ON THE SMOOTH UNKNOTTING CONJECTURE IN DIMENSION FOUR

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ABSTRACT. We will reduce the smooth unknotting conjecture in dimension four to the special case and discuss also about the reduced case. The starting point is that the fundamental group of the complement is cyclic and then we have a generic one-parameter family of smooth maps of S^2 into $\mathbb{R}^4$ which connects the given surface knot to the standard one with only cusp births and deaths.

The important step is a Markov type theorem; we can translate the image of such a one-parameter family into a one-parameter family of the singular surface braids, which will be described by a deformation of their chart diagrams on S^2. This part is due to Seiichi Kamada.

Such a deformation of chart diagrams is a special object consisting of 2-dimensional surfaces in $S^2 \times \mathbb{R}$. In the family of simple singular surface braids case the trace of the self-intersection points and the boundaries has no singular points with respect to the deformation parameter. In fact, we can cut off the part which contains only one self-intersection point and satisfies the simple condition. If this part can be modified to the deformation of non-singular surface braids, the conjecture is solved affirmatively by the induction on the number of self-intersection points.

In this way the conjecture is reduced to the case of the deformation of the simple chart diagrams with one node assuming that one end has a canceling node and another end is a 'trivial' chart with one node. We will discuss some ideas about this case, too.

1. INTRODUCTION

A smooth embedding of S^2 into $\mathbb{R}^4$ is called smoothly unknotted if it is diffeomorphic to the standard embedding. In this case the fundamental group of the complement is an infinite cyclic group. So, the question is whether the converse is true: if the fundamental group of the complement of a smooth embedding of S^2 into $\mathbb{R}^4$ is cyclic, is it smoothly unknotted? Kawauchi [7] proved that when it is considered as an embedding into S^4 by adding a point at infinity, the complement has the homotopy type of S^1 and Freedman [2] proved that it is homeomorphic to the standard embedding. The purpose of this paper

is to discuss how to get an affirmative answer to the following smooth unknotting conjecture in dimension four.

Conjecture. *Let $S^2 \subset \mathbb{R}^4$ be a smooth knot such that the fundamental group of the complement is an infinite cyclic group. Then it is smoothly unknotted.*

For the classical knot Dehn's lemma was proved by Papakyriakopoulos [11] in 1957. The higher dimensional knot whose complement is of the same homotopy type with the standard embedding is proved to be unknotted topologically by Stallings [12] in 1963 and smoothly by Levine [8] and [9] in 1965 and 1970 except the four-dimensional case.

2. Review of the terminology concerning surface braids

The basic reference is Kamada's book [3] and a surface braid may be also called a 2-dimensional braid. The special kind of locally flat surfaces in 4-space are treated there but it is not difficult to make them PL and then they are smoothable due to Cerf [1]. Note that a surface braid can represent only an orientable surface and we do not know yet any counterexample for the smooth unknotting of orientable surfaces in 4-space.

A surface braid of degree m is an oriented surface $(= 2\text{-manifold})$ S embedded properly and locally flatly in $D_1^2 \times D_2^2$ such that the restriction map $pr_2|S : S \to D_2^2$ of the second factor projection is a branched covering map of degree m and $\partial S = \{m \text{ interior points}\} \times \partial D_2^2$. See Definition 14.1 at p.105 of [3].

A singular surface braid is a properly immersed oriented surface with almost the same property as in the case of a surface braid but admitting at most one double point in $pr_2^{-1}(y)$ when y is not a branched point. See Definition 34.3 at p.272 of [3] for the exact definition.

A simple (singular) surface braid is a (singular resp.) surface braid whose associated branched covering is simple, that is, for each branched point $y \in D_2^2$ there is a unique singular point x with $pr_2(x) = y$ and the local degree at x is two. See Definition 14.8 at p.108 of [3].

A closed (singular) surface braid in $\mathbb{R}^4$ is an oriented (singular resp.) surface link in $\mathbb{R}^4$ which is the closure $\hat{S} = S \cup (\amalg m D^2)$ of S contained in $N(S^2) = D_1^2 \times S^2 \subset \mathbb{R}^4$. The closure can be thought as a surface braid not in $D_1^2 \times D_2^2$ but in $D^2 \times S^2$. See Definitions 23.1-2 at pp.179-180 of [3].

Let S and S' be closed (singular) surface braids in $D^2 \times S^2$.

(2) S and S' are equivalent if they are ambient isotopic by an isotopy $\{h_u\}_{u \in [0,1]}$ of $D^2 \times S^2$ such that for each $u \in [0,1]$, h_u is fiber-preserving.

(3) S and S' are braid ambient isotopic if they are ambient isotopic by an isotopy $\{h_u\}_{u \in [0,1]}$ of $D^2 \times S^2$ such that for each $u \in [0,1]$, $h_u(S)$ is a (singular resp.) surface braid.

See Definition 14.10 at p.109 (and also Definition 14.2) of [3].

A chart of degree m is a finite graph Γ on D_2^2 (or on S^2) whose edges are oriented and labeled satisfying the following conditions:

(0) The graph is disjoint from the boundary of D_2^2.

(1) Every vertex has degree one, four or six.

(2) The labels of edges are integers in $\{1, 2, \ldots, m-1\}$.

