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On the decomposition numbers of J_4

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In this talk, I explained the way to calculate decomposition numbers by computers. I use this for determining the decomposition numbers of the block of defect 2 in the fourth Janko group J_4.

1 Notations

1.1 Class functions

Let G be a finite group and p is a prime. We denote by $G_{p'}$ a set of p-regular elements in G. We denote by $\mathrm{Cl}(Y)$ a set of class functions on Y where Y is G or $G_{p'}$. For a p–block A of G, the set of irreducible ordinary characters and one of indecomposable projective characters of G belong to A are denoted by $\mathrm{Irr}(A)$ and $\mathrm{IPr}(A)$, respectively. Both of them are subsets of $\mathrm{Cl}(G)$. Let $\mathrm{IBr}(A)$ be the set of irreducible Brauer characters of G belong to A which is a subset of $\mathrm{Cl}(G_{p'})$. The number l denotes the size of $\mathrm{IBr}(A)$. We can regard $\mathrm{Cl}(G_{p'})$ as a subset $\mathrm{Cl}(G)$ via $\phi(x) = 0$ for $\phi \in \mathrm{Cl}(G_{p'})$, $x \in G\backslash G_{p'}$. For general facts about blocks and characters we refer to [3] and [5].

Let $X \subset \mathrm{Cl}(G)$ and R be $\mathbb{N}$ or $\mathbb{Z}$. We denote the set of R-linear combinations of X by the following.

$$\langle X\rangle_R := \left\{ \sum_{\chi\in X} a_\chi \chi \,\middle|\, a_\chi \in R \right\}$$

Then we can see that

- $\langle \mathrm{Irr}(A)\rangle_{\mathbb{N}}$: a set of ordinary characters in A
- $\langle \mathrm{IBr}(A)\rangle_{\mathbb{N}}$: a set of Brauer characters in in A
- $\langle \mathrm{IPr}(A)\rangle_{\mathbb{N}}$: a set of projective characters in A
- $\langle \mathrm{Irr}(A)\rangle_{\mathbb{Z}}$: a set of **generalized** ordinary characters in A
- $\langle \mathrm{IBr}(A)\rangle_{\mathbb{Z}}$: a set of **generalized** Brauer characters in A
- $\langle \mathrm{IPr}(A)\rangle_{\mathbb{Z}}$: a set of **generalized** projective characters in A

Since $\mathrm{Irr}(A)$, $\mathrm{IBr}(A)$ and $\mathrm{IPr}(A)$ are linearly independent, $\mathrm{Irr}(A)$, $\mathrm{IBr}(A)$ and $\mathrm{IPr}(A)$ are $\mathbb{Z}$-basis of $\langle \mathrm{Irr}(A)\rangle_{\mathbb{Z}}$, $\langle \mathrm{IBr}(A)\rangle_{\mathbb{Z}}$ and $\langle \mathrm{IPr}(A)\rangle_{\mathbb{Z}}$, respectively.

1.2 Decomposition numbers of A

Let $\chi \in \langle \mathrm{Irr}(A) \rangle_{\mathbb{N}}$. We define

$$\hat{\chi}(x) := \begin{cases} \chi(x) & : x \in G_{p'} \\ 0 & : \text{else} \end{cases}$$

then $\widehat{\mathrm{Irr}(A)} := \{\hat{\chi} \mid \chi \in \mathrm{Irr}(A)\}$ is a subset of $\langle \mathrm{IBr}(A) \rangle_{\mathbb{N}}$. (See Theorem 6.17 in [5])

Definition 1.1. *Let $\chi \in Irr(A)$ then there are non-negative integers $d_{\chi\phi}$ such that*

$$\hat{\chi} = \sum_{\phi \in IBr(A)} d_{\chi\phi}\phi.$$

These numbers $d_{\chi\phi}$ are **decomposition numbers** *of A.*

Proposition 1.2. $\Psi_\phi := \sum_{\chi \in Irr(A)} d_{\chi\phi}\chi \Rightarrow IPr(A) = \{\Psi_\phi \mid \phi \in IBr(A)\}$

We denote the inner product of characters by the following. Let $\lambda, \mu \in \mathrm{Cl}(G)$; $\langle \lambda, \mu \rangle := \frac{1}{|G|} \sum_{g \in G} \lambda(x)\bar{\mu}(x)$ where $\bar{\mu}$ is a complex conjugate character of μ. It is easy to see the following facts.

