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How to Unify Interactions? – Independence and Dependence in Physics –*

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Abstract

In this article, a new scheme is proposed for unifying the four interactions governing nature, i.e., strong, weak, electromagnetic and gravitational ones, based on the triangular matrix features of the coupling scheme which constitutes mathematically a composition series of the characteristic dynamical laws and their underlying stabilized hierarchical domains.

1 Introduction: Micro-Macro Duality & Quadrality Scheme

In this article, a new scheme is proposed for unifying the four interactions governing nature, i.e., strong, weak, electromagnetic and gravitational ones, based on the triangular matrix structure of the coupling scheme which constitutes mathematically a composition series of the characteristic dynamical laws and their underlying stabilized hierarchical domains. To explain it, we first need the theoretical framework of quadrality scheme [1] based upon Micro-Macro duality [2, 3].

1) Micro-Macro duality [2, 3]: as a mathematical version of "***quantum-classical correpsondence***", ***Micro-Macro duality*** describes bi-directional relations between microscopic ***sectors*** as quasi-equivalence (= unitary equivalence up to multiplicity) classes of ***factor states*** of observable algebra and macroscopic ***inter-sectorial*** level in terms of geometrical structures on central spectrum $Spec(\mathfrak{Z})$:

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← Visible *Macro* of	***independent objects***		$\cdots$	→	***Inter-sectorial***
$\cdots$ γ_N	***sectors*** γ		γ_2	γ_1	*Spec*(3)
$\vdots$	$\vdots$		$\vdots$	$\vdots$	↑ ***Intra-sectorial***
$\cdots$ π_{γ_N}	π_γ		π_{γ_2}	π_{γ_1}	‖
$\vdots$	$\vdots$		$\vdots$	$\vdots$	↓ invisible ***Micro***

According to Fourier & Galois dualities, these Micro and Macro are in duality, meaning that the data given at Macro level is derived from the structural analysis of Micro and vice versa. This implies also the bi-directionality between Induction & Deduction. Essence of duality in Fourier transform $(\mathcal{F}f)(\gamma) = \int \overline{\gamma(g)} f(g)dg)$ ($f \in L^1(G)$) is formulated by Fourier-Pontryagin duality $G \rightleftarrows \hat{G}$ between a locally compact abelian group G and its dual group $\hat{G}$ consisting of all the characters $\chi : G \to \mathbb{T}$. Via extension to compact cases due to Tannaka & Krein, the most general form can now be found in Tatsuuma-Enock-Schwartz theorem of the duality between a locally compact non-abelian group G and its representation category $Rep(G)$ consisting of "all" the representations. The corresponding version is formulated by Takesaki (and/or Takai) for dynamical system with a (non-commutative) algebra $\mathcal{F}$ and with an action $\tau : \mathcal{F} \curvearrowleft G$ of G on $\mathcal{F}$ in such a form (in C*- or W*-versions, respectively) as

$$\mathcal{F}^G \rtimes_{\hat{\tau}} \hat{G} = \mathcal{F} \quad : \textbf{\textit{Recovery of }} \mathcal{F} \textbf{\textit{ from G-invariants }} \mathcal{F}^G;$$

$$\mathcal{F} \rtimes_\tau G = \mathcal{F}^G \otimes \mathcal{K}(L^2(G)) \text{ or } \mathcal{F}^G \otimes B(L^2(G)).$$

2) Quadrality scheme [1]: as a combination of two kinds of Micro-Macro dualities in "horizontal" and "vertical" directions, a general methodological framework can be formulated for theoretical descriptions of physical phenomena which I call ***quadrality scheme***:

Macro: visible levels	***Spec***(trum)= classifying space	
classification / emergence ↗↶	Fourier- V ↑↓ I	↘ (quantum) fields (as logical ext)
States ⇆ GNS	← (***Hilbert***) ***Modules*** →	⇆ ***Alg***(ebra of observables)
("co-fields") ↘	-Galois dualities ↑↓ *Gal*	↷↗ co-emergence
	Dyn(amics)	object system :Micro

Here, the map V maps modules or representations to the classifying space specifying the components of spectrum contained in modules and the inverse map I assigns the corresponding modules to subsets (or varieties) in the classifying space of sectors. Moreover, this ***quadrality*** scheme is equivalent to "***adjunction***" as a categorical formulation of ***duality***! [i.e., Duality $\overset{\text{dual}}{\rightleftarrows}$ Quadrality]

It is remarkable that this quadrality structure overlaps with the basic structure controlling the four interactions appearing in particle physics as follows:

Gravity		
Electromagnetism	(Abelianization = thermal relaxation $\leftrightarrows$ Non-abelianization)	Weak force
		Strong force

,

in which the meaning of "unification of four forces" need not be restricted to such a *simple-minded* version as converging to *single* entity, like the fashionable one. Instead, their mutual relations may well be understood alternatively in their *integrated organization* in nature and the corresponding theory, where they occupy mutually *different places*, playing *different roles* inherent in each, through which a *unified totality* of nature and its theoretical explanations are achieved. From this viewpoint, the standard simple "unification" pursued in "Geometrization of Physics" seems to be too narrow to incorporate these non-trivial aspects.