(3) For each degree-six vertex, three consecutive edges are oriented inward and the others are oriented outward, and the six edges are labeled i and $i + 1$ alternately for some i. This vertex is called a white vertex. Also the degree-one vertex is called a black vertex.

(4) For each degree-four vertex, diagonal edges have the same label and are oriented coherently, and the labels i and k of the diagonals satisfy $|i - k| > 1$.

See Definition 18.7 at p.135 of [3].

A chart determines an associated simple surface braid through motion pictures and also through braid monodromies up to equivalence and vise versa. Moreover, two charts of degree m present the same, up to equivalence, simple surface braid if and only if they are related by a finite sequence of CI-, CII- and CIII-moves and ambient isotopies of D_2^2 (or S^2). See Chapter 18 especially Theorem 18.20 at p.142 of [3]. CI-, CII- and CIII-moves are illustrated as in Figure 1 in §7.

A singular chart admits also degree-two vertex, called a node. And for each degree-two vertex, the two edges have the same label but the orientations are opposite. Moreover, two singular charts of degree m describe the same, up to equivalence, simple singular surface braid if and only if they are related by a finite sequence of the chart moves ($=$ C-moves $=$ CI-, CII-, CIII-, CIV and CV-moves) and ambient isotopies of D_2^2 (or S^2). See § 34.4 especially Theorem 34.5 at p.274 of [3]. CIV- and CV-moves are illustrated as in Figure 2 in §7.

A free edge of a (singular) chart is an edge whose end points are black vertices. See Definition 20.5 at p.156 of [3].

A (singular) surface braid S' of degree $m + 1$ is obtained from a (singular resp.) surface braid S of degree m by a stabilization if $\Gamma(S')$ is a union of $\Gamma(S)$ and a new free edge with label m, where $\Gamma(S)$ and $\Gamma(S')$ denote the charts for S and S' respectively. A destabilization is the inverse of a stabilization. See Definition 25.5 and Exercise 25.11 at pp.188-190 of [3]. In fact, we may put the free edge with label m in any region of $S^2 - \Gamma(S)$.

A nomad of degree m is a union of $(m-1)$ number of free edges whose labels are $1, 2, \ldots, m-1$. It represents the trivial surface knot of genus 0 and may be called a trivial chart but a trivial surface braid has already defined to be one equivalent to the product $\{m \text{ interior points}\} \times D_2^2$ (Definition 14.1 at p.107 of [3]).

A non-simple (singular) surface braid can also be described by non-simple (singular) chart by admitting some fusions of black vertices. The chart moves can be easily generalized to this case. Moreover, braid isotopies can be described by the chart moves admitting fusions and fissions of black vertices. Of course there are several conditions to get fusion but we do not care about the conditions and any possibility of attaching sheets to black vertices in this paper. For the details see [5].

Note that the cusp death point is described by the fusion of a simple black vertex and a simple node with opposite signs. The result is also a simple black vertex with the same sign as the node. The cusp at the birth point is described by the inverse.

We will use the charts in S^2 and can neglect the conjugations in Markov's Theorem. See Theorems 25.12 and 25.14 at p.190 of [3].

3. THEOREMS AND A RESTRICTED CONJECTURE

The following Theorems 1 to 3 and affirmative answer to Conjecture 4 will solve the smooth unknotting conjecture.

Theorem 1. *Let $f : S^2 \to \mathbb{R}^4$ be a smooth embedding such that the fundamental group of the complement is an infinite cyclic group. Then, there is a generic one-parameter family of smooth maps from S^2 to $\mathbb{R}^4$ which connects the given surface knot to the standard one with only cusp births and deaths (for singularities of smooth maps).*

The proof of this fact is not difficult, because any finger moves in this dimension are isotopic to trivial ones by the given condition and each of them is easily deformed into two cusps. The proof is given in [10].

Moreover, we may assume that each self-intersection double point appears near the end and keeps the same place and then disappears near another end.

Further, after introducing self-intersection double points we may assume that the singular surface knots near two ends are ambient isotopic, that is, there is a generic one-parameter family of smooth maps from S^2 to $\mathbb{R}^4$ without any singular points with respect to the deformation parameter.

The Markov type theorem for a singular surface knot will be as follows.

Theorem 2. *Let an ambient isotopy of a singular surface knot, that is, an ambient isotopy of a generic immersion of S^2 in $\mathbb{R}^4$ be given. We may suppose that the double points are not moved. Assume that the two ends are given by closed simple singular surface braids in $D^2 \times S^2$. Then, two end singular surface braids are related by a finite sequence of braid ambient isotopies (which keep the nodes disjoint from the other vertices by definition), and stabilizations and destabilizations.*

A Markov type theorem for a surface knot is proved in [3] as Theorem 25.14 at p.190 and this is its generalization to a singular surface knot. The proof will be given somewhere else by Kamada.

The result gives a one-parameter family of singular surface braids whose traces of double points are disjoint non-singular curves with respect to the deformation parameter except at the cusp birth and death points. Also the part between two level singular surface braids give an ambient isotopy if it contains no cusp singularity. Hereafter such a family will be called a one-parameter *cusp* family of singular surface braids.