Proposition 1.3. *(i)* $\Psi_\phi \in IPr(A)$, $\phi' \in IBr(A) \Rightarrow \langle \Psi_\phi, \phi' \rangle = \delta_{\phi\phi'}$

(ii) $\varphi \in \langle IBr(A) \rangle_{\mathbb{N}}$, $m_\phi := \langle \Psi_\phi, \varphi \rangle \Rightarrow \varphi = \sum_{\phi \in IBr(A)} m_\phi \phi$

(iii) $\Psi \in \langle IPr(A) \rangle_{\mathbb{N}}$, $n_\phi := \langle \Psi, \phi \rangle \Rightarrow \Psi = \sum_{\Psi_\phi \in IPr(A)} n_\phi \Psi_\phi$

From the above propositions, we can get the decomposition numbers from indecomposable projective characters Ψ_ϕ as the following.

$$d_{\chi\phi} = \langle \Psi_\phi, \hat{\chi} \rangle$$

Proposition 1.4. *Let $\Psi \in \langle Irr(A) \rangle_{\mathbb{N}}$. If Ψ is 0 on $G \backslash G_{p'}$ then $\Psi \in \langle IPr(A) \rangle_{\mathbb{Z}}$. In addition if $\langle \Psi, \phi \rangle \geqq 0$ for all $\phi \in IBr(G)$ then Φ is a projective character of G.*

The following notations will allow us to simplify some proofs. Let $\{1 = g_1, g_2 \cdots g_s\}$ be representative of conjugacy classes of G. Let C be $s \times s$-diagonal matrix whose (i,i)-entry is $|C_G(g_i)|^{-1}$. Let $\mathcal{M} := \{\lambda_1, \cdots, \lambda_m\}$ and $\mathcal{N} := \{\mu_1, \cdots, \mu_n\}$ be subsets of $\mathrm{Cl}(G)$. We denote $m \times s$-matrix $\{\lambda_i(g_j)\}$ by $[\mathcal{M}]$. We also denote $m \times n$-matrix $\{\langle \lambda_i, \mu_j \rangle\}$ by $\langle \mathcal{M}, \mathcal{N} \rangle$. From the definition of the inner product we can see that $\langle \mathcal{M}, \mathcal{N} \rangle = [\mathcal{M}]\, C\, [\bar{\mathcal{N}}]^t$ where $\bar{\mathcal{N}} := \{\bar{\mu_1}, \cdots, \bar{\mu_n}\}$.

2 Calculations of Basic Sets

2.1 A basic set and a system of atom

Definition 2.1. *Let $X = IBr(A)$ or $IPr(A)$ and $\mathcal{S}$ be $\mathbb{Z}$-basis of $\langle X\rangle_{\mathbb{Z}}$.*

$$\begin{array}{lcl} \mathcal{S} : \text{a } \textbf{basic set } w.r.t.\ X & \Leftrightarrow & \mathcal{S} \subset \langle X\rangle_{\mathbb{N}} \\ \mathcal{S} : \text{a } \textbf{system of atom } w.r.t.\ X & \Leftrightarrow & X \subset \langle \mathcal{S}\rangle_{\mathbb{N}} \end{array}$$

Note that if $\mathcal{S}$ is a basic set and a system of atom w.r.t. X then $\mathcal{S} = X$.