It is also interesting to note that the ***four kinds of QP-independence*** can be accommodated in the quadrality scheme, according to their mutual relations in such a way as:

Emergence $\nearrow$	Monotone independence	
Bosonic tensor type independence	$\leftrightarrows\uparrow\downarrow\leftrightarrows$	Boolean independence
	Free independence	$\nearrow$ Co-emergence

Here the emergence of monotone independence from tensor type one is due to Dr. H. Saigo's "Quantum-Classical Correspondence between Harmonic Oscillator and Arcsin Law" involving ***emergence*** as an essential step: harmonic oscillator as a typical model of ("quantum-decomposed") statistical independence $\overset{\textit{emergence}}{\twoheadrightarrow}$ arcsin law= CLT of monotone independence. This is similar to the Bogoliubov transformation: $a \to a+\sqrt{\lambda}$, $a^* \to a^*+\sqrt{\lambda}$, from a harmonic oscillator to Poisson distribution (see Accardi-Obata's textbook).

A quadrality scheme

$\mathcal{A} = \langle E \vert E\rangle = FE$	$\twoheadleftarrow$	$F = E^*$
$\downarrow$	$\searrow\!\!\nwarrow$	$\uparrow$
E	$\twoheadrightarrow$	$\mathcal{X} = \vert E\rangle\langle E\vert = EF$

can be represented by the linking algebra $\lambda(E) := \begin{pmatrix} \mathcal{A} & E^* = F \\ E & \mathcal{X} \end{pmatrix}$ of a Hilbert bi-module E with left and right acttions, respectively, of the algebra $\mathcal{X}$ of microscopic variables and of the observable algebra $\mathcal{A}$: $\mathcal{X} \curvearrowright E \curvearrowleft \mathcal{A}$, equipped with $\mathcal{A}$-valued right inner product $E^*E = \mathcal{A}$ and $\mathcal{X}$-valued left one $\mathcal{X} = EE^*$. The linking algebra $\lambda(E)$ can be defined by a subalgebra of the algebra $\mathcal{L}(E)$ of adjointable maps from E to E. When E has such a unit element I that $\langle I|E\rangle \cong |E\rangle$ and $|I\rangle\langle E| \cong \langle E|$, the fullness of E^*E in $\mathcal{A}$ as well as of EE^* in $\mathcal{X}$ (= "imprimitivity") can be expressed by the exact sequences, $\mathcal{A} \hookrightarrow E \twoheadrightarrow \mathcal{X}$ and $\mathcal{A} \twoheadleftarrow E \hookleftarrow \mathcal{X}$, the former of which can be understood as the bundle structure and the latter as a connection defined by a splitting of the former. This construction of a linking algebra $\lambda(E)$ starting from a Hilbert module E can be inverted, in a sense, by considering the Grassmannian Hilbert module as a module version of Grassmannian manifolds in such forms as

$$\lambda(E)/\left(\begin{array}{|c|c|}\hline \mathcal{A} & 0 \\ \hline E & \mathcal{X} \\ \hline\end{array}\right) = \left(\begin{array}{|c|c|}\hline I_{\mathcal{A}} & F \\ \hline 0 & I_{\mathcal{X}} \\ \hline\end{array}\right),$$

$$\lambda(E)/\left(\begin{array}{|c|c|}\hline \mathcal{A} & F \\ \hline 0 & \mathcal{X} \\ \hline\end{array}\right) = \left(\begin{array}{|c|c|}\hline I_{\mathcal{A}} & 0 \\ \hline E & I_{\mathcal{X}} \\ \hline\end{array}\right).$$

The conceptual meaning of these formulae can be understood through such an example as Prof. Mikio Sato's formulation of the set $Hom_{\mathcal{D}}(\mathcal{A} \leftarrow \mathcal{X}) = Hom(\mathcal{A} \leftarrow \mathcal{X})^{\mathcal{D}} = F^{\mathcal{D}}$ of $\mathcal{A}$-valued solutions of a system $Pf = 0$ of linear partial differential equations with a differential map $P : \mathcal{D}^m \to \mathcal{D}^n$, the latter of which can be boiled down to specifying a subalgebra $\mathcal{X} = coker(P) = \mathcal{D}^n/\mathcal{D}^m P$ of a CCR algebra $\mathcal{D}^n$ [4]. In this way, off-diagonal terms in the linking algebra of a Hilbert bimodules can be reformulated into Grassmannian Hilbert-modules: the $\mathcal{A}$-$\mathcal{X}$-bimodule F plays the role of visualizing the unknown solution of equations $\mathcal{X}$ in the known $\mathcal{A}$ and the $\mathcal{X}$-$\mathcal{A}$-bimodule E can work in the opposite direction to find the equation $\mathcal{X}$ from the known solutions in $\mathcal{A}$.