Theorem 3. *Assume that the lower end of the given one-parameter cusp family of singular surface braids is simple. We assume that there is one cusp death point next to the lower end, which means the fusion of the node and a simple black vertex occurs as the last move. Then, the family can be modified so that the corresponding cusp birth point is pushed down to just above the lowest cusp death point, at least lower than the other cusp death points. Remark that each of two end singular surface braids is unchanged up to equivalence. In particular, we have no non-simple surface braids under the moved new cusp birth point.*

Conjecture 4. *Let a one-parameter cusp family of simple singular surface braids between two non-singular surface braids be given. Assume that the chart of the lower end surface braid is 'trivial' described by a nomad and that the trace of double points is connected. Then, the one-parameter cusp family can be changed into the one-parameter family of non-singular simple surface braids keeping two ends unchanged.*

The proof of Theorem 3 is a main part of this paper. The idea of the proof depends on the chart description of the one-parameter cusp family of singular surface braids and will be explained later in §5.

Theorem 2 works only for the orientable but possibly singular surfaces. In the case of higher genus Theorem 3 is valid but the discussion about Conjecture 4 becomes too complicated.

4. OUTLINE OF DISCUSSION PROVING THE MAIN CONJECTURE

First we get a generic one-parameter family of smooth maps from S^2 to $\mathbb{R}^4$ which connects the given surface knot to the standard one with only cusp births and deaths by Theorem 1. We can move the times of cusp births to the very beginning and those of cusp deaths to the very end. This means that after introducing the same numbers of self-intersection double points by the cusp births from each end, two singular knots at the new ends are ambient isotopic to each other.

Moreover, after introducing the self-intersection double points, we may assume that these self-intersection points do not move during the ambient isotopy. In fact, the cusp birth and death are paired by tracing

the self-intersection points and moving the cusp death points appropriately we can assume that each pair of the cusp birth and death points is located at the same point in $\mathbb{R}^4$ and then we can deform each trace curve of self-intersection double points which is non-singular with respect to the deformation parameter to the straight line fixing the both ends because $1 + 1 < 5$.

We may assume that the given surface knot is a closed surface braid in $D^2 \times S^2 \subset \mathbb{R}^4$ and the standard embedding is a closed 'trivial' surface braid. Since introducing a cusp is given by a fission of a black vertex into a pair of node and black vertex as explained in §2, we can apply Markov type Theorem 2 to the intermediate ambient isotopy of the one-parameter cusp family of singular surface knots above and get a one-parameter cusp family of singular and possibly non-simple surface braids connecting a simple surface braid representing the given surface knot and the 'trivial' surface braid described by a nomad.

Now we take the lowest cusp death point; if there are several ones we take one of them and move its time lower, that is, we make the fusion of a pair of node and black vertex a little later. Then, the corresponding cusp birth point can be pushed down lower than the other cusp death points by Theorem 3. Note that the assumption of Theorem 3 can be satisfied by our construction.

Taking a level surface braid a little above the new lowest birth point as a new upper end, we get a one-parameter cusp family of simple singular surface braids whose lower end is the 'trivial' surface braid described by a nomad and whose trace of double points is connected.

So, if Conjecture 4 is affirmative, we can apply it to the lower part above and replacing this part we get a one-parameter cusp family of singular and possibly non-simple surface braids connecting a simple surface braid representing the given surface knot and the 'trivial' surface braid described by a nomad with one less number of connected components of double points. By an induction on the number of connected components of double points, we get a one-parameter family of non-singular, possibly non-simple surface braids connecting a simple surface braid representing the given surface knot and the 'trivial' surface braid described by a nomad. This means that they are related by a finite sequence of braid ambient isotopies, stabilizations and destabilizations. Therefore, they are ambient isotopic in $\mathbb{R}^4$ by Theorem 25.14 of [3]. So, the given surface knot is unknotted.

5. PROOF OF THEOREM 3

We may assume that the one-parameter cusp family of singular and possibly non-simple surface braids is given by a one-parameter cusp family of the charts for singular and possibly non-simple surface braids. In fact, each chart move is parametrized and the chart description of the one-parameter cusp family is a 2-dimensional object consisting of

the trace of edges and vertices in $S^2 \times \mathbb{R}$ whose intersection with $S^2 \times t$ gives a chart admitting also the diagram on the way of chart moves. Note that the 'boundaries' of this object are the trace of black vertices and only the simple boundaries touch with the trace of nodes at the cusp birth or death points. The trace of black vertices and node are the main objects because they determine the braid monodromies.

At first we would like to move the stabilizations and destabilizations out of the range so that we can consider the one-parameter cusp family to be one of singular surface braids of constant degree. To do so we need to move the time of each stabilization to the very beginning. We start with a stabilization with the smallest degree n. This is described by a newly appeared free edge with label n at the stabilization time. If there is no destabilizations (of degree n) upper than it, there is no problem to push up the free edge to the very beginning. When our rising free edge meets the time of destabilization, if the free edge with label n which disappears as a destabilization locates in the same complementary region of the singular surface braid of degree n consisting of the components other than each free edge with label n, we can connect these free edges with label n directly; then a pair of stabilization and destabilization of degree n will disappear and get a local family of singular surface braids of degree $n + 1$. Even if two free edges are in the distinct regions, these charts are related by a finite sequence of chart moves as in Exercise 25.11 at pp.189-190 of [3]. So, by inserting a parametrized family for these chart moves into the original one-parameter cusp family, the condition above is satisfied and a pair of stabilization and destabilization of degree n disappears. Hence, we can push up all the stabilizations to the very beginning by an induction on the number of stabilizations of degree n and an induction on n. All the destabilizations are also pushed down to the very end.

