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A Survey on Decision Time

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Abstract

This paper reviews the literature on decision time, which has been considered as essential in many fields including psychology and marketing science. After summarizing the studies in these fields as well as in economics, we also examine the methodologies employed therein. Finally, we introduce Koida (2001), a recent attempt to incorporate decision time into the traditional decision theory.

Keywords: decision time, psychology, marketing science, decision theory, conflict, incomplete preferences
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1 Introduction

Decision time[1] has been considered as an essential element in many fields such as psychology, marketing science, and neuroscience because it provides valuable information on the decision maker's (DM) cognitive process: suppose that decision problem A' is derived from decision problem A by adding an extra stage. Then, the decision time for the problem A' minus the decision time for the problem A can be interpreted as the time required to process the task induced by the extra stage. By varying the combination of such decision problems, we can obtain much information on how decision can be reached in DM's mind. This viewpoint prevails in psychology and marketing science (for an extensive survey, see, e.g., the references provided by Tyebjee 1979) and is also discussed in economics (Gabaix et al. 2006).

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[1]Decision time is also referred to as *response time* or *contemplation time*, depending on the field involved.

Decision time is also crucial in other situations. For example, if the DM searches the alternatives in the feasible set one by one instead of considering them all at once, the time that the DM is willing to take before making a decision may depend on the composition of the alternatives (as well as the number of alternatives) in the feasible set. On the other hand, if two alternatives are equally attractive and thus cause conflict in her mind, the DM may have difficulty in choosing one of them, which has an impact on decision time.

The objective of this paper is to review the literature relevant to decision time in psychology, marketing science, and economics, as well as the methodological tools employed therein. The crucial point is that we attempt to unify the implications derived from each field, which tend to be discussed separately. Psychological and marketing-scientific approaches provide more empirical findings, whereas economic approach provides more rigorous foundations. Thus, we expect that by synthesizing these approaches, a formal description of the empirical facts on decision time can be obtained. In the later part of this paper, we introduce Koida (2011), a recent development on decision time from a decision-theoretic point of view, on the basis of this idea.

The remainder of this paper is organized as follows. The next section surveys studies relevant to decision time in psychology, marketing science, and economics, and reviews basic results. We examine methodological ideas to analyze decision time in Section 3, anticipating the explanation of Koida (2011) in Section 4. Concluding remarks are made in Section 5.

2 Decision Time in Various Fields of Science

In this section, we review studies on decision time in psychology, marketing science, and economics, and summarize basic results.

2.1 Psychology and Marketing Science

We can attribute the first study specifying the relationship between decision time and the nature of a decision problem to Cartwright (1941). He proposed a model in which two alternatives create the opposite "forces" in mind that is determined by the attractiveness of the alternative. The two forces eventually drive the DM to choose an alternative, depending on the balance between them. For instance, if the DM is about to choose between an apparently good alternative and a mediocre alternative, the force corresponding to the former is much stronger than the one corresponding to the latter. Eventually, the DM reaches a decision to choose the former after a relatively short decision time. On the other hand, if she is forced to choose between almost equally appealing alternatives, the velocities of the forces induced by them are nearly equal. Then, the DM takes time to make a decision between them. As we will discuss, this view conforms with the studies of conflict involved in decision making, which is one of the central themes in psychology.

Tyebjee (1979) examined in a brand choice problem the relationship between decision time and the following two psychological measures: the first was the degree of conflict that a decision problem induces and the second was the DM's involvement in a decision problem. The experiments that he conducted imply that the degree of conflict did have a significant impact on decision time, whereas the influence on decision time by involvement was not significant. Moreover, he indicated that decision time becomes longer if conflict intensifies not only in the sense of the attractiveness of alternatives becoming closer, but also in the sense of the number of alternatives in the feasible set increasing.

On the other hand, the authors such as Tversky and Shafir (1992) and Dhar (1997) consider choice deferral, i.e., the DM not making a choice ex ante and does make a choice ex post (or at some later period), instead of directly considering decision time. Tversky and Shafir conduct a variant of the experiment considered by Huber et al. (1982) and demonstrate that if a "no-choice" option is available, the DM is more likely to exploit it in a situation involving conflict than in a situation involving no conflict. In the second experiment of their study, they indicate that the DM is more likely to choose a no-choice option as the number of alternatives in the feasible set increases, which reinforces the Tyebjee's results that we mentioned above.