2 Coupling Patterns of Four Interactions

Applying this formulation, we can find an interesting physical meaning of the above relations in the following new observations for "unification of forces" [IO, in preparation]. The process of "co-emergence" of the algebra of observables from the dynamical flows is seen, in the table of coupling patterns below, to carry such a characteristic feature as a lower triangular matrix structure of the interactions:

Alg \ Dyn	gravity	weak	el.mg.	strong
gravit.'al field	♦	0	0	0
el. mg. field	♦	?	0	0
neutrino ν	*	♦	0	0
charged leptons	*	*	♦	0
hadrons	*	*	*	*

: *lower triangular matrix*

To incorporate this information into the construction of the relevant dynamical systems corresponding to different kinds of interactions, we consider a functor φ of dynamical co-emergence from the category of ordered types of force = {g(ravity) $\leftarrow$ w(eak) $\leftarrow$ e(l.mg.) $\leftarrow$ s(trong)} to a $\otimes$-category generated by algebras, respectively, $\mathcal{M}_h$ of hadrons, $\mathcal{M}_l$ of charged leptons, $\mathcal{M}_W$ of W, Z-bosons, $\mathcal{M}_\nu$,of neutrinos ν, $\mathcal{M}_e$ of electromagnetic and $\mathcal{M}_g$ of gravitational fields:

Objects φ(i) for i= g, w, e, h, are respective automorphism groups due to each force whose acting patterns are indicated above:

$$\text{Morphisms:}\ \begin{cases} \varphi(\mathrm{g}) \curvearrowright \mathcal{M}_g \otimes \mathcal{M}_e \otimes [\mathcal{M}_\nu \otimes \mathcal{M}_W \otimes \mathcal{M}_l] \otimes \mathcal{M}_h \\ \varphi(\mathrm{w}) \curvearrowright \qquad\qquad [\mathcal{M}_\nu \otimes \mathcal{M}_W \otimes \mathcal{M}_l] \otimes \mathcal{M}_h \\ \varphi(\mathrm{e}) \curvearrowright \qquad\qquad\qquad [\mathcal{M}_W \otimes \mathcal{M}_l] \otimes \mathcal{M}_h \\ \varphi(\mathrm{s}) \curvearrowright \qquad\qquad\qquad\qquad\qquad \mathcal{M}_h \\ \varphi(\mathrm{i} \leftarrow \mathrm{j}) = [\varphi(\mathrm{i}) \leftarrow \varphi(\mathrm{j})] \end{cases},$$

where the morphisms φ(i$\leftarrow$j)= [φ(i)$\leftarrow$ φ(j)] describe the mutual relations among different forces and fields due to the off-diagonal terms. The co-emergence flows from Dynamics to Algebra are expressed here by the left-actions of dynamics on the respective algebras, which can be seen as the table of coupling patterns in the following upper triangular form:

Dyn \ Alg	gravit.'al field	el. mg. field	neu-trino ν	charged leptons	hadrons
g(ravity)	♦	♦	*	*	*
w(eak)	0	?	♦	*	*
e(l.mg.)	0	0	0	♦	*
s(trong)	0	0	0	0	*

: *upper triangular matrix*

Here the direction $\nwarrow$ from the right bottom to the left top can be regarded as the ***historical formation of stabilized hierarchical domains*** with the first appearance ♦ of ***stabilized forms of matter*** and with the associated ***repeatable laws*** established to run. The opposite arrow $\searrow$ from the left top to the right bottom can be viewed as the directions of deepening human ***recognitions*** and ***controllability***, which always involve the dilations to uncover microscopic agents causing unknown phenomena at the macroscopic levels. The divergent $\nwarrow\uparrow$ and the convergent $\overleftarrow{\nwarrow}$ can be seen to constitute the historical paths of nature. The question mark (?) above shows different behaviours of electromagnetic field at different scales:

$$? = \begin{bmatrix} 0\text{: behaviours as purely abelian gauge field in } \textbf{\textit{Macro}} \\ \blacklozenge\text{: } \textbf{\textit{Weinberg-Salam mixing}} \text{ of } (e) \leftrightarrows (w) \text{ in } \textbf{\textit{Micro}} \end{bmatrix}$$

At the bottom row (s(trong) | 0 0 0 0 *), the level of ***hadrons*** as the only carriers of strong forces shows a sharp contrast between the ***extremely high instability*** of individual members and the ***persistent existence*** of the level itself as a whole under the influence of all different forces shown in the rightmost column: $\begin{pmatrix} \text{hadrons} \\ \hline * \\ * \\ * \\ * \end{pmatrix}$. It may be interesting to find the answer to the question where we have seen similar patterns elsewhere, in such a form as the level of ***bacteria*** in the organic world [5]! This situation should be contrasted with the top line, (gravity | * * * * *), showing the ***universality*** of gravitational ***attractions.***

The above ***upper triangular block*** structure,

$\mathcal{A}_1$	$\mathcal{F}_{12}$	$\mathcal{F}_{13}$	$\cdots$	$\mathcal{F}_{1r}$
0	$\mathcal{A}_2 = \mathcal{X}_1$	$\mathcal{F}_{23}$	$\cdots$	$\vdots$
0	0	$\ddots$	$\mathcal{F}_{r-2,r-1}$	$\mathcal{F}_{r-2,r}$
$\vdots$	$\cdots$	$\cdots$	$\mathcal{A}_{r-1} = \mathcal{X}_{r-2}$	$\mathcal{F}_{r-1,r}$
0	$\cdots$	$\cdots$	0	$\mathcal{X}_{r-1}$

