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On Asymptotic Properties of the Parameters of Differentiated Product Demand and Supply Systems When Demographically-Categorized Purchasing Pattern Data are Available

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In this exposition, we give a setup for deriving asymptotic theorems for Petrin (2002) extension of Berry, Levinsohn, and Pakes (BLP, 1995) framework to estimate demand and supply models with additional moments. The additional moments contain the information relating the consumer demographics to the characteristics of the products they purchase. In the paper of the same title distributed on the day of presentation, the extended estimator is shown to be consistent, asymptotically normal, and more efficient than BLP estimator with further assumptions. The paper also contains discussion on the conditions under which these asymptotic theorems hold for the random coefficient logit model as well as the extensive simulation studies that confirm the benefit of the additional moments in estimating the random coefficient logit model of demand in the presence of oligopolistic suppliers.

1 Introduction

Some recent empirical studies in industrial organization and marketing extend the framework proposed by Berry, Levinsohn and Pakes (1995, henceforth BLP (1995)) by integrating information on consumer demographics into the utility functions in order to make their demand models more realistic and convincing. Wide availability of public sources of information such as the Current Population Survey (CPS) and the Integrated Public Use Microdata Series (IPUMS) makes these studies possible. Those sources give us information on the joint distribution of the U.S. household's demographics including income, age of household's head, and family size. For example, Nevo (2001)'s examination on price competition in the U.S. ready-to-eat cereal industry uses individual's income, age and a dummy variable indicating if s/he has a child in the utility function. Sudhir (2001) includes household's income to model the U.S. automobile demand in his study of competitive interactions among firms in different market segments.

In analyzing the U.S. automobile market, Petrin (2002) goes further and links demographics of new-vehicle purchasers to characteristics of the vehicles they purchased. Petrin adds a set of functions of the expected value of consumer's demographics given specific product characteristics (e.g. expected family size of households that purchased minivans) as additional moments to the original moments used in BLP (1995) in the GMM estimation. Specifically, he matches the model's probability of new vehicle purchase for different income groups to the observed purchase probabilities in the Consumer Expenditure Survey (CEX) automobile supplement. He also matches model prediction for average household characteristics of vehicle purchasers such as family-size to the data in CEX automobile supplement. Petrin presupposes readily accessible and publicly available market information on the population average.[1] He maintains that "the extra information plays the same role as consumer-level data, allowing estimated substitution patterns and (thus) welfare to directly reflect demographic-driven differences in tastes for observed characteristics" (page, 706, lines 22-25). His intention, it seems, is to reduce the bias associated with "a heavy dependence on the idiosyncratic logit "taste" error"(page 707, lines 5-6). He explains that "the idea for using these additional moments derives from Imbens and Lancaster (1994). They suggest that aggregate data may contain useful information on the average of micro variables" (page 713, lines 27-29).[2]

It should be noted that these additional moments are subject to simulation and sampling errors in BLP estimation. This is because the expectations of consumer demographics are evaluated conditional on a set of exogenous product characteristics $\boldsymbol{X}$ and an unobserved product quality $\boldsymbol{\xi}$, where the $\boldsymbol{\xi}$ is evaluated with the simulation error induced by BLP's contraction mapping as well as with the sampling error contained in observed market shares. In addition, market information against which the additional moments are evaluated itself contains another type of sampling error. This is because the market information is typically an estimate for the population average demographics obtained from a sample of consumers (e.g. CEX sample), while observed market shares are calculated from another sample of consumers. This error also affects evaluation of the additional moments. In summary each of the four errors (the simulation error, the sampling error in the observed market shares, the sampling error *induced when researcher evaluates the additional moments*, and the sampling error *in the market information itself*) as well as stochastic nature of the product characteristics affects evaluation of the additional moments.

The estimator proposed by Petrin appears to assume that we are able to control impacts

[1]Berry, Levinsohn, and Pakes (2004), on the other hand, uses detailed consumer-level data, which include not only individuals' choices but also the choices they would have made had their first choice products not been available. Although the proposed method should improve the out-of-sample model's prediction, it requires proprietary consumer-level data, which are not readily available to researchers, as the authors themselves acknowledged in the paper: the CAMIP data "are generally not available to researchers outside of the company" (page 79, line 30).

