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Independence, Coupling and Dependence from the viewpoint of Micro-Macro duality

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Abstract

We explain the basic notions pertaining to the scattering processes in terms of asymptotic fields and of S-matrix as a kind of central limit theorem. These notions can be used for understanding the highly dynamical behaviours of strongly interacting hadrons, from the viewpoint of the duality involving independence, coupling and dependence, which may have some interesting relations with the monotone and/or free independences.

1 Introduction: Hadrons and Bacteria as "Unsung Heros" behind Nature

In all the physical nature, the ***hadronic world*** is characterized by its ***extreme activity*** and its ***longest history of existence*** (of the level as a whole); we cannot imagine and verify the possibility of historical period without its activities and existence; for instance, the history of universe with evolution of stars starts from ***protons*** as the most typical hadrons. Similar situation can be found in the roles played by the ***bacterial*** levels in the biological context, as was emphasized by Stephen Jay Gould in his book, "Full House – The Spread of Excellence from Plato to Darwin" (Harmony Books, 1996)[1]. This kind of aspects will be seen to be crucial and indispensable for our satisfactory understanding of the ***consistency*** between ***repeatable laws*** and their ***historical developments without repetitions***, as seen in the cosmological and biological evolutions. At the end, we try to examine this problem in the realm of quantum fields and hadrons, from the viewpoint of ***Micro-Macro duality*** [1], which will hopefully be useful for unified understanding of nature according to the longitudinal axis of its historical processes and to the transverse ones of coexisting network structures

[1]This interesting book was brought to my attention by Prof. I. Yamato at Tokyo University of Science, to whom the present author expresses his deep gratitude.

spanned by various bridges among different hierarchical regimes in nature. After explaining Micro-Macro duality, we recollect the formulation of scattering amplitudes (= S-matrix functional) in terms of quantum fields, on the basis of which basic features of hadrons are examined.

While there are no strict boundaries between micro- and macroscopic levels in nature, it is important to specify such a boundary in a scientific discussion of a given restricted domain, for the purpose of which the notion of "sectors" plays a crucial role. The essence of the sector structures found in various areas in nature can be summarized in the context of Micro-Macro duality as follows, where a "sector" is interpreted as quasi-equivalence class of factor states [1]:

⟵ Visible *Macro*	of ***independent objects***	⋯	⟶	***Inter-sectorial***
⋯ γ_N	***sectors*** γ	γ_2	γ_1	$Sp(3)$
⋮	⋮	⋮	⋮	↑ ***Intra-sectorial***
⋯ π_{γ_N}	π_γ	π_{γ_2}	π_{γ_1}	‖
⋮	⋮	⋮	⋮	↓ invisible ***Micro***

,

or, in a little more elaborated form:

[[***Independence***]]	gravity ↷ spacetime $\mathcal{O}$: Spec	
El.-mag. interactions ↷	↑↓	
Rep : S-matrix	⇆ Micro-Macro ***coupling*** ⇆	hadronic sectors of ***resonances*** : Alg
	↑↓	↻ ***weak interactions***
	Hadronic intra-sector = ***Regge trajectories*** with ***strong int.*** : Dyn	[[***Dependence***]]

.

A good physical example of the mathematical notion of ***statistical independence*** can be found in the form of *asymptotic fields* arising in the scattering theory of relativistic quantum fields through the asymptotic condition, which is nothing but a version of ***Central Limit Theorem*** [3] in the physical context. Once the independent objects are successfully identified, the essence of the most important tasks in mathematical and physical descriptions of natural phenomena can be found in the problem concerning the gaps between ***idealized*** (= ***approximate***) ***world of independence*** and ***realistic interacting world of dependence***, which are to be filled up by the scheme of ***coupling*** = ***interactions*** [3]. In the case of scattering theory of QFT, the basic scheme can be understood in the following diagram:

out	S-matrix & PCT	in	
$Ad\Theta^{out} \curvearrowright \phi^{out}(x)$	AdS & $Ad\Theta$ $\leftrightarrows$ AdS^{-1} & $Ad\Theta$	$\phi^{in}(x) \curvearrowleft Ad\Theta^{in}$	Asymp. fields : Macro
$t \to +\infty$ ↖	asymp.cond.	↗ $t \to -\infty$	
	$\varphi_H(x) \curvearrowleft Ad\Theta$		Interacting fields : Micro

.