,

can be related with a ***generalized flag manifold***:

$$\begin{aligned} &GM(m_1|m_2|\cdots|m_r) \\ &= U(m_1+m_2+\cdots+m_r)/\left(U(m_1)\times U(m_2)\times\cdots\times U(m_r)\right) \\ &:= GM(m_1|m_2+\cdots+m_r)\times GM(m_2|m_3+\cdots+m_r)\times\cdots\times GM(m_{r-1}|m_r) \\ &= GL(m_1+\cdots+m_r;\mathbb{C})/\begin{bmatrix} GL(m_1;\mathbb{C}) & M(m_1,m_2;\mathbb{C}) & \cdots & M(m_1,m_r;\mathbb{C}) \\ 0 & GL(m_2;\mathbb{C}) & \cdots & M(m_2,m_r;\mathbb{C}) \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & M(m_{r-1},m_r;\mathbb{C}) \\ 0 & 0 & 0 & GL(m_r;\mathbb{C}) \end{bmatrix} \end{aligned}$$

whose denominator is a ***Borel parabolic subalgebra*** describing a nested solvability structure of ***composition series.***

Dual to the above ***co-emergence***: (***Alg*** $\curvearrowleft$ ***Dyn***), is the ***emergence***: (***Alg*** $\curvearrowleft$ ***Dyn***)* =(***Spec*** $\Leftarrow$ ***States***) which can be seen as follows:

Spec \ States	gravit.'al field	el. mg. field	ν	charged leptons	hadronic states
gravitational force $\Gamma^{\lambda}_{\mu\nu} \curvearrowright \{x^{\mu}\}$ = index of emergence	: initial objects ♦	*	*	*	*
weak force $\Leftarrow I_3$	0	?*	♦	*	*
el.mg.force $\Leftarrow Q$	0	0	0	♦	*
strong force $\Leftarrow (I, B, S)$ of Regge trajectories	0	0	0	0	*: terminal objects

: upper triangular (with electric charges $Q = I_3 + (B + S - L)/2$)

Moreover, hadrons are "***terminal*** objects" dual to the spacetime points x^{μ} as ***initial*** objects which is equivalent to "***confinement***"!! (because, in this context, "quarks" are nothing but constituents of weak-electro-magnetic currents J_{μ}, J^{V-A}_{μ} to couple with $A_{\mu}, W^{\pm}_{\mu}$ and Z_{μ} without need for any reality!)

2.1 History of universe vs. repeatable laws

We can interpret the upper off-diagonal terms F_{ij} as a composite system of diagonals $\mathcal{A}_1, \cdots, \mathcal{A}_r$ and $\mathcal{X}_1, \cdots, \mathcal{X}_{r-1}$ characterized by ***exact sequences*** $\mathcal{A}_i \twoheadleftarrow F_{i,i+1} \hookleftarrow \mathcal{X}_i$, equipped with a structure of left-$\mathcal{A}_i$ right-$\mathcal{X}_i$ ***Hilbert modules*** ${}_{\mathcal{A}_i}F_{i,i+1\mathcal{X}_i}$ (with $\mathcal{A}_i = F_{i,i+1}F^*_{i,i+1}$-valued left inner product and $\mathcal{X}_i = F^*_{i,i+1}F_{i,i+1}$-valued right inner product). $F_{i,i+1}$ can also be regarded as a ***left-adjoint functor*** $\mathcal{A}_i \overset{F_{i,i+1}}{\longleftarrow} \mathcal{X}_i$. Thus, the upper triangularity with 0 in the lower left corner implies the ***presence of uni-directional arrows*** $\nwarrow$, which provide a monotone scaling parameter (similarly to Lieb-Yngvason's axiomatic derivation of entropy from adiabatic ordering) of what can be interpreted as ***historical uni-directionality inherent already at the Micro-level.***

To understand the mathematical structure relevant to the mutual relations between micro- and macroscopic levels, what is crucial is to clarify the processes of ***emergence*** of the latter from the former: they can be understood here as ***condensation of order parameters*** arising from ***symmetry breaking*** (***spontaneous*** or ***explicit***). Since any transformation associated with an explicitly broken symmetry changes (by definition) the physical constants characteristic of a physical system described by a specific theory, the notion of explicitly broken symmetries has been excluded systematically from the traditional schemes for mathematical and/or theoretical treatments of a "fixed" physical theory. From a more general viewpoint, however, nothing prohibits us to consider many different theories at once and the use of explicitly broken symmetry tansformations can be systematized in such a context into a **method of variation of natural constants** to relate different physical theories; in fact, such a method has been used in [6] in a efficient way to identify (inverse) temperature with an order parameter of explicitly broken scale invariance. In this context we suppose a ***structure of Theory Bundle*** with a base space consisting of physical constants on each point of which fibre is given by a physical theory with fixed physical constants specified by the base point [7]. In this formulation, such a transformation can easily and consistently be performed as mapping a given physical theory with a fixed set of physical constants into a different theory with another fixed one. By doing this, we can not only compare different theories but also pursue the historical formations of stabilized domains such as the worlds of nuclei and of atoms equipped with repeatable and reproducible physical laws in the evolution processes of the universe. Along the line of the latter problem, we can discuss the following problem:

[Historical formation of ***Stabilized Domains***] $\rightleftarrows$ [***Realization of repeatable laws***]= duality between [*History* = ***processes*** governed by ***randomness*** to form ***hierarchical domains***] and [***hierarchical domains***= **worlds of necessity controlled by laws**].