Proposition 2.2. *Let $\mathcal{BS} := \{\varphi_i\}$ and $\mathcal{PS} := \{\Psi_i\}$ be basic sets w.r.t. $IBr(A)$ and $IPr(A)$, respectively. Then there are system of atoms $\mathcal{BA} := \{\Psi_j^*\}$ and $\mathcal{PA} := \{\varphi_j^*\}$ w.r.t. $IBr(A)$ and $IPr(A)$ such that*

(i) $\langle \Psi_i, \Psi_j^*\rangle = \langle \varphi_i, \varphi_j^*\rangle = \delta_{ij}$

(ii) $\phi \in IBr(A), m_i := \langle \Psi_i, \phi\rangle \Rightarrow \phi = \sum_{i=1}^{l} m_i \Psi_i^*$

(iii) $\Psi \in IPr(A), n_i := \langle \Psi, \varphi_i\rangle \Rightarrow \Psi = \sum_{i=1}^{l} n_i \varphi_i^*$

Proof: From the definition of $\mathcal{BS}$ and $\mathcal{PS}$, there are $l \times l$-matrices U and V over $\mathbb{N}$ such that $[\mathcal{BS}] = U[\mathrm{IBr}(A)]$ and $[\mathcal{PS}] = V[\mathrm{IPr}(A)]$. Since $\mathcal{BS}$ and $\mathcal{PS}$ is $\mathbb{Z}$-basis, U^{-1} and V^{-1} are matrices over $\mathbb{Z}$. Let we define $\mathcal{BA}$ and $\mathcal{PA}$ which are satisfied $[\mathcal{BA}] = V^{-1}[\mathrm{IBr}(A)]$ and $[\mathcal{PA}] = U^{-1}[\mathrm{IPr}(A)]$. Then $\mathcal{BA}$ and $\mathcal{PA}$ are $\mathbb{Z}$-basis of $\langle \mathrm{IBr}(A)\rangle_{\mathbb{Z}}$ and $\langle \mathrm{IPr}(A)\rangle_{\mathbb{Z}}$, respectively. From Proposition 1.3 (i), $\langle \mathrm{IPr}(A), \mathrm{IBr}(A)\rangle = I_l$ where I_l is the identity matrix. Thus $\langle \mathcal{PA}, \mathcal{BS}\rangle = [\mathcal{PA}]C[\bar{\mathcal{BS}}]^t = U^{-1}\langle \mathrm{IPr}(A), \mathrm{IBr}(A)\rangle U = I_l$ and similarly for $\langle \mathcal{BA}, \mathcal{PS}\rangle$. So (i) is followed. Since $\mathcal{BA}$ is $\mathbb{Z}$-basis of $\langle \mathrm{IBr}(A)\rangle_{\mathbb{Z}}$, there are some integers m_j such that $\phi = \sum_{j=1}^{l} m_j \Psi_j^*$ for any $\phi \in \mathrm{IBr}(A)$. Thus $\langle \Psi_i, \phi\rangle = \sum_{j=1}^{l} m_j \langle \Psi_i, \Psi_j^*\rangle = m_i$ from (i). In particular, $m_i = \langle \Psi_i, \phi\rangle \geqq 0$ because of Proposition 1.3 (i) and $\Psi_i \in \langle \mathrm{IPr}(A)\rangle_{\mathbb{N}}$. So $\mathcal{BA}$ is a system of atom w.r.t. $\mathrm{IBr}(A)$. We can apply the same argument to $\mathcal{PA}$. □

There are the following relations among $\langle \mathcal{PA}\rangle_{\mathbb{N}}$, $\langle \mathrm{IPr}(A)\rangle_{\mathbb{N}}$ and $\langle \mathcal{PS}\rangle_{\mathbb{N}}$ ($\langle \mathcal{BS}\rangle_{\mathbb{N}}$, $\langle \mathrm{IBr}(A)\rangle_{\mathbb{N}}$ and $\langle \mathcal{BA}\rangle_{\mathbb{N}}$).