[2]Original intension of Imbens and Lancaster is to improve efficiency for a class of the extremum estimators. There is a difference between Petrin's and Imbens and Lancaster's approaches in sampling process to construct original and additional sample moments. Petrin combines the sample moments calculated *over products* with additional moments calculated *over individuals*, while Imbens and Lancaster use the moments calculated over the same individuals.

from the first four errors. However, it is not apparent if Petrin estimator is consistent and asymptotically normal (CAN) without such a control. Furthermore, it is not known either if how many and in what way individuals need to be sampled in order for Petrin estimator to be more efficient than BLP estimator. We write this paper to formalize Petrin's idea and provide the conditions under which Petrin estimator not only has CAN properties, but is more efficient than BLP estimator. Then we implement extensive simulation studies and confirm benefits of the additional moments in estimating the random coefficient logit model of demand.

We assume econometrician samples two sets of individuals independent of each other, one to simulate market shares of products and the other to evaluate the additional moments, in order to avoid intractable correlations between the two sets of individuals. We also assume the additional market information on demographics of consumers are calculated from a sample independent of these two samples. We follow the rigorous work of Berry, Linton, and Pakes (2004) (hereinafter, BLP (2004)) in which the authors presented the asymptotic theorems applicable to the random coefficient logit models of demand in BLP (1995).

In section 2, we operationalize Petrin's extension of BLP framework and define the sampling and simulation errors in the GMM objective function.

2 System of Demand and Supply with Additional Moments

In this section, we give precise definition of the product space, and reframe the estimation procedure of BLP when combining the demand and supply side moments with the additional moments relating consumer demographics to the characteristics of products they purchase. Since our approach extends BLP (2004), notations and the most of definitions are kept as identical as possible to those in BLP (2004).

2.1 Demand Side Model

The discrete choice differentiated product demand model formulates that the utility of consumer i for product j is a function of demand parameters $\boldsymbol{\theta}_d$, observed product characteristics $\boldsymbol{x}_j$, unobserved (by econometrician) product characteristics ξ_j, and random consumer tastes ν_{ij}. Given the product characteristics $(\boldsymbol{x}_j, \xi_j)$ for all (J) products marketed, consumers either buy one of the products or choose the "outside" good. Each consumer makes the choice to maximize his/her utility. Different consumers assign different utility to the same product because their tastes are different. The tastes follow the distribution P^0.

Although most product characteristics are not correlated with the unobserved product characteristics $\xi_j \in \Re$, $j = 1, \ldots, J$, some of them (e.g. price) are. We denote the vector of observed product characteristics $\boldsymbol{x}_j = (\boldsymbol{x}'_{1j}, \boldsymbol{x}'_{2j})'$ where $\boldsymbol{x}_{1j} \in \Re^{K_1}$ are exogenous and not correlated with ξ_j, while $\boldsymbol{x}_{2j} \in \Re^{K_2}$ are endogenous and correlated with ξ_j. We assume the set of exogenous product characteristics $(\boldsymbol{x}_{1j}, \xi_j), j = 1, \ldots, J$ are random sample of product characteristics of size J from the underlying population of product characteristics.[3] Thus, $(\boldsymbol{x}_{1j}, \xi_j)$ are assumed independent across j, while $\boldsymbol{x}_{2j}$ are not in general across j since they are endogenously determined in the market as functions of others' and its own product characteristics. The ξ_j's are assumed to be mean independent of $\boldsymbol{X}_1 = (\boldsymbol{x}_{11}, \ldots, \boldsymbol{x}_{1J})'$ and to have a finite conditional variance as

$$\mathrm{E}[\xi_j|\boldsymbol{X}_1] = 0 \qquad \text{and} \qquad \sup_{1\leq j\leq J} \mathrm{E}[\xi_j^2|\boldsymbol{x}_{1j}] < \infty \tag{1}$$

with probability one. A set of observed product characteristics for all the products is denoted by $\boldsymbol{X} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_J)'$.

The conditional purchase probability σ_{ij} of product j is a map from consumer i's tastes $\boldsymbol{\nu}_i \in \Re^v$, a demand parameter vector $\boldsymbol{\theta}_d \in \Theta_d$, and the set of characteristics of all products $(\boldsymbol{X}, \boldsymbol{\xi})$, and is thus denoted as $\sigma_{ij}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\nu}_i; \boldsymbol{\theta}_d)$. We assume $\sigma_{ij}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\nu}_i; \boldsymbol{\theta}_d) > 0$ for all possible values of $(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\nu}_i, \boldsymbol{\theta}_d)$. BLP framework generates the vector of market shares, $\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P)$, by aggregating over the individual choice probability with the distribution P of the consumer tastes $\boldsymbol{\nu}_i$ as