So, we may assume that we are given a one-parameter cusp family of singular surface braids of degree m which satisfy the same condition although the end charts for singular surface braids may be stabilized .

Next we modify the chart for the singular surface braid at the time of the corresponding cusp birth. The cusp birth, which is a fission of a simple black vertex into a pair of a node and a simple black vertex with the same label, has the label n with $n \leq m - 1$. If $n < m - 1$, we take two concentric simple loops with the same orientation and the labels $n + 1, n$ near the cusp birth (or a pair of black vertex and a node) by CI-moves. Then, we push the cusp (the black vertex first and then the node) into the simple loops with labels $n + 1, n$ in this order and get a white vertex by CIII- and CV-moves; the label of cusp birth is changed to $n + 1$. By an induction we can assume that the corresponding cusp birth has the highest label $m - 1$.

Now we take a small interval neighborhood of the cusp birth time. We stabilize this part of the one-parameter cusp family for the singular

surface braids and by the technique shown in the following Figure 3 we can move the node on the newly introduced free 'edge' and become a cusp birth with label m. We repeat this process three times and push down the free edges with labels $m, m+1, m+2$ carefully along the trace of the double point as in the following Figure 4 so that the cusp birth can be pushed down just above the corresponding cusp death. Then, we get the result. Note that we only need to observe not to link with the trace of the black vertices and nodes.
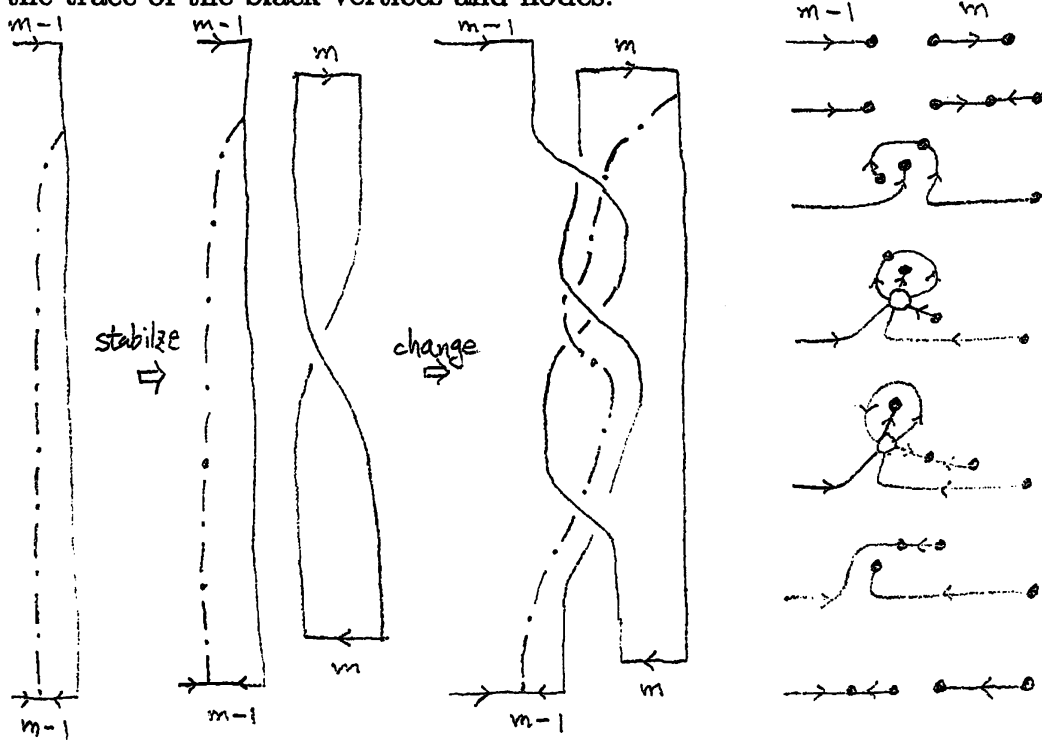
FIGURE 3. Move of the cusp to the stabilization free edge
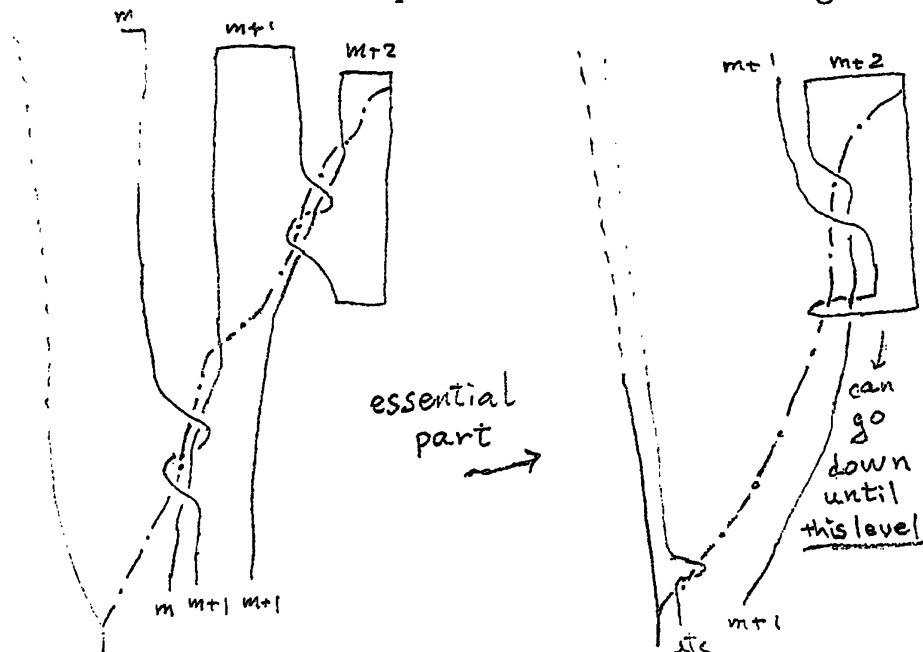
FIGURE 4. The cusp birth which can be pushed down