$$\begin{array}{ccc} \langle \mathcal{PA}\rangle_{\mathbb{N}} \ni \varphi_j^* & \langle \varphi_i, \varphi_j^*\rangle = \delta_{ij} & \varphi_i \in \langle \mathcal{BS}\rangle_{\mathbb{N}} \\ \cup & & \cap \\ \langle \mathrm{IPr}(A)\rangle_{\mathbb{N}} \ni \Psi_\phi & \langle \Psi_\phi, \phi'\rangle = \delta_{\phi\phi'} & \phi \in \langle \mathrm{IBr}(A)\rangle_{\mathbb{N}} \\ \cup & & \cap \\ \langle \mathcal{PS}\rangle_{\mathbb{N}} \ni \Psi_i & \langle \Psi_i, \Psi_j^*\rangle = \delta_{ij} & \Psi_j^* \in \langle \mathcal{BA}\rangle_{\mathbb{N}} \end{array}$$

First of all, we construct $\mathcal{PS}$, $\mathcal{PA}$, $\mathcal{BS}$ and $\mathcal{BA}$. We determine $\mathrm{IBr}(A)$ and $\mathrm{IPr}(A)$ by improving the two basic sets $\mathcal{BS}$ and $\mathcal{PS}$.

2.2 Parts of characters

Definition 2.3. *Let* $X = IBr(A)$ *or* $IPr(A)$, *and* $\mathcal{T} = \{\lambda_i\}$ *be a system of atom w.r.t.* X. *Let* $\mu \in \langle X \rangle_{\mathbb{N}}$ *and* $\mu = \sum_{k=1}^{l} m_k \lambda_k$ $(m_k \in \mathbb{N})$

$$\mu' : \text{a } \mathbf{part} \text{ of } \mu \Leftrightarrow \mu' = \sum_{k=1}^{l} m'_k \lambda_k \qquad (0 \leqq m'_k \leqq m_k, m'_k \in \mathbb{N})$$

Since indecomposable direct summands Ψ_ϕ of Ψ are in $\langle \mathcal{T} \rangle_{\mathbb{N}}$, they are parts of Ψ. So it might be possible to investigate indecomposable direct summands of Ψ from parts of Ψ. From Proposition 1.3, we can get the following proposition. We apply this proposition for checking indecomposability of projective characters.

Proposition 2.4. *Let* $\Psi \in \langle IPr(A) \rangle_{\mathbb{N}}$. *if for any parts* Ψ' *of* Ψ *where* $\Psi' \neq 0, \Psi$, *there is a* $\varphi \in \langle IBr(A) \rangle_{\mathbb{N}}$ *such that* $\langle \Psi', \varphi \rangle \lneqq 0$ *or* $\langle \Psi - \Psi', \varphi \rangle \lneqq 0$ *then* $\Psi \in IPr(A)$.

3 Investigation of projective summands

3.1 Multiplicity of projective direct summands

Let B and P be a set of all **computed** Brauer and projective characters and $\mathcal{BS} = \{\varphi_i\}$ and $\mathcal{PS} = \{\Psi_j\}$ be basic sets. We can obtain $\mathcal{BS}$ and $\mathcal{PS}$ as subsets B and P.

Definition 3.1. *Let* $\mathcal{J} \subset \{1, \cdots, l\}$ *such that* $i \in \mathcal{J} \Leftrightarrow \Psi_i \in IPr(A)$. **Possible multiplicities** *of direct summands* Ψ_i *in* Ψ_j *are*

$$m_{ij} := \begin{cases} \infty & (i \notin \mathcal{J}) \\ \delta_{ij} & (i, j \in \mathcal{J}) \\ max\{n \in \mathbb{N} \mid \langle \Psi_j - n\Psi_i, \phi \rangle \geqq 0, \forall \phi \in B\} & \\ & (i \in \mathcal{J}, j \notin \mathcal{J}) \end{cases}$$

Note that m_{ij} is depend on P, B and $\mathcal{PS}$.

Definition 3.2. *Let* $\Psi \in \langle IPr(A) \rangle_{\mathbb{Z}}$.

Ψ: **multiplicity free** $\Leftrightarrow$ $\Psi = n_1\varphi_1^* + \cdots + n_l\varphi_l^*$ *such that* $n_i \in \{0, 1\}$ *where* $\mathcal{PA} = \{\varphi_i^*\}$ *is a system of atom corresponding to* $\mathcal{PS}$ *in Proposition 2.2.*

It is easy to see that if $\Psi \in \langle \mathrm{IPr}(A) \rangle_{\mathbb{N}}$ is multiplicity free then $\langle \Psi, \Psi_\phi \rangle$ is 0 or 1 for $\Psi_\phi \in \mathrm{IPr}(A)$.