The Precise meaning of the "Central Limit Theorem" can be seen in "***Micro-Macro Duality***" ***in QFT*** in the following sense:

[[Macro]]	
Asymp. fields ϕ^{as} as ***indep.*** objects s.t. $(\Box + m^2)\phi^{as} = 0$ $\iff p^2 = m^2$: on-shell cond.	↖ ***asymp. cond.*** = ***CLT***
GLZ-Fock expansion of φ_H ↘	***Interacting fields*** φ_H Yang-Feldman eqn: $\varphi_H = \Delta_{ret} * J_H + \phi^{in}$
Coupling $J_H := (\Box + m^2)\varphi_H$	[[Micro]]

or,

Macro	S-matrix & PCT	Micro
ϕ^{as}: universal	*asymp.cond.* $\leftrightarrows$	φ_H : generic
$p^2 = m^2$ $\iff$ $(\Box + m^2)\phi^{as} = 0$	GLZ-Fock expansion of φ_H in ϕ^{as}	$(\Box + m^2)\varphi_H = J_H$ $\iff$ $\varphi_H = \Delta_{ret} * J_H + \phi^{in}$

.

(Remark: there is another local-net version of independence based on the so-called *nuclearity condition* in Algebraic QFT.)

It is the aim of the present article to clarify the precise meaning of the notions and the diagrams appearing above.

2 What does Einstein's Formula $\langle\langle E = mc^2 \rangle\rangle$ Mean?: "Unit" of *Independence* = Free Particles

In Quantum Probability, several versions of ***independence*** generalizing bosonic tensor type have been proposed, developed and classified with interesting results [2]. Here my naïve questions are: *on which physical ground* do they emerge and *what physical meaning* do they have? For Gaussian (=bosonic CCR & fermionic CAR) case(s), the following is my partial answer in the context of relativistic QFT [3]: ***emergence of independence***=

a "Central Limit Theorem" via ***asymptotic condition***, $\varphi_H(x) \overset{x^0=t\to\mp\infty}{\to} \phi^{in/out}(x)$, from ***non-independent*** interacting Heisenberg fields φ_H to ***independent*** free *asymptotic fields* $\phi^{as} = \phi^{in/out}$ *& asymptotic states.* To formulate this problem in a clear-cut way, the notion of the "particles" characterized by the mass-shell condition plays essential roles whose familar version can be found in Einstein's famous formula $E = mc^2$.

Owing to such serious and actual consequences as atomic bombs and nuclear power plants, ***Einstein's famous equality*** $\langle\!\langle E = mc^2 \rangle\!\rangle$ of energy & mass has always been regarded as one of the most fundamental notions of the special theory of relativity. Properly speaking, however, this is a simple and trivial sort of misunderstanding, because this formula is meaningful ***only for asymptotic fields/ states*** as the "***on-shell condition***" to extract ***1-particle modes*** (!!) from the ***interacting*** Heisenberg fields φ_H: if it were not for the ***interactions*** of Heisenberg fields φ_H, any kind of nuclear reactions as the sources of radioactivity cannot take place, and hence, the formula $\langle\!\langle E = mc^2 \rangle\!\rangle$ itself yields ***no actual events***, good or bad!! Its genuine theoretical meaning is simply the condition to define ***independent= free= non-interacting*** asymptotic fields/ states, $p^2 = p_\mu p^\mu = m^2$ containing independent = free = non-interacting particles. The resulting asymptotic fields ϕ^{as}, provide a ***vocabulary for describing state changes*** taking place in the scattering processes: [***asymptotic in-states*** $\overset{\text{S-matrix}}{\Longrightarrow}$ ***out-states***]. For lack of interactions, however, ***on-shell*** asymptotic fields ϕ^{as} by themselves can***not*** ignite scattering processes, and hence, we need to introduce ***off-shell interacting Heisenberg fields*** φ_H, which violate Einstein's formula $\langle\!\langle E = mc^2 \rangle\!\rangle$!

In fact, taking m as "moving mass" $m = \dfrac{m_0}{\sqrt{1-v^2/c^2}}$, we have

$$\begin{aligned}
&E = mc^2 = \frac{m_0}{\sqrt{1-v^2/c^2}}c^2 \\
&\Longrightarrow (m_0c)^2 = (\frac{E}{c})^2(1-v^2/c^2) = (\frac{E}{c})^2 - \left(\frac{m_0}{\sqrt{1-v^2/c^2}}\vec{v}\right)^2 \\
&= (\frac{E}{c})^2 - (\vec{p})^2 \\
&\Longrightarrow p^2 = p_\mu p^\mu = (m_0c)^2,
\end{aligned}$$

where $\dfrac{m_0}{\sqrt{1-v^2/c^2}}\vec{v} =: \vec{p}$ is the relativistic momentum and $p^\mu = (\dfrac{E}{c}, \vec{p})$ is the 4-mementum. The actual meaning of $p^2 = p_\mu p^\mu = (\dfrac{E}{c})^2 - (\vec{p})^2 = (m_0c)^2$ can be seen as follows:

i) ***mass-shell*** (or, ***on-shell***) ***condition*** to characterize a mass hyperboloid in p-space of 4-momenta $p_\mu \in \hat{\mathbb{R}}^4$ carried by free 1-particle states with rest mass m_0.