A simple mathematical model of this situation can be found in the decomposition of broken symmetry group G into a compact subgroup H describing an unbroken symmetry as a kinematically ***repeatable law*** and a homogeneous space G/H describing a ***stabilized domain*** consisting of degenerate vacua mutually connected by broken dynamical transitions (due to Goldstone modes) which arises from a historical process of emergence due to condensations.

3 Emergence of Relativistic Spacetime

At this point we notice the similarity of the relevant structures involved in this picture to the basic features of "punctuated equilibria" [8] found in the evolution processes in the biological world. We confirm this viewpoint by examining the typical and, perhaps, most important case of emergence of the spacetime, among transition processess bridging various regimes controlled by different forces. From this angle, the etiology of the current fashionable "unification programme" can be found in "Geometrization of Physics", whose origin goes back to Einstein's unsuccessful "Unified Field Theory" to unify electromagnetism and gravity in spacetime geometry: unfortunately, the physical origin of gravity and spacetime cannot be unveiled from this viewpoint: for instance, what does *spacetime* mean when we are faced with totally *indeterminate future*??

To overcome such conceptual difficulty, we propose here a scenario for deriving gravity and spacetime as *epigenetic* secondary notions *emerging from microscopic regimes* of (microscopic) matter motions. To this end, the essence of the following discussion is just to explain the following diagram consisting of the structures relevant to the emergence of special- and general-relativistic spacetimes (see below).

The basic ingredients necessary for this purpose are as follows:

i) ***Independence*** = freely falling frames as "sectors" without gravity containing only strong, weak & electromagnetic couplings

ii) ***Coupling***= gravitational force $\Gamma^{\lambda}_{\mu\nu}$defined as Levi-Civita connection to connect different free-falling fames as "sectors" at the *meta-level, and,*

iii) ***Dependence***= the composite system arising from the above physical systems constructed by three kinds of forces (strong, weak & electromagnetic) coupled with each other by the gravitational force.

By re-examining how general-relativistic spacetime emerges from the physical processes in Micro quantum systems, we clarify here under which condition the notion of "spacetime" can be meaningful from the viewpoint of "Micro-Macro duality".

Spec = spacetime $\{x^{\mu}\}$	***Regime of general relativity***	General Covariant Functorial Sym G
$g_{\mu\nu}$	$\Gamma^{\lambda}_{\mu\nu} \curvearrowright$ free-falling frames $\{x^{\mu}\}$ at different pt's	$\uparrow$ induced rep Ind^{G}_{H}
$R, R_{\mu\nu}$ $\uparrow\downarrow$	$\uparrow m_{grav} = m_{inert}$ ***Equiv. principle*** of inside & outside of ***sector*** $\{x^{\mu}\}$ ***with no gravity***	Unbroken Sym $H = \mathcal{P}^{\uparrow}_{+}$
Einstein eqn : $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$	$\begin{bmatrix} x \\ \mid \\ p \end{bmatrix}$ duality	$\searrow\nwarrow$ local spacetime $\swarrow\nearrow$ *emergence* $1/c$
$\|\|$ $\kappa\omega(T_{\mu\nu})$ state ω: $\uparrow\downarrow$	***Maxwell eqn*** e $F_{\mu\nu} \leftarrow J_{\mu}$ $\uparrow\downarrow \quad \uparrow\downarrow$ A_{μ} $\xrightarrow{\textit{covariant}}$ *derivatives* ψ	(material path) $\searrow\nwarrow$ W-S angle $\searrow\nwarrow$ θ_{WS}
$\uparrow\downarrow$		Weak Interactions
$T_{\mu\nu}$	: ***Dynamics***	Strong Interactions

3.1 Symmetry Breaking & Condensed States

Breakdown of a symmetry G of a dynamical system $\mathcal{F} \curvearrowleft G$ in a state $\omega \in E_{\mathcal{F}}$ is characterized [2] by non-invariance of the "central extension" of ω on the centre $\mathfrak{Z}_{\pi_\omega}(\mathcal{F}) := \pi_\omega(\mathcal{F})'' \cap \pi_\omega(\mathcal{F})'$ under the corresponding G-action on $\mathfrak{Z}_{\pi_\omega}(\mathcal{F})$. In this case, Galois closedness of $\mathcal{F}^G$ is broken, which is recovered by dynamical system $\mathcal{F} \curvearrowleft H$ described by a compact Lie subgroup H of G corresponding to unbroken symmetry: $\mathcal{F} = \mathcal{F}^H \rtimes \widehat{H}$ according to the general method of Galois extension by means of crossed products [9, 2]. Then, the sector structure is determined by the factor spectrum $\widehat{\mathcal{F}^H} = Spec(\mathfrak{Z}(\mathcal{F}^H)) = \hat{H}$ as a group dual consisting of irreducible unitary represetations of H. On the basis of these key roles played by the symmetry breaking, we can understand the emergence of macroscopic phenomena as ***condensation effects*** coming from the microscopic dynamics, whose precise mathematical formulation can be given by the application of ***forcing method*** whose common use has benn restricted so far in the contexts of foundations of mathematics (and which also implies the Born rule [5]).