By conducting experiments in a different setting, Dhar demonstrates that intensifying conflicts both in the sense of the increasing number of alternatives in the feasible set and of the decreasing difference in the attractiveness of alternatives lengthen decision time (which also conforms with Tyebjee (1979)), whereas the impact of the number of conflicting attributes is insignificant.

Overall, these psychological and marketing-scientific studies successfully depict the relations between decision time and various psychological effects. We should note, however, that they often lack an axiomatic foundation and even a formal definition of key concepts such as conflict. Hence, we need to formalize their ideas to interpret decision time in an economic context as a measure of some reasoning process.

2.2 Economics

2.2.1 Search

One of the economic models relevant to decision time is search since it can be easily interpreted as a description of the internal cognitive process of the DM until she reaches a final decision.

This strand of studies was initiated by the seminal work by Stigler (1961), and recently, Gabaix et al. (2006) and Caplin and Dean (2011) treat decision time in this context more explicitly. These studies assume that the DM considers an alternative in the feasible set one by one rather than all at once,[2] and chooses the "optimal" one from a certain point of view. The time elapsed to make a final decision is interpreted as decision time.

An important class of strategies to choose such an optimal alternative is that the DM terminates search if the value of the current alternative exceeds a threshold (reservation utility) while she continues otherwise. Obviously, the higher the reservation utility is, the longer the decision time becomes.

Major factors that affect reservation utility are summarized twofold. First, in search under uncertainty, the higher the variance of alternatives that the DM confronts is, the higher the reservation utility becomes (see, e.g., Kohn and Shavell 1974). This is because a higher variance implies for the DM a better chance to encounter an extremely good alternative (as well as an extremely bad alternative) in the future, which can be paraphrased that decision time becomes longer as the heterogeneity of alternatives intensifies.

Second, decision time in a search model depends on the perfectness of recall of the alternatives that the DM encountered in the past: if the DM has perfect recall, i.e., she can choose the best alternative among all the alternatives that she encountered in the past, she is willing to take a longer time to throughly search as many alternatives in the feasible set as possible. On the other hand, if she has no recall, i.e., the alternatives that she encountered in the past cannot be chosen, the reservation utility is relatively low because the DM is willing to secure the current alternative if it is satisfactory to a certain degree. Karni and Schwartz (1977) investigate the case of imperfect recall, i.e., the DM can get back to the alternatives that she encountered with the probability that is positive but not necessarily equal to one, in which case the reservation utility is between the levels corresponding to perfect recall and no recall.

It is true that the search approach provides a tractable way to relate economic models with decision processes. However, some empirical studies contradict the results derived from the search models. In particular, Dhar (1997) demonstrates that the heterogeneity of alternatives in the feasible set tends to shorten decision time as opposed to the implications indicated in the previous paragraph.[3]

[2]Thus, the DM may be unaware of the existence of some alternatives in the feasible set, even if they yield higher utility than the one that is eventually chosen. In other words, the DM who follows this search procedure has limited consideration sets.

[3]Precisely, Dhar considers choice deferral in his model rather than decision time itself.

2.2.2 Learning

Natenzon (2010) pioneers a leaning model that can accommodate the attraction effect (Huber et al. 1982) in the short run, and the similarity hypothesis (Tversky 1972) in the long run. The key idea is as follows: suppose that alternatives x, y, and z yield the same utility while x and z are similar (the similarity is measured by the correlation between random variables). At an earlier stage of the learning process in his model, the similar alternatives x and z provide more accurate information about the utility that they yield than the dissimilar alternative y, which renders the probability that y is chosen nearly zero. On the other hand, after learning continues for a sufficiently long time, the similar alternatives x and z tend to be treated as a group rather than two independent alternatives, and each of $\{x, z\}$ and y is chosen with nearly equal probability, which implies at infinity that y is chosen with higher probability than x and z.

Decision time (contemplation time, in his terminology) in this model is defined by the length of time after which decision is reached. However, it is *exogenous* rather endogenous, i.e., the DM is forced to make a decision at a predetermined period. Thus, his model cannot specify the relationship between the composition of alternatives in the feasible set and the time that the DM *spontaneously* spends before reaching a decision.