6. BRIEF DISCUSSION TO PROVE CONJECTURE 4

Since the one-parameter cusp family contains only simple singular surface braids, the traces of black vertices are disjoint and hence we think that the black vertices are fixed on the standard places and not moved during the deformation. On the other hand the trace of node starts from some black vertex and goes down by winding the traces of black vertices and then reach some black vertex.

Since the lower end is a 'trivial' chart or equivalently a nomad, we should take the trace of node backward from the lower end in a generic position. We may assume that the trace of node ends at the black vertex with label 1 by the technique shown in the following Figure 5. (The dotted line indicates a part of the trace of the node.) Originally
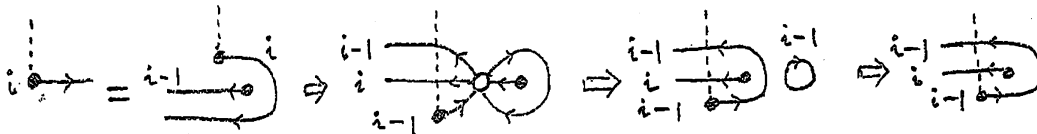
FIGURE 5. Moves to change the end black vertex

the trace of node starts at some black vertex but we start a little later, that is, after the fission into a node and a black vertex. So, the monodoromy at the starting point may be assumed to be σ_i^2. (Alternatively if σ_i^{-2} the argument will be valid by reversing the orientation of all the edges.) The starting chart diagram is a 'trivial' one, that is, one that a free edge of a nomad with label i is changed into a segment consisting of one node, two inward oriented edges and two end black vertices with negative sign.

Now we project generically the trace of node on the starting chart. Then, there might be many self-intersections for the trace of node. But by the method of [6] which pushes the self-intersections over the starting node, we can eliminate the self-intersections of the trace of node. Moreover, after eliminating the self-intersections we get a word by reading the crossing with the oriented edges of the starting 'trivial' chart. Read σ_i (or σ_i^{-1}) when the trace crosses the edge with orientation from right to left (or from left to right resp.) and with label i.

It is proved by Kamada (Lemma in §3.2.5 of [4]) that if this word is considered as an element of the braid group

$$B_m = \left\langle \sigma_1, \ldots, \sigma_{m-1} \; \middle| \; \begin{array}{rll} \sigma_i\sigma_j\sigma_i &= \sigma_j\sigma_i\sigma_j & (|i-j|=1), \\ \sigma_i\sigma_k &= \sigma_k\sigma_i & (|i-k|>1) \end{array} \right\rangle$$

then the monodromy can be changed only by the following three types:

(1) By adding a path $\sigma_j^\delta \sigma_i^\delta$ the change $\sigma_j^2 \to \sigma_i^2$ occurs because $\sigma_j^\delta \sigma_i^\delta \sigma_j^2 \sigma_i^{-\delta} \sigma_j^{-\delta} = \sigma_i^2$ for $|i - j| = 1$ and $\delta = \pm 1$. We have also the inverse operation.

(2) A path σ_i^δ can be added or deleted to the node with the label i because $\sigma_i^\delta \sigma_i^2 \sigma_i^{-\delta} = \sigma_i^2$ for $\delta = \pm 1$.

(3) A path σ_k^δ can be added or deleted to the node with the label i because $\sigma_k^\delta \sigma_i^2 \sigma_k^{-\delta} = \sigma_i^2$ for $|i - k| > 1$ and $\delta = \pm 1$.

It is not so difficult to find a finite sequence of chart moves to add which makes the word trivial in the braid group keeping the ending black vertex unchanged and the starting chart 'trivial' without simple loops by a similar method as in Figure 5. Note that the trace of node starts from the node with label 1 and ends at the black vertex with label 1 in this case.

Therefore, from the word we get a van-Kampen diagram with a starting and ending point, hexagons, cubes and bilateral edges. The word on the van-Kampen diagram is read starting from the starting point and rounding the boundaries in the reverse-clockwise direction .