For this purpose, we need to extend the standard Doplicher-Roberts method for reconstructing the field algebra $\mathcal{F} = \mathcal{F}^H \rtimes \widehat{H}$ from the observable algebra $\mathcal{F}^H$ with the unbroken symmetry described by H, to incorporate the situation of a broken symmetry in the following way [2]: with $\tilde{\mathcal{F}} := \mathcal{F}^H \rtimes \widehat{G} = \mathcal{F} \rtimes \widehat{(H\backslash G)}$ called an ***augmented algebra*** [2], we have a ***split*** bundle exact sequence $\mathcal{F}^H \underset{\hookrightarrow}{\overset{\tilde{m}}{\leftarrow}} \tilde{\mathcal{F}} \underset{\twoheadrightarrow}{\hookleftarrow} \tilde{\mathcal{F}}/\mathcal{F}^H \simeq \widehat{G}$. In this situation, the ***minimality*** of $\tilde{\mathcal{F}}$ is guaranteed by the ***G-central ergodicity***, i.e., G-ergodicity of the centre $\mathfrak{Z}_{\tilde{\pi}}(\tilde{\mathcal{F}})$ in the representation $\tilde{\pi}$ given by the GNS representation of $\omega_0 \circ \tilde{m}$ induced from the vacuum state ω_0 of $\mathcal{F}^H$ [2]. Then we have the following commutativity diagram of algebra extension:

into $\swarrow$	$\widetilde{\mathcal{F}}^G = \mathcal{F}^H$: unbroken observables	$\searrow^{\text{into}}$
extended alg. of observables : $\widetilde{\mathcal{F}}^H$	$\searrow\searrow^{\text{into}}$ $\Downarrow^{\text{into}}$ into $\swarrow$	$\mathcal{F}$
onto $\downarrow$	$\widetilde{\mathcal{F}}$: augmented alg.	$\downarrow^{\text{onto}}$
onto $\downarrow$	$\swarrow^{\text{onto}}$ $\Downarrow^{\text{onto}}$ onto $\searrow\searrow$	$\downarrow^{\text{onto}}$
$\widehat{H\backslash G}$	$\hookrightarrow$ $\widehat{G}$ $\twoheadrightarrow$	$\widehat{H}$

,

which is dual to the diagram for sectors:

unbroken sectors:	$\widehat{H} \simeq \widehat{\widetilde{\mathcal{F}}^G} = \widehat{\mathcal{F}^H}$	
$\Downarrow^{\text{into}}$	$\uparrow^{\text{onto}}$ onto $\Uparrow$	$\nwarrow$ onto
sector bundle:	$G \underset{H}{\times} \widehat{H} \simeq \widetilde{\mathcal{F}}^H$ $\Uparrow$	$\widehat{\mathcal{F}}$
$\Downarrow^{\text{onto}}$	into $\uparrow$ into $\nwarrow\nwarrow$ into $\Uparrow$	$\uparrow^{\text{into}}$
degenerate vacua:	$G/H \simeq \widehat{\widetilde{\mathcal{F}}} \twoheadleftarrow G$: broken	$\hookleftarrow H$

,

where $\overset{\frown}{\mathcal{F}} = Spec(\mathfrak{Z}(\mathcal{F}))$ denotes the factor spectrum of $\mathcal{F}$, etc.

The physical essence of extension $\mathcal{F}^G \Longrightarrow \mathcal{F}^H$ from the G-fixed point subalgebra $\mathcal{F}^G$ to the H-fixed one $\mathcal{F}^H$ can now be understood as the "extension of a coefficient algebra $\mathcal{F}^G$" by (the dual of) G/H which parametrizes the degenerate vacua: $\mathcal{F}^H = \widetilde{\mathcal{F}^G} = [(\mathcal{F} \rtimes \widehat{(H\backslash G)})]^G = \mathcal{F}^G \rtimes \widehat{(H\backslash G)}$. In this extension, a part G/H of originally ***invisible*** G has become ***visible*** through the ***emergence of degenerate vacua*** parametrized by G/H due to the ***condensation of order parameters*** $\in G/H$ associated with **S**(ponteneous) **S**(ymmetry) **B**(reaking) of G to H.

As a result, observables $A \in \mathcal{A}$ acquire G/H-dependence: $\widetilde{A} = (G/H \ni \dot{g} \longmapsto \widetilde{A}(\dot{g}) \in \mathcal{A}) \in \mathcal{A} \rtimes \widehat{(H\backslash G)}$, which should just be interpreted as an example of the ***logical extension*** [10] transforming a "***constant*** object" $(A \in \mathcal{A})$ into a "***variable*** object" $(\widetilde{A} \in \mathcal{A} \rtimes \widehat{(H\backslash G)})$ having ***functional dependence*** on the universal ***classifying space*** G/H for multi-valued ***semantics***(, as is familiar in non-standard and Boolean-valued analysis).