$$\sigma_j(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P) = \int \sigma_{ij}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\nu}_i; \boldsymbol{\theta}_d) dP(\boldsymbol{\nu}_i)$$

where P is typically the empirical distribution of the tastes from a random sample drawn from P^0. Note that these market shares are still random variables due to the stochastic nature of the product characteristics $\boldsymbol{X}$ and $\boldsymbol{\xi}$. If we evaluate $\sigma_j(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P)$ at $(\boldsymbol{\theta}_d^0, P^0)$, where $\boldsymbol{\theta}_d^0$ is the true value, we have the "conditionally true" market shares $\boldsymbol{s}^0$ given the product characteristics $(\boldsymbol{X}, \boldsymbol{\xi})$ in the population, i.e. $\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d^0, P^0) \equiv \boldsymbol{s}^0$.

Equation in the form of $\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P) = \boldsymbol{s}$ can, in theory, be solved for $\boldsymbol{\xi}$ as a function of $(\boldsymbol{X}, \boldsymbol{\theta}_d, \boldsymbol{s}, P)$. BLP (1995) provides general conditions under which there is a unique solution for

$$\boldsymbol{s} - \boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P) = \boldsymbol{0} \tag{2}$$

[3] BLP(2004) and we develop the econometric properties of an estimator where the presumption is that there is one "national" market and the asymptotics in the data arise as the number of products gets large. Recent empirical studies in IO estimate demand functions by using regional-level data. For these cases, the asymptotics—not in the number of products in a market, but the number of markets—can in principle be easy to obtain because their market shares do not converge to zero. However, we found that the moments calculated for each regional market are likely to be correlated even if we assume each consumer chooses a product only from one regional market. This means, in practice, that asymptotic covariance matrix of the estimated parameter becomes harder to obtain. More importantly, asymptotics so obtained are *conditional* on the characteristics of products available in the market, while our asymptotics are *unconditional*.

for every $(\boldsymbol{X}, \boldsymbol{\theta}_d, \boldsymbol{s}, P) \in \mathcal{X} \times \Theta_d \times \mathcal{S}_J \times \mathcal{P}$, where $\mathcal{X}$ is a space for the product characteristics $\boldsymbol{X}$, $\mathcal{S}_J$ is a space for the market share vector $\boldsymbol{s}$, and $\mathcal{P}$ is a family of probability measures. If we solve (2) at any $(\boldsymbol{\theta}_d, \boldsymbol{s}, P) \neq (\boldsymbol{\theta}_d^0, \boldsymbol{s}^0, P^0)$, the independence assumption for the resulting $\xi_j(\boldsymbol{X}, \boldsymbol{\theta}_d, \boldsymbol{s}, P)$ no longer holds because the two factors deciding the ξ_j—the market share s_j and the endogenous product characteristics $\boldsymbol{x}_{2j}$ for product j—are endogenously determined through the market equilibrium (e.g. Nash in prices or quantities) as a function of the product characteristics not only of its own but also of its competitors. However, if we solve the identity $\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d^0, P^0) = \boldsymbol{s}^0$ with respect to $\boldsymbol{\xi}$ under the conditions to guarantee the uniqueness of the $\boldsymbol{\xi}$ in (2), we are able to retrieve the original $\xi_j(\boldsymbol{X}, \boldsymbol{\theta}_d^0, \boldsymbol{s}^0, P^0)$, which we assume are independent across j.

2.2 Supply Side Model

In this paper we take into account supply side moment condition unlike not in BLP (2004). The framework is based on BLP (1995). Here, we give the model and define notations.

The supply side model formulates the pricing equations for the J products marketed. We assume an oligopolistic market where a finite number of suppliers provide multiple products. Suppliers $(m = 1, \ldots, F)$ are maximizers of profit from the combination of products they produce. Assuming the Bertrand-Nash pricing for supplier's strategy provides the first order condition for the product j of the manufacturer m as

$$\sigma_j(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P) + \sum_{l \in \mathcal{J}_m} (p_l - c_l) \partial \sigma_l(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P)/\partial p_j = 0 \quad \text{for } j \in \mathcal{J}_m,$$

where $\mathcal{J}_m$ denotes a set of the products offered by manufacturer m, and p_j and c_j are respectively price and marginal cost of product j. This equation can be expressed in matrix form

$$\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P) + \boldsymbol{\Delta}(\boldsymbol{p} - \boldsymbol{c}) = \boldsymbol{0} \tag{3}$$

where $\boldsymbol{\Delta}$ is the $J \times J$ non-singular gradient matrix of $\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P)$ with respect to $\boldsymbol{p}$ whose (j, k) element is defined by

$$\Delta_{jk} = \begin{cases} \partial \sigma_k(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P)/\partial p_j, & \text{if the products } j \text{ and } k \text{ are produced by the same firm;} \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