In the word, essentially in the boundary of some hexagon, there is a part of path $\sigma_i^\delta \sigma_j^\delta \sigma_i^{-\delta} \sigma_j^{-\delta}$ ($|i - j| = 1$, $i, j \geq 2$) illustrated as in the following Figure 6. The modification shown there makes the length of
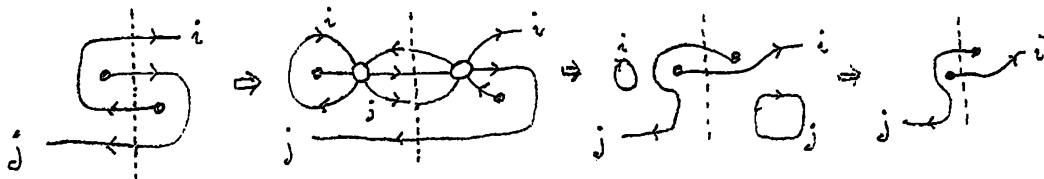

FIGURE 6. Essential move to eliminate the hexagonal relations

the word less at least by two, keeping to have a van-Kampen diagram. So, by an induction we can eliminate all the hexagonal relations except
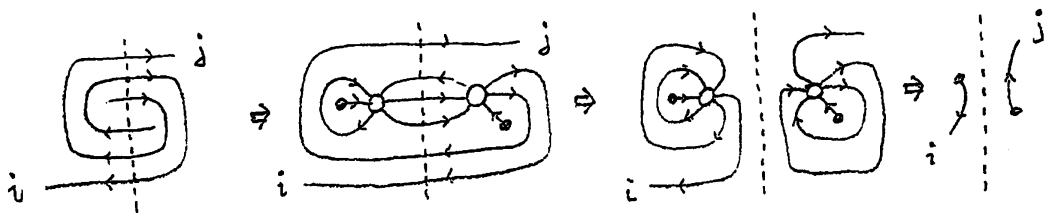

FIGURE 7. Typical application to eliminate the 6-relation

$\sigma_1 \sigma_2 \sigma_1 = \sigma_2 \sigma_1 \sigma_2$ in the word. Note that the relations $\sigma_i \sigma_k = \sigma_k \sigma_i$ for $|i - k| > 1$ are also eliminated by an easier method (possibly except the case $i = 1$ and $k = 3$). There are still no essential simple loops which means that any segment with label i encircled by a simple loop with label j with $|i - j| = 1$, $i, j \geq 2$ has no intersection with the trace of node.

Now we have to divide the cases of relation figures into two types: (A) The two ends of the trace of node for one hexagonal relation are in the same complementary region or can be connected to the final positions and (B) Otherwise. (Some examples are shown in Figure 8 in §7.) If there is no type B, almost the same argument as the previous one is applicable. Even if there is some type B, we can eliminate the hexagons in the remaining van-Kampen diagram but the ending point may not coincide with the starting point. The final condition is that the word essentially contains only σ_1 and σ_2 without trivial relation $\sigma_i \sigma_i^{-1}$ and hexagonal relation does not simplify the word and that there is no essential simple loops in the chart. Then, we can essentially eliminate the elements $\sigma_i^{\pm 1}$ ($i \geq 3$) in the word.

The sequences of examples which satisfy the final condition are enumerated (some of them are illustrated in Figure 9 in §7) and their words are simplified to nul. I think all the examples are exhausted essentially. Note that this does not mean that some example of type B is actually concerned.
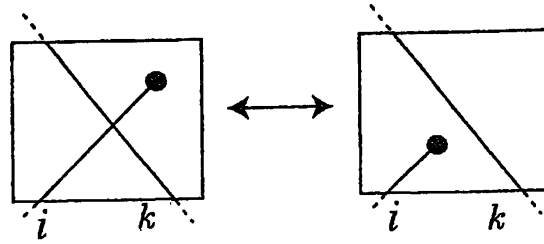
Finally the word would become nul. Then, we can easily understand that the chart after the cusp death is equivalent to the 'trivial' one described by a nomad. To make sure see the Figure 10 in §7.