By this condition an orbit family $p^2 = m^2 > 0$ can be picked up among the four: $p^2 \gtreqless 0, p_\mu = 0$ (: vacua), of Poincaré group $\mathcal{P}_+^\uparrow = \mathbb{R}^4 \rtimes SL(2,\mathbb{C})$ in the Wigner's construction of unitary representations induced from "little groups" ($SU(2)$, $E(2)$, $SU(1,1)$);

ii) through "first quantization" $p_\mu \rightarrow i\hbar\partial_\mu = i\hbar(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla})$, we have the Klein-Gordon equation $[\hbar^2\partial_\mu\partial^\mu + (m_0 c)^2]\phi(x) = 0$ of a free scalar field $\phi(x)$ with rest mass m_0.

iii) The existence of positive/negative energy solutions $E = \pm\sqrt{(\vec{p}c)^2 + (m_0c^2)^2}$ of $(\frac{E}{c})^2 - (\vec{p})^2 = m_0^2c^2$ leads to the creation & annihilation operators, particle-antiparticle pairs, time reversal $\boldsymbol{T}$ and ***PCT invariance.***

Thus, the famous equivalence $E = mc^2$ between energy E and mass m gives only *partial* information for dynamical descriptions of relativistic quantum fields, with ***off-shell*** apects being neglected in spite of their vital importance for non-trivial scattering processes, particle decays and productions, etc., etc.!

2.1 Free=*independent* vs. interacting = *non-independent*

It is also remarkable that the free asymptotic fields ϕ can be decomposed into the sum of creation and annihilation operators $a(\vec{p}).a^*(\vec{q})$. Namely, free quantum field $\phi(x)$ as quantized solution of Klein-Gordon equation $(\Box + m^2)\phi = 0$ describes "particle pictures" in terms of creation and annihilation operators: $\phi(x) \rightleftarrows$ creation and annihilation operators $a(\vec{p}).a^*(\vec{q})$:

$$\begin{aligned}
\phi(x) &= \int \frac{d^3p}{\sqrt{(2\pi)^3 2\omega_{\vec{p}}}}(a(\vec{p})\exp(-ip_\mu x^\mu) + h.c.), \\
a^*(f) &:= i\int \phi(x)\overleftrightarrow{\partial_0} f(x)d^3x = \int a^*(\vec{p})\tilde{f}(\vec{p})d^3p \\
&= [a(f)]^*, \\
[a(f).a^*(g)] &= \int \overline{\tilde{f}(\vec{p})}\tilde{g}(\vec{p})d^3p = \langle \tilde{f}, \tilde{g}\rangle, \\
[\phi(x).\phi(y)] &= \int \frac{d^4p}{(2\pi)^3}\varepsilon(p^0)\delta(p^2 - m^2)\exp(-ip(x-y)) \\
&=: i\Delta(x-y;m^2),
\end{aligned}$$

with $\omega_{\vec{p}} := \sqrt{\vec{p}^2 + m^2}$ in the "natural unit system" with $\hbar = c = 1$ (rest mass m_0 is denoted by m, henceforth).

It is customary for most physicists to regard quantum fields $\phi(x)$ with $a^*(\vec{p}), a(\vec{p})$ as sufficient objects for describing wave-particle dualism inherent in elementary particles. Perpetual creation and annihilation processes of particles, however, require ***interactions*** among elementary particles, which is not consistent with the linearity of free field equation. In fact, the contents of the famous ***Haag theorem*** is that Poincaré (or even, Galilei)-covariant quantum fields related to free fields by a unitary transformation are only free fields, which means that it is meaningless to formulate interacting Heisenberg fields by means of a unitary transformation of free fields (as is common in perturbative approaches). Note that this is in sharp contrast to qunatum systems with finite degrees of freedom.

On the other hand, to describe relativistic scattering processes of elementary particles, we need the following three items: Poincaré-covariant quantum fields/ their interactions/ free fields. Free fields are necessary because it provide us with indispensable vocabulary for the description of scattering processes, where an initial state with incoming free particles is changed into a final one with outgoing particles. According to the above Haag theorem, however, we cannot discuss directly the relation between interacting Heisenberg and free fields. Instead, the ***unitary S-matrix appears between two free asymptotic fields,*** $\phi^{in}(x)$ and $\phi^{out}(x)$ in the form of a basis change $S_{\beta,\alpha} := \langle \beta, out|\alpha, in\rangle$ between in-state basis $|\alpha, in\rangle$ and out-state basis $|\beta, out\rangle$:

out	S-matrix & PCT	in	
$Ad\Theta^{out} \curvearrowleft \phi^{out}(x)$	AdS & $Ad\Theta$ $\leftrightarrows$ AdS^{-1} & $Ad\Theta$	$\phi^{in}(x) \curvearrowleft Ad\Theta^{in}$	Asymp. fields : Macro
$t \to +\infty$ $\nwarrow$	asymp.cond.	$\nearrow$ $t \to -\infty$	
	$\varphi_H(x) \curvearrowleft Ad\Theta$		Interacting fields : Micro