2.2.3 Decision Theoretic Approach

Some studies in decision theory are relevant to decision time. They mainly focus on choice deferral, i.e., the DM postponing a decision to the future, rather than decision time itself.

The literature is divided into the following two approaches. The first (e.g., Kopylov 2009) employs an incomplete preference. Let $\succsim$ and $\succsim'$ be the ex ante and ex post (generally incomplete) preferences, respectively, and suppose that we have neither $x \succsim y$ nor $y \succsim x$ for some alternatives x and y. If, in addition, we have neither $x \succsim' y$ nor $y \succsim' x$, the ranking between x and y is undetermined both ex ante and ex post. However, if we have either $x \succsim' y$ or $y \succsim' x$, the DM does not make a decision ex ante while she does ex post, which can be interpreted that the DM takes time to reach a decision between x and y.

Note that a crucial premise for this interpretation is the *consistency axiom*, which states that the ranking is settled *ex post* if the ranking between alternatives is settled *ex ante*. Under this axiom, once decision is reached between some alternatives, the DM will not be ever indecisive between them in the future. We can uniquely define decision time by exploiting this axiom as we will discuss later.

The second approach (e.g., Buturak and Evren 2010) uses a choice function instead of a preference. For a feasible set X of alternatives, let $C(X)$ denote the alternative chosen from X. Then, $C(X) = \phi$ implies that the DM chooses nothing and possibly defers choice until the next opportunity arrives. Note that unlike the first approach, this allows to consider a choice among more than two alternatives.

Both approaches hint a tractable description of decision time within the context of the traditional decision theory. However, these models have the following two limitations. First, they can not explicitly treat decision time since they focus on a one-step deferral of choice rather than postponing a decision for multiple periods. Because of this, we are unable to, say, conduct the comparative statics of decision time with respect to the nature of decision problems. Second, although they provide a behavioral foundation for decision time (choice deferral) such as the consistency axiom, internal cognitive processes that affect decision time are not necessarily specified in these models. In Section 4, we attempt to overcome these difficulties by extending these approaches.

3 Methodologies

Anticipating the discussion in Section 4, we summarize key psychological or economic notions relevant to decision time in this section.

3.1 Conflicts

As we mentioned in Section 2.1, many psychologists regard conflict as a major source for lengthening decision time. Festinger (1957) first pointed out the importance of conflict in psychology, while Cartwright (1941) proposed a model to bridge conflict and decision time. Tyebjee (1979) conducted an experiment to test the implication of these theories. Now, we formalize these ideas.

Consider the experiment conducted by Tversky and Shafir (1992): there involves conflict in choice between alternatives x and y, i.e., x is more attractive in terms of attribute 1 (x_1) while y is more attractive in terms of attribute 2 (x_2). Next, consider an alternative x' whose attractiveness is close to that of x but is dominated by x in terms of both attributes 1 and 2 (Figure 1). Permitting the subjects to choose a no-choice option, the authors first ask them to choose an alternative from the feasible set $A = \{x, y, \phi\}$ (ϕ denotes a no-choice option) in the first treatment, and from $B = \{x, x', \phi\}$ in the second treatment. The authors demonstrate that the ratio of subjects who choose ϕ from A is significantly higher than the ratio of those who choose ϕ from B, which implies that a choice involving conflict takes more time than a choice involving no conflict.