We define the marginal cost c_j as a function of the observed cost shifters $\mathbf{w}_j$ and the unobserved (by econometrician) cost shifters ω_j as

$$g(c_j) = \mathbf{w}_j' \boldsymbol{\theta}_c + \omega_j \tag{5}$$

where $g(\cdot)$ is a monotonic function and $\boldsymbol{\theta}_c \in \Theta_c$ is a vector of cost parameters. While the choice of $g(\cdot)$ depends on application, we assume $g(\cdot)$ is continuously differentiable with a finite derivative for all realizable values of cost. Suppose that the observed cost shifters $\mathbf{w}_j$ consist of exogenous $\mathbf{w}_{1j} \in \Re^{L_1}$ as well as endogenous $\mathbf{w}_{2j} \in \Re^{L_2}$, and thus we write $\mathbf{w}_j = (\mathbf{w}'_{1j}, \mathbf{w}'_{2j})'$ and $\boldsymbol{W} = (\mathbf{w}_1, \ldots, \mathbf{w}_J)'$. The exogenous cost shifters include not only the cost variables determined outside the market under consideration (e.g. factor price), but also the product design characteristics suppliers cannot immediately change in response to fluctuation in demand. The cost variables determined at the market equilibrium (e.g. production scale) are treated as endogenous cost shifters. As in the formulation of $(\boldsymbol{x}_{1j}, \xi_j)$ on the demand side, we assume a set of exogenous cost shifters $(\mathbf{w}_{1j}, \omega_j)$ is a random sample of cost shifters from the underlying population of cost shifters. Thus $(\mathbf{w}_{1j}, \omega_j)$ are assumed to be independent across j, while $\mathbf{w}_{2j}$ are in general not independent across j as they are determined in the market as functions of cost shifters of other products. Similar to the demand side unobservables, the unobserved cost shifters ω_j are assumed to be mean independent of the exogenous cost shifters $\mathbf{W}_1 = (\mathbf{w}_{11}, \ldots, \mathbf{w}_{1J})'$, and satisfy with probability one,

$$\mathrm{E}[\omega_j | \mathbf{W}_1] = 0, \quad \text{and} \quad \sup_{1 \le j \le J} \mathrm{E}[\omega_j^2 | \mathbf{w}_{1j}] < \infty. \tag{6}$$

Define $\boldsymbol{g}(\boldsymbol{x}) \equiv (g(x_1), \ldots, g(x_J))$. Solving the first order condition (3) with respect to $\boldsymbol{c}$ and substituting for (5) give the vector of the unobserved cost shifters

$$\boldsymbol{\omega}(\boldsymbol{\theta}, \boldsymbol{s}, P) = \boldsymbol{g}(\boldsymbol{p} - \boldsymbol{m}_g(\boldsymbol{\xi}(\boldsymbol{X}, \boldsymbol{\theta}_d, \boldsymbol{s}, P), \boldsymbol{\theta}_d, P)) - \boldsymbol{W}\boldsymbol{\theta}_c, \tag{7}$$

where

$$\boldsymbol{m}_g \equiv -\boldsymbol{\Delta}^{-1}\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P) \tag{8}$$

represents the vector of profit margins for all the products in the market. Hereafter, we suppress the dependence of ξ_j and ω_j on $\boldsymbol{X}$ and $\boldsymbol{W}$ to express $\xi_j(\boldsymbol{\theta}_d, \boldsymbol{s}, P)$ and $\omega_j(\boldsymbol{\theta}, \boldsymbol{s}, P)$ respectively for notational simplicity. Notice that the parameter vector $\boldsymbol{\theta}$ in $\boldsymbol{\omega}$ contains both the demand and supply parameters, i.e. $\boldsymbol{\theta} = (\boldsymbol{\theta}'_d, \boldsymbol{\theta}'_c)'$. Since the profit margin $m_{g_j}(\boldsymbol{\xi}, \boldsymbol{\theta}_d, P)$ for product j is determined not only by its unobserved product characteristics ξ_j, but by those of the other products in the market, these ω_j are in general dependent across j when $(\boldsymbol{\theta}, \boldsymbol{s}, P) \neq (\boldsymbol{\theta}^0, \boldsymbol{s}^0, P^0)$. However, when (7) is evaluated at $(\boldsymbol{\theta}, \boldsymbol{s}, P) = (\boldsymbol{\theta}^0, \boldsymbol{s}^0, P^0)$, we are able to recover the original $\omega_j, j = 1, \ldots, J$, and they are assumed independent across j.