To treat Heisenberg fields $\varphi_H(x)$, we recapitulate briefly the essence of Wightman axioms for relativistic quantum fields (in the vacuum representation $(\mathcal{P}, \mathfrak{H}, U, \Omega)$) in the form of relativistic covariance, local commutativity, cyclicity or ergodicity of vacuum vector and spectral condition:

a) [Heisenberg fields] = operator-valued distributions $\mathcal{D}(\mathbb{R}^4) \ni f \longmapsto \varphi^i_H(f)$ with values being (unbounded) closable operators acting on Hilbert space $\mathfrak{H}$ is defined on the 4-dimensional Minkowski spacetime $(\mathbb{R}^4, \eta)$, where η is the Minkowski metric: $\eta(x, y) := x \cdot y = x^0 y^0 - \vec{x} \cdot \vec{y}$ for $x = (x^0, \vec{x})$.

b) [relativistic covariance]: local net $\mathcal{P} : \mathcal{K} \ni \mathcal{O} \longmapsto \mathcal{P}(\mathcal{O})$ of *-algebras $\mathcal{P}(\mathcal{O})$ generated by local fields $\varphi^i_H(f) = \int \varphi^i_H(x) f(x) d^4x$ with $f \in \mathcal{D}(\mathcal{O})$ and their polynomials defined on the net $\mathcal{K}$ of double cones $\mathcal{O}$ in the Minkowski spacetime constitute a non-commutative covariant dynamical system,

$$\begin{aligned}\alpha_{a,\Lambda}(\varphi^i_H(x)) &= U(a, \Lambda)\varphi^i_H(x)U(a, \Lambda)^{-1} \\ &= s(\Lambda)^i_j \varphi^i_H(\Lambda^{-1}(x - a)), \\ \alpha_{a,\Lambda}(\mathcal{P}(\mathcal{O})) &= \mathcal{P}(\Lambda\mathcal{O} + a),\end{aligned}$$

under the action α, $\mathcal{P}^\uparrow_+ \ni (a, \Lambda) \longmapsto \alpha_{a,\Lambda} \in Aut(\mathcal{P}(\mathbb{R}^4))$, of Poincaré group $\mathcal{P}^\uparrow_+ = \mathbb{R}^4 \rtimes L^\uparrow_+$ (or, its universal covering $\mathbb{R}^4 \rtimes SL(2, \mathbb{C})$) defined by the semi-direct product of spacetime translation group $\mathbb{R}^4$ and (proper) Lorentz group $L_+^\uparrow := \{\Lambda = (\Lambda^\mu_\nu);\ \Lambda x \cdot \Lambda y = x \cdot y, \Lambda^0_0 > 0, \det(\Lambda) = +1\}$ (or its universal covering $SL(2,\mathbb{C})$).

$\mathcal{P}_+^\uparrow \ni (a,\Lambda) \longmapsto U(a,\Lambda) \in \mathcal{U}(\mathfrak{H})$ is a unitary representation of $\mathcal{P}_+^\uparrow$ on $\mathfrak{H}$, and $s(\Lambda)^i_j$ is a finite-dimensional representation of Lorentz group $L_+^\uparrow$ associated with each field multiplet $(\varphi^i_H(x))_i$.

c) [local commutaitivity]: absence of propagation of physical effects exceeding the light velocity due to Einstein causality, implies the local commutativity of Heisenberg fields $\varphi^i_H(f)$:

$$[\varphi^i_H(f_1), \varphi^j_H(f_2)] = 0 \quad \text{if } (suppf_1)\times(suppf_2)$$

where $\mathcal{O}_1 \times \mathcal{O}_2$ means that any pair of points $x \in \mathcal{O}_1, y \in \mathcal{O}_2$ are spacelike separated: $(x-y)^2 < 0$.

Remark: By this condition, the Fourier transform of a Wightman function $\omega_0(\varphi^{i_1}_H(x_1)\cdots\varphi^{i_r}_H(x_r))$ as a correlation function of φ^i_H in the vacuum state $\omega_0(\cdot) \doteq \langle\Omega|(\cdot)\Omega\rangle$ defined in the next d) admits an analytic continuation into a holomorphic function in the complex energy-momentum space, from which dispersion relations follow.

d) [vacuum state and spectrum condition]:

d-i) energy-momentum spectrum $Sp(U(\mathbb{R}^4))$ of spacetime translations $\mathbb{R}^4$ realized on $\mathfrak{H}$ is within the forward light cone, $Sp(U(\mathbb{R}^4)) \subset \overline{V_+}$ in p-space $\widehat{\mathbb{R}^4}$, and the lowest energy is realized by eigenvalue 0 of the vacuum vector Ω: $U(x) := U(x,1) = \int_{p\in\overline{V_+}} \exp(ipx)dE(p)$; $U(x)\Omega = \Omega$.