3.2 A bit of Brauer characters

Definition 3.3. *Let* $\varphi \in \mathcal{BS}$ *and* $\Psi_i \in \mathcal{PS}$ *such that* $\langle \Psi_i, \varphi \rangle \gneqq 0$
In case that $i \in \mathcal{J}$ *so* Ψ_i *is* **indecomposable**.

$$\varphi' = \sum_{k=1}^{l} n'_k \Psi^*_k \text{ : a } \textbf{bit} \text{ of } \varphi \Leftrightarrow \begin{cases} \varphi' \text{ : a part of } \varphi \\ n'_k = \begin{cases} 1 & (k = i) \\ 0 & (k \in \mathcal{J} \backslash \{i\}) \end{cases} \\ n'_k \leqq m_{ik} \qquad (k \notin \mathcal{J}) \\ \forall \Psi \in P;\ \langle \Psi, \varphi' \rangle \geqq 0 \text{ and} \\ \qquad \langle \Psi, \varphi - \varphi' \rangle \geqq 0 \end{cases}$$

If $\Psi_i = \Psi_\phi$ then the irreducible Brauer character ϕ is a bit of φ.

Definition 3.4. *Let* $\varphi \in \mathcal{BS}$ *and* $\Psi_i \in \mathcal{PS}$ *such that* $\langle \Psi_i, \varphi \rangle \gneqq 0$
In case that $i \notin \mathcal{J}$ *but* Ψ_i *is* **multiplicity free**.

$$\varphi' = \sum_{k=1}^{l} n'_k \Psi^*_k \text{: a } \textbf{bit} \text{ of } \varphi \Leftrightarrow \begin{cases} \varphi' \text{ : a part of } \varphi \\ n'_k = 1 \qquad (k = i) \\ n'_k \leqq m_{ki} \qquad (k \in \mathcal{J}) \\ \forall \Psi \in P; \langle \Psi, \varphi' \rangle \geqq 0 \text{ and} \\ \qquad \langle \Psi, \varphi - \varphi' \rangle \geqq 0 \end{cases}$$

If $\Psi_i = \sum_{\phi \in \mathrm{IBr}(A)} \alpha_\phi \Psi_\phi$ is multiplicity free then there are some ϕ $(\alpha_\phi = 1)$ which are bits of φ.

3.3 Calculations of possible direct summands

Definition 3.5. *Let* Ψ_i *be indecomposable or multiplicity free in* $\mathcal{PS}$ *and* $\varphi \in \mathcal{BS}$ *such that* $\langle \Psi_i, \varphi \rangle \gneqq 0$. *For* $\Psi \in P$,

$$m(\Psi_i, \Psi, \varphi) := min\{\langle \Psi, \varphi' \rangle \mid \varphi' \text{ : a bit of } \varphi \text{ w.r.t. } \Psi_i\}$$

Let Ψ_ϕ be one of summands of Ψ_i. Since some φ' must be ϕ, $m(\Psi_i, \Psi, \varphi)$ is a lower bound of the multiplicity of projective summand Ψ_ϕ in Ψ from 1.3(iii). Thus next proposition is followed.

Proposition 3.6. *Let* $Z(\Psi_i, \Psi) := \{m(\Psi_i, \Psi, \varphi) \mid \varphi \in \mathcal{BS}, \langle \Psi_i, \varphi \rangle \gneqq 0\}$ *then* $\Psi - z\Psi_i \in \langle IPr(A) \rangle_{\mathbb{N}}$ *where*

$$z := \begin{cases} max(Z(\Psi_i, \Psi)) & \Psi_i \text{ : indecomposable} \\ min(Z(\Psi_i, \Psi)) & \Psi_i \text{ : multiplicity free} \end{cases}.$$

By this proposition, we can get a lower bound of the multiplicity of projective summand Ψ_i in Ψ. If we get enough B and P, this lower bound gets near to the multiplicity of projective summand. But there is no guarantee that we can get the multiplicity of projective summand.

