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ON THE KOTTWITZ-SHELSTAD NORMALIZATION OF TRANSFER FACTORS FOR AUTOMORPHIC INDUCTION FOR GL_n (JOINT WORK WITH K. HIRAGA)

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This note is a report on a joint work with Kaoru Hiraga. Details will appear elsewhere.

Automorphic induction for GL_n over a p-adic field is an example of endoscopic transfer and its character identity was established by Henniart and Herb [2], up to a constant. We discuss a relation of this constant to the Kottwitz-Shelstad transfer factor [5], in particular, to the normalization using ε-factors.

Let F be a non-archimedean local field of characteristic zero. Let $G = \mathrm{GL}_n(F)$ and $\mathbf{a} \in \mathrm{H}^1(W_F, Z(\hat{G}))$, where W_F is the Weil group of F and $Z(\hat{G})$ is the center of the dual group of G. Let $(H, \mathcal{H}, s, \xi)$ be an endoscopic data for $(G, \mathrm{id}, \mathbf{a})$ (see [5]). Then we have a map

$$\begin{aligned}\mathrm{Tran}_H^G : &\{(\text{stable}) \text{ invariant distributions on } H\}\\ &\longrightarrow \{\text{twisted invariant distributions on } G\}\end{aligned}$$

defined as follows.

Let ω be the character of $F^\times$ associated to $\mathbf{a}$. We write $\omega(g) = \omega(\det g)$ for $g \in G$. For a (strongly) regular semisimple element $\gamma \in G$ such that $G_\gamma \subset \ker \omega$ and $f^G \in C_c^\infty(G)$, put

$$O_\gamma^\omega(f^G) = \int_{G_\gamma \backslash G} \omega(g) f^G(g^{-1}\gamma g)\, dg,$$

where G_γ is the centralizer of γ in G. Similarly, for a (strongly) G-regular semisimple element $\gamma_H \in H$ and $f^H \in C_c^\infty(H)$, put

$$O_{\gamma_H}(f^H) = \int_{H_{\gamma_H} \backslash H} f^H(h^{-1}\gamma_H h)\, dh,$$

where H_{γ_H} is the centralizer of γ_H in H. Here we choose suitable Haar measures on G, G_γ, H, and H_{γ_H}. By a result of Waldspurger [7], for each $f^G \in C_c^\infty(G)$, there exists $f^H \in C_c^\infty(H)$ such that

$$O_{\gamma_H}(f^H) = \sum_{\gamma} \Delta(\gamma_H, \gamma) O_\gamma^\omega(f^G)$$

for all G-regular semisimple elements $\gamma_H \in H$. Here the sum is taken over a set of representatives for the conjugacy classes of $\gamma \in G$ whose norm is γ_H and Δ is a transfer factor (see [5]). Since G is quasi-split over F, we can normalize Δ using Whittaker data and ε-factors as in [5, §5.3]. For an invariant distribution D on H, we define a twisted invariant distribution $\mathrm{Tran}_H^G(D)$ by

$$\mathrm{Tran}_H^G(D)(f^G) = D(f^H)$$

for $f^G \in C_c^\infty(G)$.

On the other hand, by a result of Henniart and Herb [2], for each irreducible tempered admissible representation π_H of H, there exist an irreducible tempered admissible representation π of G and a constant $c \in \mathbb{C}^\times$ such that $\pi \otimes \omega \cong \pi$ and

$$\mathrm{Tran}_H^G(\Theta_{\pi_H}) = c \cdot \Theta_\pi^\omega.$$

Here $\Theta_{\pi_H}(f^H) = \mathrm{trace}(\pi_H(f^H))$ for $f^H \in C_c^\infty(H)$ and $\Theta_\pi^\omega(f^G) = \mathrm{trace}(\pi(f^G) \circ \mathcal{A}_\omega)$ for $f^G \in C_c^\infty(G)$, where $\mathcal{A}_\omega : \pi \otimes \omega \to \pi$ is an isomorphism as vector spaces. Since π is generic, we can normalize $\mathcal{A}_\omega$ using Whittaker functionals. By a result of Henniart and Lemair [3], the constant c does not depend on the representations.

Our main result is as follows.

Theorem 1. *We have*

$$c = 1.$$

Remark 2. An analogous result for $F = \mathbb{R}$ was proved by Henniart [1].

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