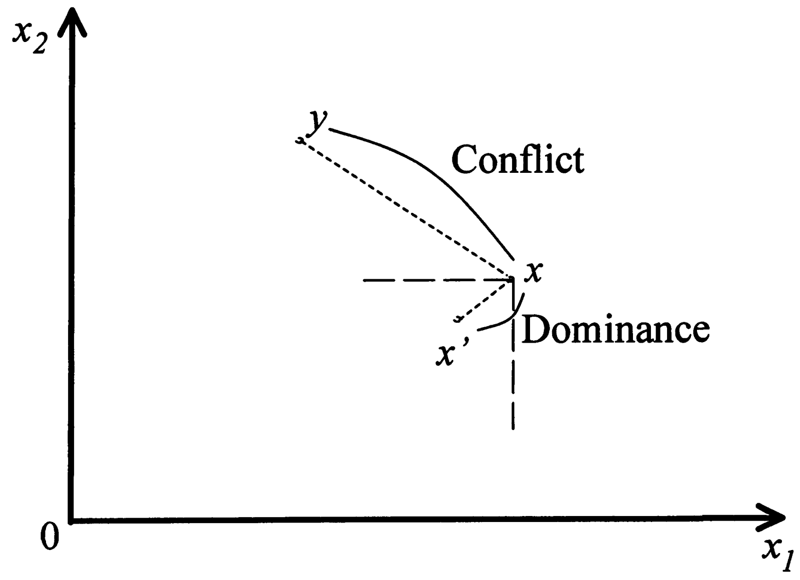

Figure 1: Conflict and Dominance

Moreover, the common findings in psychology that we discussed in Section 2.1 imply that decision time becomes longer if (a) the difference in the attractiveness of conflicting alternatives decreases, and (b) the number of alternatives in the feasible set increases. By using the notation above, the condition (a) implies that the ratio of those who choose ϕ from $\{x, y, \phi\}$ increases as the utilities that x and y yield become closer, while (b) implies that the ratio of those who choose ϕ from $\{x, y, z, \phi\}$ is higher than that of those who choose ϕ from $\{x, y, \phi\}$, where neither of x, y, and z dominates another.

3.2 Incomplete Preferences

In decision theory, incomplete preferences have been used to describe indecisiveness between two alternatives (Aumann 1962; Dubra, Maccheroni, and Ok 2004). In particular, Dubra, Maccheroni, and Ok propose *expected multi-utility theory*, in which alternative x is preferred to y if and only if x yields higher expected utility than y for *all* the utility functions u in a certain set $\mathcal{U}$. Although they do not apply their model to choice deferral or decision time, it can be interpreted as a description of indecisiveness due to conflicting values in DM's mind (in the case of single-person decision making), or among the members of a group (in the case of group decision making).

On the other hand, Kopylov (2009) models choice deferral by employing incomplete preferences. As we mentioned in Section 2.2.3, the key premise for his characterization is the consistency axiom. It is first proposed by Gilboa, Maccheroni, Marinacci, and Schmeidler

(2010) to characterize the relation between Knightian uncertainty models à la Bewley (1986) and à la Gilboa and Schmeidler (1989). Kopylov also considers choice defferal in this context, whereas imposing this axiom is crucial to the model that we will discuss in the next section.

4 A Quantitative Study: Koida (2011)

As we have seen, several approaches have been proposed to analyze decision time. In this section, we introduce Koida (2011), a recent attempt to unify psychological and economic (decision-theoretic) approaches.

Let X be the finite set of prizes and denote by $\mathcal{P}(X)$ the set of alternatives (lotteries). Since X is finite, we identify a von Neumann-Morgenstern utility function $u : X \rightarrow \Re_+$ with a utility vector $u' = (u_1, \cdots, u_{|X|}) \in \Re^{|X|}$ such that $u_i = u(x_i)$ for $i = 1, \cdots, |X|$ and a signed measure p on X with a vector $p' = (p_1, \cdots, p_{|X|}) \in \Re^{|X|}$. Then, the expected utility $\int_X u(x)dp(x)$ can be identified with the inner product $u \cdot x$. (We use these interpretations interchangeably in the following.) For time $t = 0, 1, \cdots$, let $\succsim_t$ be the preference relation on $\mathcal{P}(X)$ at time t.

The author extends expected multi-utility theory to this dynamic setting and derives the following representation of $\succsim_t$ for all t:[4]

$$p \succsim_t q \text{ if and only if } \int u(x)dp(x) \geq \int u(x)dq(x) \text{ for all } u \in \mathcal{U}_t,$$

where $\mathcal{U}_t$ is a closed and convex cone of utility vectors. In other words, p is preferred to q at time t if and only if p yields higher expected utility than q for *all* the utility functions in $\mathcal{U}_t$.

A salient property of this representation is that the preference depends on *vector differences* between two alternatives. Figure 2 illustrates this point. For alternatives p, q, p', q', p'', and q'', let $s = q - p$, $s' = q' - p'$, and $s'' = q'' - p''$ be the vector differences of two alternatives, u_1 and u_2 be the (extreme) elements of $\mathcal{U}_t$, and h_1 and h_2 be the hyperplanes that have normal vectors u_1 and u_2. (Note that each utility vector u in $\mathcal{U}_t$ corresponds to a hyperplane h whose normal vector is u, and $\int u(x)ds(x) \geq 0$ if s is in the positive half space induced by h.)