Remark: Similarly to p-analyticity due to local commutativity, x-space analyticity of a Wightman function $\omega_0(\varphi^{i_1}_H(x_1)\cdots\varphi^{i_r}_H(x_r))$ follows from spectrum condition, which provides powerful tools for structural analysis.

d-ii) The equivalence holds among cyclicity $\overline{\mathcal{P}(\mathbb{R}^4)\Omega} = \mathfrak{H}$ of $\Omega \Longleftrightarrow$ irreducibility of $\mathcal{P}(\mathbb{R}^4) \Longleftrightarrow$ uniqueness of vacuum (: $U(x)\Psi = \Psi \Longrightarrow \Psi \propto \Omega$) $\Longleftrightarrow$ cluster property:

$$|\omega_0(A(x)B(y)) - \omega_0(A)\omega_0(B)| \to 0 \text{ as } (\vec{x}-\vec{y})^2 \to \infty.$$

where $A(x) := \alpha_x(A) = U(x)AU(x)^*$, $B(y) := \alpha_y(B)$ are the spacetime translates of local observables $A, B \in \mathcal{P}(\mathcal{O})$ by $x, y \in \mathbb{R}^4$, respectively. This condition follows from partition of unity due to spectral resolution of spacetime translations $U(x)$:

$$1 = |\Omega\rangle\langle\Omega| + \sum_i (\textbf{\textit{1-particle singularities on mass-shell}}\ p^2 = m_i^2)$$
$$+ \text{(absolutely continuous } p\text{-spectra)}$$

and is equivalent to the validity of the ergodicity of a unique vacuum vector Ω invariant under spacetime translations $U(x)$ in combination with the local commutativity.

3 Asymptotic Condition & Yang-Felman Equation

From the above ***cluster property*** combined with local commutativity:

$$\langle\Omega|A\alpha_{\vec{x}}(B)\Omega\rangle \overset{\vec{x}\to\infty}{\to} \langle\Omega|A\Omega\rangle\langle\Omega|B\Omega\rangle,$$

the ***asymptotic condition*** $\varphi_H(x) \overset{x^0=t\to\mp\infty}{\to} \phi^{in/out}(x)$ (as weak convergence) follows. In sharp contrast to this cluster property for interacting Heisenberg fields as *asymptotic factorization* valid only in the asymptotic limit, the asymptotic fields ϕ^{as} materialize the *kinematical* factorization (= independence) of correlations *without* taking asymptotic limit, which is just equivalent to the validity of "Wick theorem":

$$\omega_0(\phi^{as}\phi^{as}\cdots\phi^{as}) = \sum \omega_0(\phi^{as}\phi^{as})\cdots\omega_0(\phi^{as}\phi^{as}),$$

as the expansion of n-point functions into the sum of products of 2-point functions. This is nothing but the "quasi-freeness" of ω_0 w.r.t. ϕ^{as} constituting the contents of ***independence*** of Gaussian type.

It is also remarkable that ϕ^{as} contains creation and annihilation operators $a(\vec{p}), a^*(\vec{q})$ as ***infinite number of conserved quantities***: for any solution $f(x)$ of the Klein-Gordon equation $(\Box + m^2)f = 0$, we have

$$J_\mu(f) := i\phi(x)\overleftrightarrow{\partial_\mu} f(x);$$
$$\partial^\mu J_\mu(f) = i\partial^\mu[\phi(x)\overleftrightarrow{\partial_\mu} f(x)] = i\phi(x)\overleftrightarrow{\Box} f(x) = 0,$$

among which we find $a(\vec{p}) = \int dS^\mu J_\mu(f)$, $a^*(\vec{p}) = \int dS^\mu J_\mu(\bar{f})$ for $f(x) := \exp(-ip_\mu x^\mu)$.