4 Use MOC for J_4

MOC(**MO**dular **C**haracters)-system[7] is the program which calculates irreducible Brauer characters and indecomposable projective characters from ordinary characters. MOC is developed by G. Hiss, C. Jansen, K. Lux and R. Parker in 1993. Recently MOC is implemented in GAP system by F. Noeske.

Let G be J_4 and H and K be maximal subgroups of G which are isomorphic to $2^{11}:M_{24}$ and $2_+^{1+12}.3M_{22}:2$. The character table of J_4 can be found in the ATLAS[1]. The character tables of H and K are known and stored in the GAP library of character tables [2]. All 3-decomposition numbers of H and K have been determined [6]. So we can get enough Brauer and projective characters of G by tensored characters of G and induced characters from H and K.

4.1 the block of J_4

Let A_2 be the block of defect 2 in G. The set of ordinary irreducible characters $\mathrm{Irr}(A_2)$ is $\{\chi_{14}, \chi_{21}, \chi_{25}, \chi_{27}, \chi_{28}, \chi_{30}, \chi_{31}, \chi_{35}, \chi_{41}\}$.

For a projective character Ψ of G, we denote by $\Psi.A_2$ the projective summand which belong to A_2. Let ψ_4 and ψ_6 be projective characters in blocks of defect 1 of G. Let χ_3 and χ_4 be ordinary characters in G. Let ξ_3, ξ_{13}, η_8 and η_{15} be projective characters in blocks of defect 1 of H and K.

We can get the following projective characters in A_2.

$\Psi_1 := (\psi_4 \otimes \chi_3).A_2$, $\Psi_2 := (\psi_4 \otimes \chi_4).A_2$, $\Psi_3 := (\psi_6 \otimes \chi_3).A_2$, $\Psi_4 := (\xi_3^G).A_2$, $\Psi_5 := (\xi_{13}^G).A_2$, $\Psi_6 := (\eta_8^G).A_2$ and $\Psi_7 := (\eta_{15}^G).A_2$.

Next table show that coefficients of linear combinations by $\mathrm{IBr}(A_2)$.

Proj.	Coeff. of ordi. char. in A_2											
Ψ_1	[	1	0	0	0	1	1	0	0	1	]	Mult. Free
Ψ_2	[	0	0	1	2	3	2	4	3	6	]	
Ψ_3	[	0	0	0	1	0	1	0	1	1	]	Indec.
Ψ_4	[	0	1	1	0	1	1	2	2	2	]	Mult. Free
Ψ_5	[	0	0	0	1	3	1	3	1	3	]	
Ψ_6	[	0	0	1	2	2	2	3	3	5	]	
Ψ_7	[	2	6	5	4	7	12	10	15	16	]	

We can get a multiplicity free character $\Psi_8 := \Psi_2 + \Psi_3 - \Psi_6$ by FBA(See Algorithm 5.1.3 in [4]) with Ψ_5.

Using Proposition 3.6, we also get projective characters $\Psi_9 := \Psi_8 - \Psi_3$, $\Psi_{10} := \Psi_7 - 4\Psi_3$, $\Psi_{11} := \Psi_6 - 2\Psi_8$, $\Psi_{12} := \Psi_{10} - 5\Psi_4$ and $\Psi_{13} := \Psi_{12} - 2\Psi_1$.

Thus we can get the following projective characters and check that all of them are indecomposable by Proposition 2.4.

Proj.	Coeff. of ordi. char. in A_2
Ψ_1	[1 0 0 0 1 1 0 0 1]
Ψ_{13}	[0 1 0 0 0 1 0 1 0]
Ψ_{11}	[0 0 1 0 0 0 1 1 1]
Ψ_3	[0 0 0 1 0 1 0 1 1]
Ψ_9	[0 0 0 0 1 0 1 0 1]

Thus the decomposition matrix of A_2 is the following.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

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