Since the tentative proof is too delicate, I did not name Theorem 4. But I hope this paper can be thought as a kind of first announcement.
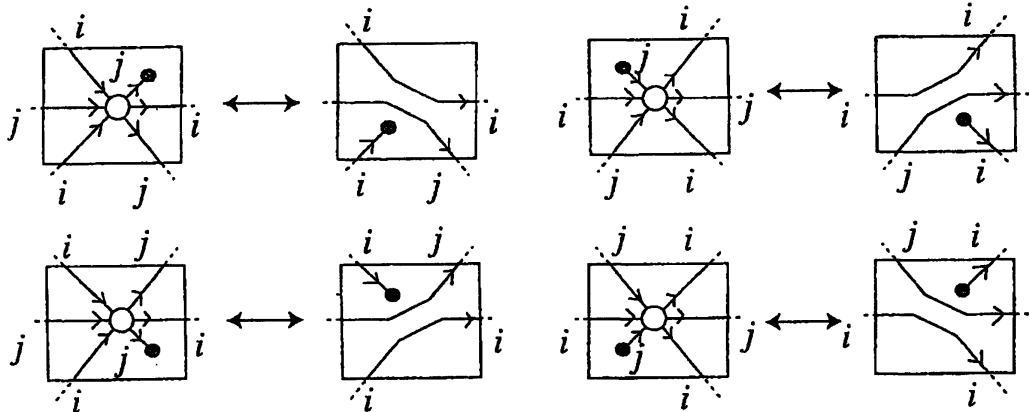
7. FIGURES 1-2 AND 8-10
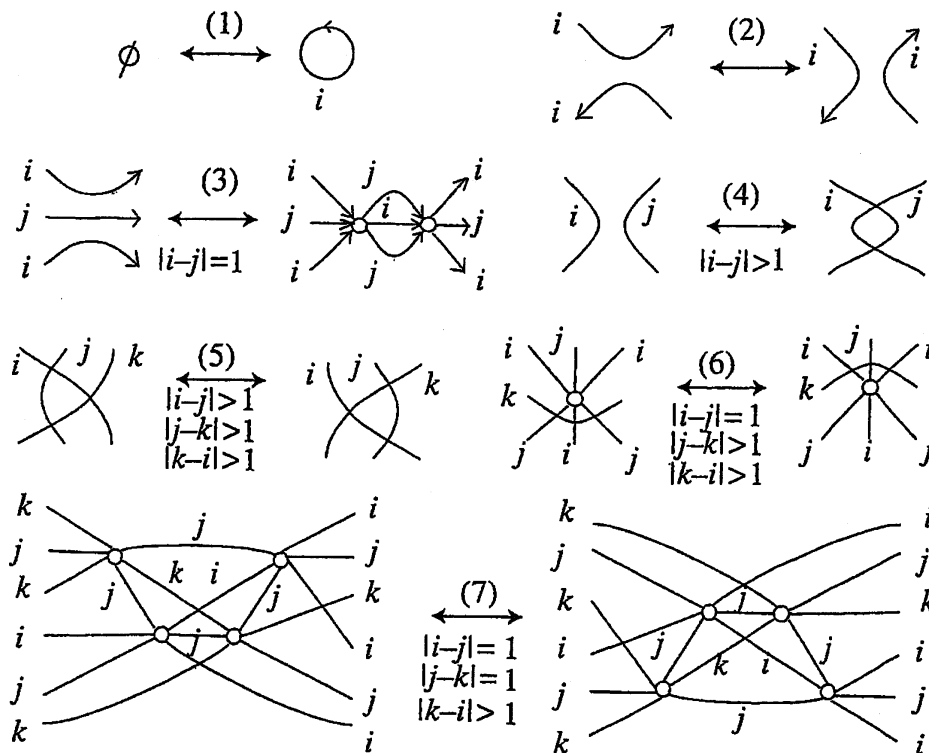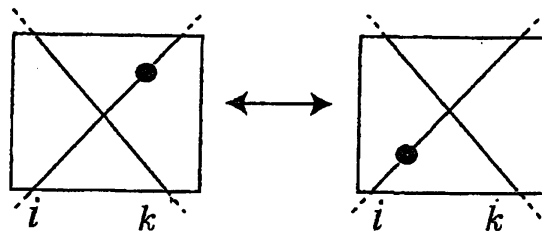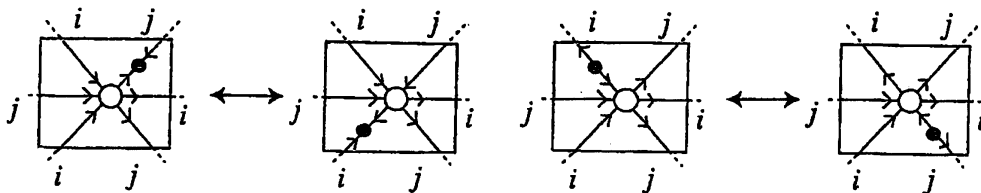

Some CI-moves from [3]

CII-move from [3]

CIII-move from [3]

FIGURE 1. Chart moves I-III

CIV-move from [3]

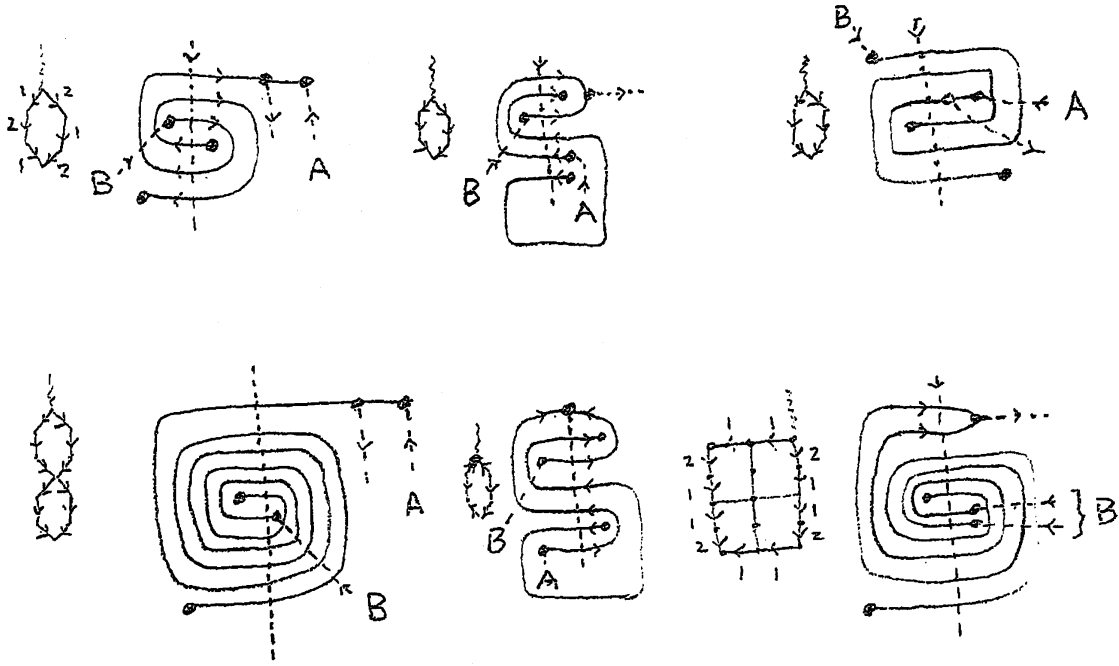CV-move from [3]