Thus, the ***independence*** embodied by asymptotic fields ϕ^{as} is seen to emerge from interacting Heisenberg fields φ_H via asymptotic condition as a kind of central limit theorem. In this context, what corresponds to "Langevin equation" is the ***Yang-Feldman equation*** to connect Heisenberg field $\varphi_H(x)$ and asymptotic field $\phi^{as}(x)$:

$$\begin{aligned}\varphi_H(x) &= \int \Delta_{ret}(x-y;m^2)J_H(y)d^4y + \phi^{in}(x)\\ &= [\Delta_{ret} * J_H + \phi^{in}](x)\\ &= \int \Delta_{adv}(x-y;m^2)J_H(y)d^4y + \phi^{out}(x)\\ &= [\Delta_{adv} * J_H + \phi^{out}](x).\end{aligned}$$

where $J_H = (\Box + m^2)\varphi_H$: ***Heisenberg source current***, $\Delta_{ret/adv}(x-y;m^2)$: retarded/advanced Green's functions (i.e., principal solutions) of Klein-Gordon equation defined by

$$\begin{aligned}(\Box_x + m^2)\Delta_{ret/adv}(x-y;m^2) &= \delta(x-y),\\ \Delta_{ret/adv}(x-y;m^2) &= 0 \quad \text{for } x_0 \lessgtr y_0.\end{aligned}$$

In the Yang-Feldman equation, the asymptotic fields $\phi^{as}(x)$ and the Heisenberg source current J_H appear, respectively, as *residue* and *quotient* in the division of φ_H by $\Delta_{ret/adv}$. More important is that J_H gives the ***residues at the on-shell pole*** $\frac{1}{p^2 - m^2}$ to determine matrix elements of scattering amplitudes.

4 "Central Limit Theorem" as Micro-Macro Duality

Along this line, we can now find the natural meaning of "central limit" in the ***universality*** guaranteed by the Haag-GLZ expansion, which is similar to the "Fock expansion" in WNA. Now, the mutual relations between the interacting Heisenberg fields φ_H (: Micro) and the asymptotic fields ϕ^{as} (: Macro) are just described by "***Micro-Macro duality***" controlled by the K-T operator W which is given by

$$\begin{aligned} W &\stackrel{\text{def}}{=} : T(\exp(iJ_H \otimes \phi^{in}) : \\ &= \sum_n \int d^4x_1 \cdots \int d^4x_n \frac{i^n}{n!} T(J_H(x_1) \cdots J_H(x_n)) \\ &\otimes : \phi^{in}(x_1) \cdots \phi^{in}(x_n) : \end{aligned}$$

and is characterized by the pentagonal relation $W_{12}(W^{as})_{23} = (W^{as})_{23}W_{13}W_{12}$ (with W^{as} being K-T operator of CCR ϕ^{as} corresponding to a ***regular representation***). From this, the S-matrix S and the Haag-GLZ-Fock expansion [4, 5] are derived:

$$\begin{aligned} S &:= (\omega_0 \otimes id)(W) =: (\omega_0 \otimes id)(T \exp(iJ_H \otimes \phi^{in}) : \\ &=: (\omega_0 \otimes id)(T \exp(iJ_H \otimes \phi^{out}) : \\ &=: \exp(\phi^{in}(\Box + m^2)\frac{\delta}{\delta J}) : \omega_0(T(\exp(iJ\varphi_H)) \restriction_{J=0} \\ B &= S^{-1} : (\omega_0 \otimes id)(T[B \otimes 1] \exp(iJ_H \otimes \phi^{in})) : \\ &=: (\omega_0 \otimes id)(T[B \otimes 1] \exp(iJ_H \otimes \phi^{out}) : S^{-1}, \end{aligned}$$

and the ***S-matrix*** S describing the state changes in the scattering processes on the vacuum state $\omega_0 = \langle \Omega | \cdots \Omega \rangle$ is an ***intertwiner*** between two free fields, in-coming $\phi^{in}$and out-going ϕ^{out}:

$$\phi^{in}(x)S = S\phi^{out}(x).$$

Thus the essence of Micro-Macro duality between φ_H and ϕ^{as} is seen in that ϕ^{as} is derived from φ_H by the asymptotic condition, and that φ_H is reconstructed from ϕ^{as} by the Haag-GLZ-Fock expansion:

$$\phi^{as} \underset{\text{Haag-GLZ-Fock}}{\overset{\text{asymp.cond.}}{\leftrightarrows}} \varphi_H.$$

In this way, the Micro-Macro duality aspect involved in the "Central Limit Theorem" can be formulated in QFT in terms of the S-matrix functional and the Haag-GLZ expansion formula.