Now, q' is preferred to p' since s' is in the intersection of positive half spaces induced by all the utility vectors in $\mathcal{U}_t$, and thus we have

$$\begin{aligned} \int u(x)ds'(x) &> 0 \\ \Leftrightarrow \int u(x)dq'(x) &> \int u(x)dp'(x) \end{aligned}$$

for all $u \in \mathcal{U}_t$. Similarly, p'' is preferred to q'' since s'' is in the intersection of negative half spaces induced by all the utility vectors in $\mathcal{U}_t$. However, neither of p nor q is preferred to the

[4] Shapley and Baucells (2008) derive a similar representation with another approach.

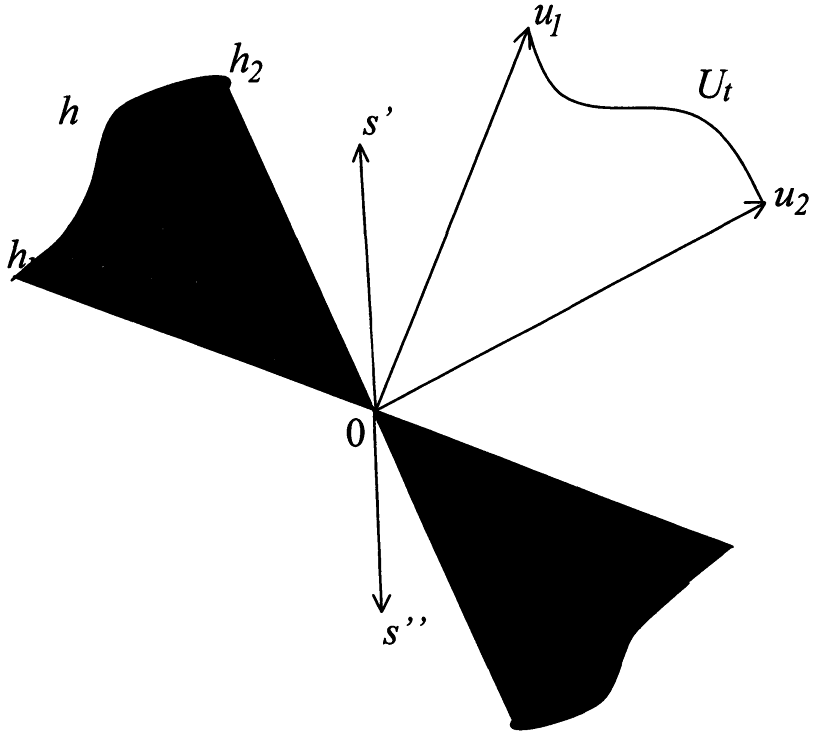

Figure 2: Indecisive Region

other since s is in the positive half spaces induced by some utility vectors (such as u_1) in $\mathcal{U}_t$ while it is in the negative half spaces induced by other utility vectors (such as u_2) in $\mathcal{U}_t$. Thus, we refer to the shaded area in the figure as the *indecisive region at time t*. The last property formalizes the intuition that the ranking between alternatives p and q is indecisive because of conflicting values such as u_1 and u_2, which is in DM's mind (in a single-person case) or in a group of DMs (in a multiple-person case).

Moreover, since we impose the consistency axiom as in Kopylov (2009), the ranking between alternatives will never be indecisive again once it is determined at some time, which implies that the indecisive region generally shrinks over time. Intuitively, this indicates that the conflict of values is resolved over time, which renders it easier for the DM to make a decision between alternatives. This property enables us to *uniquely* define decision time by the earliest time that the ranking between alternatives is decisive. Formally, we define decision time $\tau(p,q)$ over alternatives p and q such that (a) neither $p \succsim_{t-1} q$ nor $q \succsim_{t-1} p$ for all $t < \tau(p,q)$, and (b) $p \succsim_t q$ for all $t \geq \tau(p,q)$ or $q \succsim_t p$ for all $t \geq \tau(p,q)$.[5]

An advantage of this approach is that it allows for comparative statics: suppose that $\mathcal{U}_t$

[5]We define $\tau(p,q) = 0$ if $p \succsim_t q$ for all t or $q \succsim_t p$ for all t, while $\tau(p,q) = \infty$ if neither $p \succsim_t q$ nor $q \succsim_t p$ for all t.