2.3 GMM Estimation with Additional Moments

Let us define the $J \times M_1$ demand side instrument matrix $\boldsymbol{Z}_d = (\boldsymbol{z}_1^d, \ldots, \boldsymbol{z}_J^d)'$ whose components $\boldsymbol{z}_j^d$ can be written as $\boldsymbol{z}_j^d(\boldsymbol{x}_{11}, \ldots, \boldsymbol{x}_{1J}) \in \Re^{M_1}$, where $\boldsymbol{z}_j^d(\cdot) : \Re^{K_1 \times J} \to \Re^{M_1}$ for $j = 1, \ldots, J$. It should be noted that the demand side instruments $\boldsymbol{z}_j^d$ for product j are assumed to be a function of the exogenous characteristics not only of its own, but of the other products in the market. This is because the instruments by definition must correlate with the product characteristics $\boldsymbol{x}_{2j}$, and these endogenous variables $\boldsymbol{x}_{2j}$ (e.g. price) are determined by both its own and its competitors' product characteristics.

Similar to the demand side, we define the $J \times M_2$ supply side instrumental variables $\boldsymbol{Z}_c = (\boldsymbol{z}_1^c, \ldots, \boldsymbol{z}_J^c)'$ as a function of the exogenous cost shifters $(\mathbf{w}_{11}, \ldots, \mathbf{w}_{1J})$ of all the products. Here, $\boldsymbol{z}_j^c(\mathbf{w}_{11}, \ldots, \mathbf{w}_{1J}) \in \Re^{M_2}$ and $\boldsymbol{z}_j^c(\cdot) : \Re^{L_1 \times J} \to \Re^{M_2}$ for $j = 1, \ldots, J$. We also note that some of the exogenous product characteristics $\boldsymbol{x}_{1j}$ affect the price of product because they affect manufacturing cost. Thus those $\boldsymbol{x}_{1j}$ may also be used as the exogenous cost shifter $\mathbf{w}_{1j}$ if they are uncorrelated with the unobservable cost shifter ω_j.

Assume, for moment, that we know the underlying taste distribution of P^0 and that we are able to observe the true market share $\boldsymbol{s}^0$. Considering stochastic nature of the product characteristics $\boldsymbol{X}_1$ and $\boldsymbol{\xi}$, we set forth the demand side restriction as

$$\mathrm{E}_{\mathrm{x}_1,\xi}\left[\boldsymbol{z}_j^d \xi_j(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0)\right] = \mathbf{0} \tag{9}$$

at $\boldsymbol{\theta}_d = \boldsymbol{\theta}_d^0$ where the expectation is taken with respect not only to $\boldsymbol{\xi}$, but also to $\boldsymbol{X}_1$. Supply side restriction we use is

$$\mathrm{E}_{\mathrm{w}_1,\omega}\left[\boldsymbol{z}_j^c \omega_j(\boldsymbol{\theta}, \boldsymbol{s}^0, P^0)\right] = \mathbf{0} \tag{10}$$

at $\boldsymbol{\theta} = \boldsymbol{\theta}^0$. BLP framework uses the orthogonal conditions between the unobserved product characteristics (ξ_j, ω_j) and the exogenous instrumental variables $(\boldsymbol{z}_j^d, \boldsymbol{z}_j^c)$ as moment conditions to obtain the GMM estimate of the parameter $\boldsymbol{\theta}$. The sample moments for the demand and supply systems are

$$\boldsymbol{G}_J(\boldsymbol{\theta}, \boldsymbol{s}^0, P^0) = \begin{pmatrix} \boldsymbol{G}_J^d(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0) \\ \boldsymbol{G}_J^c(\boldsymbol{\theta}, \boldsymbol{s}^0, P^0) \end{pmatrix} = \begin{pmatrix} \sum_j \boldsymbol{z}_j^d \xi_j(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0)/J \\ \sum_j \boldsymbol{z}_j^c \omega_j(\boldsymbol{\theta}, \boldsymbol{s}^0, P^0)/J \end{pmatrix}. \tag{11}$$