3.2 Emergence of Spacetime as Symmetry Breaking

By replacing G/H with spacetime, the above situation can be regarded as a prototype for the origin of functional dependence of physical quantities on spacetime coordinates, due to the ***physical emergence of spacetime*** from microscopic physical world. Along this line, we prescribe the similar logical extension procedure on the observable algebra $\mathcal{F}^H$ adding G/H-dependence:

$$\mathcal{F}^H \rtimes \widehat{(H\backslash G)} = (\mathcal{F} \rtimes \widehat{(H\backslash G)})^H = \widetilde{\mathcal{F}^H}.$$

The whole sector structure of $\widetilde{\mathcal{F}^H} = (\mathcal{F}^H \rtimes \widehat{(H\backslash G)})$ can be identified with its factor spectrum $\overset{\frown}{\widetilde{\mathcal{F}^H}} = G \underset{H}{\times} \hat{H}$; this constitutes a ***sector bundle***, $\hat{H} \hookrightarrow \overset{\frown}{\widetilde{\mathcal{F}^H}} = G \underset{H}{\times} \hat{H} \twoheadrightarrow G/H$, consisting of the classifying space G/H of ***degenerate vacua***, each fibre over which describes the sector structure $\hat{H}$ of ***unbroken*** remaining symmetry H (or, more precisely, the family of conjugated groups gHg^{-1} for the vacuum parametrized by $\dot{g} = gH \in G/H$). Namely, the sector bundle can be seen as the ***connection*** or a ***splitting*** of bundle exact sequence dual to $\overset{\frown}{\mathcal{F}^H} = \hat{H} \twoheadleftarrow \overset{\frown}{\widetilde{\mathcal{F}^H}} = G \underset{H}{\times} \hat{H} \hookleftarrow G/H$ of observable triples, $\mathcal{F}^H \hookrightarrow \widetilde{\mathcal{F}^H} = \mathcal{F}^H \rtimes \widehat{(H\backslash G)} \twoheadrightarrow \widehat{(H\backslash G)}$!

Now we apply the above scheme to the situation with the group G containing both *external* (= spacetime) and *internal* symmetries. For simplicity, the latter component described by a subgroup H of G is assumed to be unbroken, and hence, the broken symmetry described by G/H represents the spacetime structure. It would be convenient (not essential, though) to take H as a normal subgroup of G. To be precise, G/H may contain such non-commutative components as spatial rotations (and Lorentz boosts) acting on spacetime, but, we simply neglect this aspect to identify G/H as spacetime itself (from which the corresponding transformation group can easily be recovered).

Then, by identifying G/H with a spacetime domain $\mathcal{R}$, we find an impressive parallelism between the commutative diagram in the previous subsection and the diagram in the Doplicher-Roberts reconstruction [9] of local field net $\mathcal{R} \longmapsto \mathcal{F}(\mathcal{R})$ from the local observable net $\mathcal{R} \longmapsto \mathcal{A}(\mathcal{R})$ (without the two bottom lines) as follows:

$$\begin{array}{ccccc} & {}_{H}\swarrow & \mathcal{F}^{H} = \widetilde{\mathcal{F}}^{G} & \searrow_{G/H} & \\ \mathcal{F} & & \Downarrow & & \widetilde{\mathcal{F}}^{H} \\ \downarrow & {}_{G/H}\searrow\searrow & \widetilde{\mathcal{F}} & \swarrow_{H} & \downarrow \\ \downarrow & \swarrow & \Downarrow & \searrow\searrow & \downarrow \\ \widehat{H} & \twoheadleftarrow & \widehat{G} & \hookleftarrow & \widehat{G/H} \end{array} \rightleftarrows \begin{array}{ccccc} & {}_{G}\swarrow & \mathcal{O}_{\rho} = \mathcal{O}_{d}^{G} & \searrow_{\mathcal{R}} & \\ \mathcal{O}_{d} & & \Downarrow & & \mathcal{A}(\mathcal{R}) \\ \downarrow & {}_{\mathcal{R}}\searrow\searrow & \mathcal{F}(\mathcal{R}) & \swarrow_{G} & \downarrow \\ \downarrow & \swarrow & \Downarrow & \searrow\searrow & \downarrow \\ \widehat{G} & \twoheadleftarrow & \widehat{G \times \mathcal{R}} & \hookleftarrow & \widehat{\mathcal{R}} \end{array}, \text{ where}$$

$\mathcal{O}_d$ is the Cuntz algebra of d-isometries.

Thus we have arrived at the stage just before gravity to be switched on, to enter General Relativity via ***Equivalence Principle***. This can naturally be formulated and understood by the above scheme in combination with induced representation. So, we should recall here the diagram at the end of the previous section, Sec.3. Here, the unbroken symmetry may be either Poincaré group $\mathcal{P}_{+}^{\uparrow}$ or rotation group $SO(3)$. G can, however, be a bigger group of symmetry transformations formulated as functors in the context of category.

