ-fuzzy measure in step 3 and let it be a comprehensive evaluation value of each result.

step 6 : We repeated the process from step 1 to step 5 for every players $i = 1, \cdots, n$. Then the fuzzy subjective game $\tilde{G}_P$ of the player P is obtained.

step 7 : We apply the conflict analysis to the FSG $\tilde{G}_P$ of the player P and obtain its SEP.

step 8 : We select one of Parato optimal points for the FSG $\tilde{G}_P$ from SEPs of $\tilde{G}_P$. Note that the set of SEPs of $\tilde{G}_P$ contains at least one Parato optimal point of $\tilde{G}_P$.

It is desirable that the player P uses a strategy indicated by the selected Parato optimal point.

Let T be a set of points which satisfy both stable equilibrium and Parato optimality. If the set T contains more than one points, the player P is at a loss which point to select. In this case we can use the method proposed in Nakai[5]. Namely, using motive distributions we calculate a realization probability of each point in T and select a point having a maximum realization probability.

5 Discussion

We propose a new method for decision making in a game situation, which taken in the following four points:

(1) subjective feelings and subjective judgments of a decision maker

(2) the ambiguity of human judgments

(3) the nonadditivity of human judgments

(4) the social efficiency (simultaneous accomplishment of both individual and social optimization)

As regards (1), we take in subjectivities of a decision maker by a motive distribution and construct a subjective game. As regards (2) and (3), we use the fuzzy integral for comprehensive evaluations of choices. As regards (4), we apply the conflict analysis to the search of stable equilibrium points with social efficiency. As a result, it is desirable that a player uses a strategy indicated by the stable equilibrium point having a maximum realization probability in his fuzzy subjective game. This method is a mathematical model which is close to a real decision making of a player. The opening problem is modeling a process of learning in a multi-stage game.

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