Moreover, the relevance of harmonic-analytic duality is evident from the Lie-algebraic structure of ***Heisenberg source currents***

$$J^i_H(x) = (\Box + m^2)\varphi^i_H = S^{-1}\frac{\delta}{i\delta\phi^{in}_i(x)}S;$$
$$\frac{\delta J^i_H(x)}{\delta\phi^j(y)} - \frac{\delta J^j_H(y)}{\delta\phi^i(x)} = i[J^i_H(x), J^j_H(y)].$$

What is closely related here is the relations among (weak) local commutativity, PCT invariance, S-matrix and Borchers classes of mutually local fields:

out	S-matrix & PCT	in	
$Ad\Theta^{out} \curvearrowright \phi^{out}(x)$	AdS & $Ad\Theta$ $\leftrightarrows$ AdS^{-1} & $Ad\Theta$	$\phi^{in}(x) \curvearrowleft Ad\Theta^{in}$	Asymp. fields : Macro
$t \to +\infty \nwarrow$	asymp.cond.	$\nearrow t \to -\infty$	
	$\varphi_H(x) \curvearrowleft Ad\Theta$		Interacting fields : Micro

From the definition of PCT transformation, $\theta(\varphi_H(x)) = \gamma\varphi_H(-x)^*$ (with $\gamma \in \mathbb{T}$) in combination with the local commutativity of φ_H, the vacuum ω_0 is seen to be invariant under θ: $\omega_0 \circ \theta = \omega_0$, which implies the existence of anti-unitary Θ to implement θ: $\theta(\varphi_H(x)) = \Theta\varphi_H(x)\Theta$ and $\Theta\Omega = \Omega$. Then the irreducibility of ϕ^{as} (following from the assumption of asymptotic completeness) implies

$$S = \Theta^{in}\Theta = \Theta\Theta^{out},$$

from such relations as $S\phi^{out}(x)S^{-1} = \phi^{in}(x) = \Theta\gamma^{-1}\phi^{out}(-x)^*\Theta = \Theta\Theta^{out}\phi^{out}(x)\Theta^{out}\Theta$. Thus, the quantum fields ***sharing the same PCT operator*** Θ shares the same S-matrix $S = \Theta^{in}\Theta = \Theta\Theta^{out}$: this explains the "ambiguities" of ***interpolating Heisenberg fields*** having the same S-matrix and is related with the notion of ***Borchers classess*** of mutually local fields sharing the same PCT operator Θ. This kind of consideration is relevant to the "inverse problem" to reconstruct interacting Heisenberg fields φ_H from the knowledge of asymptotic fields ϕ^{as} and the S-matrix S intertwining them.

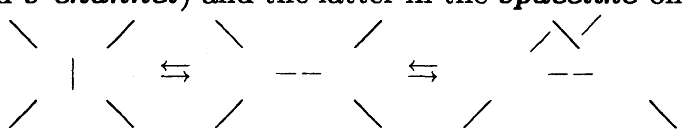
5 "Micro-Micro Duality" between Resonances and Regge Poles in Hadronic World of "Dependence = Coupling"

While the above scheme provides a good description of the scattering processes of quantum fields in terms of the asymptotic fields and the S-matrix, its pertinence highly depends on the validity of the asymptotic condition to be interpreted as a central limit theorem and on the smallness of the deviation of the Heisenberg fields from the corresponding asymptotic fields. If it were not for the notion of S-matrix based on the asymptotic fields, the following considerations on the hadronic world certainly could not have been formulated. However, the conditions crucial for the above discussions are evidently violated in the world of strongly interacting hadrons:

1) almost all hadrons are in highly unstable ***resonance states***, appearing temporarily only in the intermediate states of the scattering processes of low-lying stable (or almost stable) hadrons. This is just the essential features of ***dependence*** whose sector structure can be understood as follows:

$\longleftarrow$	El.mg.(diag)-Weak(off-diag) interactions+ ***flavour sym.*** $\curvearrowright$	$\cdots$	$\longrightarrow$	***Independence*** =Inter-hadronic ╱
γ_N	Light-mass hadrons $\gamma \in Sp(3_\pi)$	γ_2	γ_1	╱ ***Dependence*** =Intra-hadronic
				$\uparrow$ = Inside of ***hadronic sectors***
π_{γ_N}	π_γ ***Resonances***	π_{γ_2}	π_{γ_1}	$\downarrow$ $\overset{dual}{\simeq}$ ***Regge trajectories***
				$\simeq$ $\circlearrowright$ ***Strong interactions***

2) It is remarkable that the basic statistical features of the resonance states can be found in the ***Cauchy distributions*** $\propto \frac{1}{(E-E_i)^2+(\Gamma_i/2)^2} = \left|\frac{1}{E-E_i-i\Gamma_i/2}\right|^2$ w.r.t. the energy variable which correspond to the ***exponential decays*** of unstable particle states $\propto |\exp(-it(E-E_i-i\Gamma_i/2))|^2 \propto \exp(-t\Gamma_i)$ in time. Because of this instability= dependence caused by the strong interactions, the controllable stable states can be found only in ***hadrons with the lowest masses*** among those with the same (internal) quantum numbers such as the proton, neutron, and pions and so on.