FIGURE 2. Chart moves IV-V

13

FIGURE 8. Some examples of Types A and B

$$21\underbrace{121121\cdots121}_{3\times\text{even}}$$

$$21\underbrace{121121\cdots12}_{3\times\text{even}}$$

$$21\underbrace{212121\cdots212}_{3\times\text{odd}}$$

$$21\underbrace{212121\cdots121}_{3\times\text{even}}$$
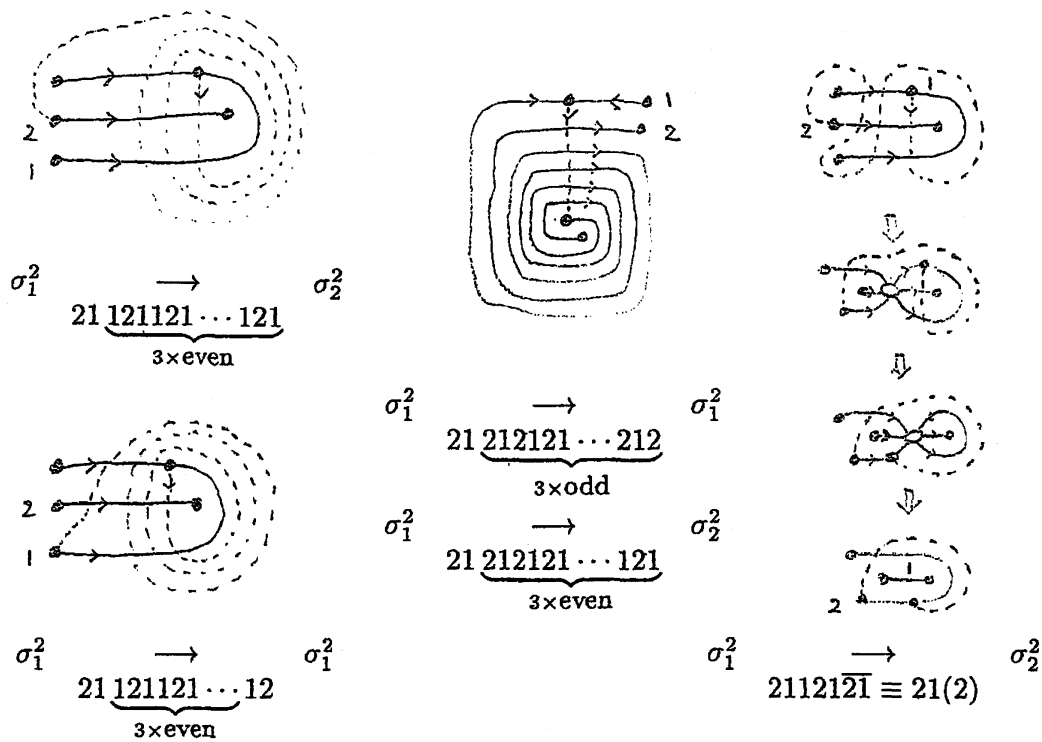
$$2112121\overline{21} \equiv 21(2)$$

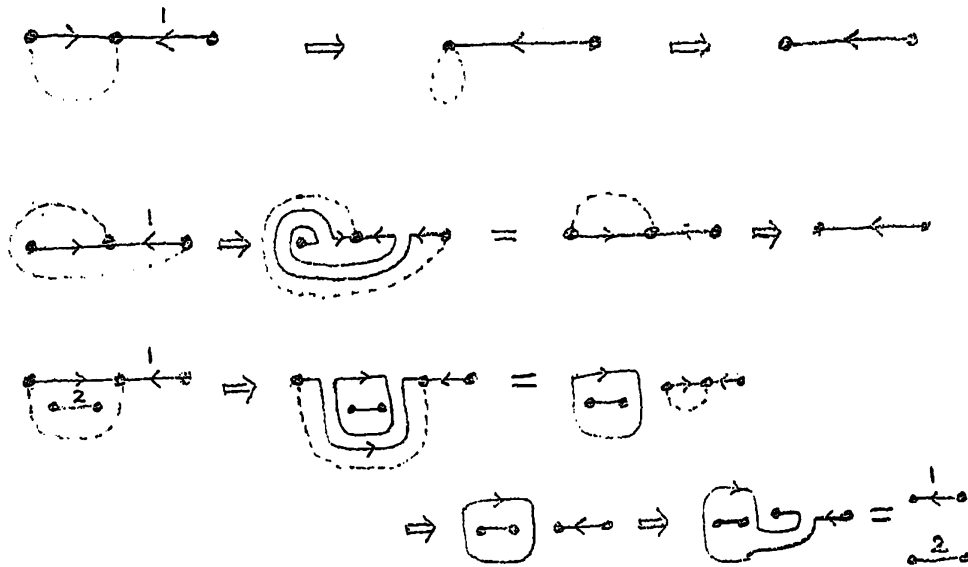FIGURE 9. Some examples satisfying the final condition

FIGURE 10. The final move to get nomad

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