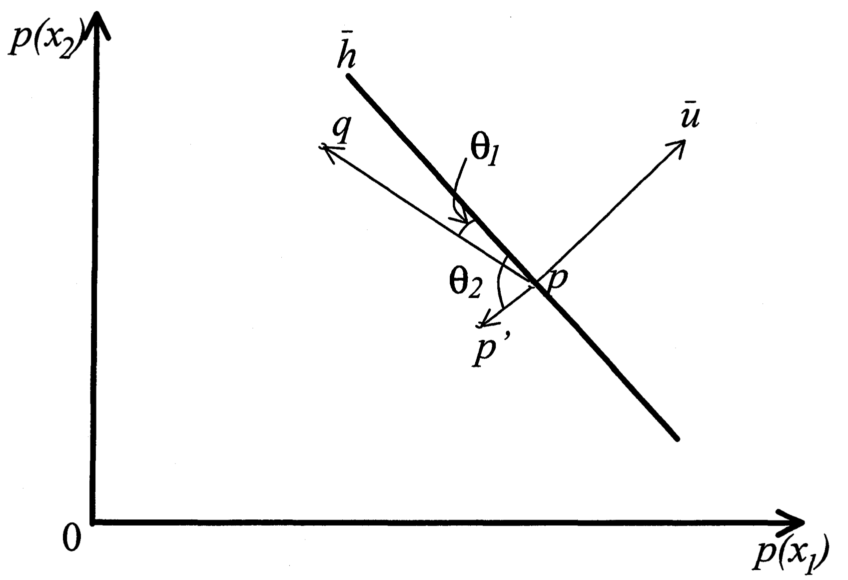

Figure 3: Direction of Conflict

are circular cones for all t, which converges to a singleton $\{\bar{u}\}$ as t goes infinity. Let $\bar{h}$ be the hyperplane whose normal vector is $\bar{u}$. Then, we indicate that the *angle* that the vector difference $q - p$ forms with the hyperplane $\bar{h}$ determines decision time $\tau(p, q)$: the smaller the angle is, the longer the decision time becomes.

This result explains the implications of Tversky and Shafir's (1992) work that we mentioned in the previous section. Let p, q, and $p' \in \mathcal{P}(X)$ be the counterparts of x, y, x' in their experiment, i.e., p yields prize x_1 with higher probability than q while q yields prize x_2 with higher probability than p, and p dominates q (Figure 3). Suppose that $\mathcal{U}_t$ converges to a singleton $\{\bar{u}\}$ in the long run that induces a hyperplane $\bar{h}$, and the angle θ_1 formed between vector difference $q - p$ and $\bar{h}$ is smaller than the angle θ_2 formed between $p' - p$ and $\bar{h}$, which is the case, say, if α' and β are indifferent under the utility function $\bar{u}$, and p' is sufficiently close to p. Then, the result mentioned in the previous paragraph indicates that the decision time for a pair (p, q) is longer than the one for a pair (p, p'). Further, in a reasonable setting in which $-u \notin \mathcal{U}_t$ for all $u \in \mathcal{U}_t$, the decision time for a dominant-dominated pair such as (p, p') equals 0.

Further, this model also conforms with the common psychological finding that identifies the degree of conflict with the proximity of the attractiveness of alternatives: let p, $q \in \mathcal{P}(X)$ be as defined above. If we fix p and make q closer to the hyperplane $\bar{h}$ (letting neither of p and q dominate the other), the angle θ_1 between vector difference $q - p$ and hyperplane $\bar{h}$ becomes smaller. This also implies that the difference in the expected utility (i.e., attractiveness) of p

and q with the utility function $\bar{u}$ also becomes smaller. Thus, for such a pair of alternatives, decision time lengthens as the difference in the expected utility of the alternatives decreases.

5 Concluding Remarks

In this paper, we have seen various approaches to characterize decision time in various fields and introduce the recent development by Koida (2011). In particular, Koida's model provides a decision theoretic (axiomatic) foundation for decision time under conflict, as well as it conforms with many empirical findings in psychology.

As the other models surveyed in this paper, this model has its own limitation. Above all, since it focuses on a binary choice, it cannot analyze the effect of an increase in the number of alternatives in the feasible set lengthening decision time, which has been indicated by many studies. Our future research will tackle this issue.

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