3) The word "***dual***" in the above diagram means the ***duality*** inside a hadronic sector($\simeq$ ***Regge trajectory*** of hadron poles with energy-dependent angular momenta $\alpha(s) = \alpha_0 + \alpha' s$) between its ***resonance poles*** & ***Regge poles***, the former of which appear in the ***time-like*** region of the S-matrix (usually called ***s-channel***) and the latter in the ***spacelike*** ones called ***t-*** or ***u-channels***:  . The former

describes particle-like ***unstable*** modes (= ***pseudo-independence***) whose ***lowest level members*** only such as proton, neutron, pions, etc., can exist in (meta-)stable ways, and the latter yields the ***interaction terms between hadrons*** reflecting the aspect of ***dependence.***

4) The meaning of ***duality*** here should properly be understood in the following two kinds of contrasts to other contexts: first, the basic common features found in the mixed moments in all kinds of ***independences*** is the ***coexistence*** of all the above three terms of types, s-, t-, u-channels. In sharp contrast to it, these three-type diagrams are here ***mutually transformed*** from one to another ***without co-existing.*** Perhaps, this can be interpreted as one of the most essential features of ***dependence*** inherent in the hadronic processes.

Next, in contrast to the usual kinds of dualities valid between objects living at different levels, the duality appearing here holds at one and the same level of strongly interacting hadrons, connecting the aspect of ***independent*** objects and that of dynamical processes as ***coupling and dependence.*** Thus, this duality can be seen as ***Micro-Micro duality.***

5) In relation with the Wigner's construction of representations of the Poincaré group $\mathcal{P}_+^\uparrow = \mathcal{H}_2(\mathbb{C}) \trianglelefteq SL(2,\mathbb{C})$ (or, $\mathbb{R}^4 \trianglelefteq SO(1,3)$), we can understood this last duality (which motivated the investigation of dual resonance and string models) as the interchange between the little group $SU(2)$ (or $SO(3)$) at the timelike momentum $p, p^2 > 0$ and that $SU(1,1)$ (or $SL(2,\mathbb{R})$) at the spacelike momentum $p, p^2 < 0$.

Reformulating this Wigner-Mackey machinery of induction based on ***little groups*** H of a semi-direct product group $G = N \trianglelefteq L$, we can see the close relation of the present hadronic duality with the (spontaneous) symmetry breakdown, degenerate vacua unified into the notion of ***augmented algebras*** [1] and with the spacetime ***emergence*** [6] as follows: the regular C*-group algebra $C_r^*(G)$ of G is isomorphic to the crossed product $C_0(\hat{N}) \rtimes L$ of $C_0(\hat{N})$ (= C*-group algebra of abelian group N) by the action of L on N, which is also related with the covariant representations of the dynamical system $C_0(\hat{N}) \curvearrowleft L$. For each character $\chi \in \hat{N}$ of N, we can consider the ***little group*** H_χ of a pure state δ_χ of $C_0(\hat{N})$ as the isotropy subgroup of the latter, which can be viewed as the group of the ***remaining unbroken symmetry*** in the pure state δ_χ.

In the case of the Poincaré group $\mathcal{P}_+^\uparrow = G$, the existence of the well-known four types of the orbits $p^2 > 0, = 0, < 0, p_\mu \equiv 0$ having little groups $SU(2), E(2), SU(1,1)$ and $SL(2,\mathbb{C})$, respectively, can be interpreted as a kind of ***phase transitions*** taking place in the process of space-time emergence, whose different ***phases*** can be mutually connected through the ***analytic continuation*** of the dynamical system $C_0(\hat{N}) \curvearrowleft L$:

$E(2)$ at $p^2 = 0$
: parabolic

$SU(2)$ at $p^2 > 0$
: elliptic

$SU(1,1)$ at $p^2 < 0$
: hyperbolic

$SL(2, \mathbb{C})$ at $p \equiv 0$

It would be useful and interesting to re-view the ***hadronic dual-resonance aspects*** encoded in the string model, from the viewpoint of ***quantum probability & its complex analysis***, in close relationship with the ***independence, coupling and dependence.***

6) In view of the ***dominant contributions*** to the S-matrix coming from the low-lying hadrons with lightest masses, it would be interesting to examine the relevance of ***monotone independence to this context of factorization of dominant components*** w.r.t. the (inverse of) energy variable s (or $1/s$) as "time parameter" in the quantum probability theory of monotone independence [7]. Along this line and also taking account of a kind of duality relation between monotone and free independences (as was informed to me by Dr.Saigo and Mr.Hasebe), we may be attracted by the possible relations of ***monotone and/or free independences*** with the energy-***level statistics of nuclei*** (among dominant figures of low-lying hadrons) formulated by ***random matrices*** which are closely related with the ***quantum chaos*** and also with the ***free probability with Wigner's semi-circle law*** as its CLT. In this special situation, the mutual ***relation between the shell model and the liquid one*** of nuclei could possibly be understood as a kind of ***duality***, similarly to that between resonance and Regge poles of hadrons.

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