4 Physical meaning of Equivalence Principle in General Relativity in the emergence process

We consider processes of spacetime emergence taking place in parallel under the influence of strong and electro-weak interactions other than gravity, each of which results in a "fiber" (= sector= pure phase) parametrized by spacetime coordinates x^{μ}. The word "fiber" here means a ***flat tangent space*** as a fiber ($T_x(M)$) of a tangent bundle ($T(M)$) on each point x^{μ} of the base space as a "spacetime manifold" (which *would* be called M but which cannot be recognized yet as such); as its physics is controlled by the three interactions other than the gravity, this fiber describes a ***free-falling frame*** without any gravitational force (the last of which has not emerged yet). In connection with our discussion up to this point, the word "sector" should be more appropriate than "fiber", we accept the latter use in order to avoid the misunderstanding of what we are concerned with here.

To be precise, what we know up to now is only the simultaneous processes of spacetime emergence at many "fiber" points x^μ each of which consists of the physical world of Poincaré covariant quantum fields governed by the strong and electro-weak interactions in the Minkowski spacetime but we do not know anything about the mutual relations among different fibers. By picking up just one specific "fiber", we focus on the local physics described by the Poincaré covariant QFT developed inside of the "free falling system without gravity", which is nothing but the physical contents of "***tangential world***" equipped with local Lorentz structure, on (or "in"?) a point (x^μ) in the emergent "base space".

Now, we pose a question: what does it mean to impose the physical requirement of "equivalence principle" between gravitational and inertial masses, $m_{grav} = m_{inert}$ on the situation after the "individual" processes of free-falling systems arising from the emergence of special-relativistic local spacetime? While the notion of "inertial mass" already exists in the "standard" physics formulated within the free-falling frames without gravity, it does not apply to the case of "gravitational mass" before our starting to discuss the situations governed by the gravitational interaction. It can be meaningful only in the context where such an attribute is assigned to a(n asymptotically) free mass point on the mass-shell, as generating the gravitational force or field as the forth one other than strong and electro-weak forces, through Einstein's gravitational equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}.$$

When we find the first (or, the 0-th approximated) roles of gravity in regulating the mutual relations among different fibers= sectors as free-falling frames, the proper range of action of the gravitational mass m_{grav} is at the level of "inter-fiber= inter-sectorial relations", but, in contrast, that of the inertial one m_{inert} is in the physics within each "fiber" (or sector). Therefore, the equivalence principle qualitatively controls in a bi-directional way the mutual duality relation between the inside and the outside of "fibers" (or, sectors)[1]: we suppose that the inter-fiber relation of free-falling frames on the "neighbouring" points x^μ and $x^\mu + \delta x^\mu$ is controlled by the connection coefficients $\Gamma^\lambda_{\mu\nu}$, as is indicated in the diagram at the beginning, which results in a force propertional to the gravitational mass m_{grav} acting on the inertial mass m_{inert}.

Then, the Newtonian equation of motion of the mass point m_{inert} with the velocity vector $v^\lambda := \dfrac{dx^\lambda}{d\tau}$ can be written as,

$$m_{inert} dv^\lambda = -v^\mu (m_{grav} \Gamma^\lambda_{\mu\nu} dx^\nu) = m_{grav} v^\mu \nabla_\mu dx^\lambda.$$

[1]From the viewpoint of emergence as a process of phase separation, the roles played by the free-falling frame in each "fiber" and by "base space" can be compared with H and G/H whose duality relation can be seen in the form of "Helgason duality". In this sense, the gravitational equivalence principle is analogous to "Helgason duality".

By the requirement of equivalence principle $m_{grav} = m_{inert}$, this reduces to the ***geodesic equation***, $\frac{dv^\lambda}{d\tau} + \Gamma^\lambda_{\mu\nu} v^\mu v^\nu = 0$, whose ***purely geometric*** form and independence of the specific mass values ensure the universality of the mass-point motions. Namely, through the validity of ***equivalence principle*** $m_{grav} = m_{inert}$, the spacetime notion x^μ acquires its own abstract universal meaning, independently of its physical origin in the mutual relations among different "fibers" of local physics consisting of three interactions, to such an extreme extent that space and "time" exist in themselves, extending from the past, the present and even the future! Eventually, the physical motions of mass points are now absorbed into a (small) part of spacetime geometry in the form of geodesic motions, without exhibiting their individuality [11].

Owing to this mechanism, we can easily forget about the ***physical origin*** of spacetime, which can, however, exhibit its existence in the situation where the validity of equivalence principle is threatened. It is also interesting to note that the above equation of motion can be rewritten in terms of momentum $p^\lambda = mv^\lambda$ into

$$dp^\lambda = p^\mu \nabla_\mu dx^\lambda,$$

which explains that mass-point motions as geodesic motion can be absorbed into the covariance (of physical motions) under the (covariantized) general coordinate transformations. If the above discussion is compared with the standard mathematical treatment of bundle structures in differential geometry, we understand that ours go from physics in the (standard) fiber to the mathematical structure of tbe bundle and base spaces in the opposite direction to the latter and that the mathematical essence of the equivalence principle lies in the G-structure of the tangent and frame bundles of the spacetime M with G being identified with the Lorentz group [12].

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