For some markets, market summaries are publicly available such as average demographics of consumers who purchased a specific type of products, even if their detailed individual-level data such as their purchasing histories are not. In the U.S. automobile market, for instance, we can obtain the data on the median income of consumers who purchased domestic, European, or Japanese vehicles from publications such as the *Ward's Motor Vehicle Facts & Figures.*

We now operationalize the idea put forth by Petrin (2002), which extends BLP framework by adding moment conditions constructed from the market summary data. First we define some words and notations. *Discriminating attribute* is the product characteristic or the product attribute that enables consumers to discriminate some products from others. When we say consumer i chooses discriminating attribute q, this means that consumer chooses a product from a group of products whose characteristic or attribute have discriminating attribute q. Discriminating attribute q is assumed to be a function of observed product characteristics $\boldsymbol{X}$. An automobile attribute "import" is one such discriminating attribute. When we say a consumer chooses this attribute, what we mean is that the consumer purchases an imported vehicle. Similarly, "minivan" and "costing between \$20,000 to \$30,000" are examples of the discriminating attribute. On the other hand, unobservable consumer's proximity to a dealership is a function of ξ only and may not be regarded as a discriminating attribute as defined. We consider a finite number of discriminating attributes ($q = 1, \ldots, N_p$) and denote a set of all the products that have attribute q as $\mathcal{Q}_q$. We assume the market share of products with discriminating attribute is positive, that is, $\Pr[C_i \in \mathcal{Q}_q | \boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0)] > 0$, where C_i denotes the choice of randomly sampled consumer i.

We next consider expectation of consumer's demographics conditional on a specific discriminating attribute. Suppose that the consumer i's demographics can be decomposed into observable and unobservable components $\boldsymbol{\nu}_i = (\boldsymbol{\nu}_i^{obs}, \boldsymbol{\nu}_i^{unobs})$. The densities of $\boldsymbol{\nu}_i$ and $\boldsymbol{\nu}_i^{obs}$ are respectively denoted as $P^0(d\boldsymbol{\nu}_i)$ and $P^0(d\boldsymbol{\nu}_i^{obs})$. Observable demographic variables such as age, family size, or, income, is already numerical, but for other demographics such as household with children, belonging to a certain age group, choice of residential area, can be numerically expressed using indicators. We denote this numerically represented D dimensional demographics as $\boldsymbol{\nu}_i^{obs} = (\nu_{i1}^{obs}, \ldots, \nu_{iD}^{obs})'$. We assume that the joint density of demographics $\boldsymbol{\nu}_i^{obs}$ is of bounded support. The consumer i's d-th observed demographic $\nu_{id}^{obs}, d = 1, \ldots, D$ is averaged over all consumers choosing discriminating attribute q in the population to obtain the conditional expectation $\eta_{dq}^0 = \mathrm{E}[\nu_{id}^{obs} | C_i \in \mathcal{Q}_q, \boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d^0, \boldsymbol{s}^0, P^0)]$. An example of this conditional expectation would be the expected value of income of consumers in the population P^0 who purchased an imported vehicle. We assume η_{dq}^0 has a finite mean and variance for all J, i.e. $\mathrm{E}_{\mathrm{x},\xi}[\eta_{dq}^0] < \infty$ and $\mathrm{V}_{\mathrm{x},\xi}[\eta_{dq}^0] < \infty$ for $d = 1, \ldots, D, q = 1, \ldots, N_p$.

Let $\Pr[d\nu_{id}^{obs} | C_i \in \mathcal{Q}_q, \boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0)]$ be the conditional density of consumer i's demographics ν_{id}^{obs} given his/her choice of discriminating attribute q and product characteristics $(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0))$. Since the conditional expectation η_{dq}^0 can be written as

$$\mathrm{E}[\nu_{id}^{obs} | C_i \in \mathcal{Q}_q, \boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0)] \tag{12}$$

$$
\begin{aligned}
&= \int \nu_{id}^{obs} \Pr[d\nu_{id}^{obs} | C_i \in \mathcal{Q}_q, \boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0)] \\
&= \frac{\int \nu_{id}^{obs} \Pr[C_i \in \mathcal{Q}_q | \boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0), \nu_{id}^{obs}] P^0(d\nu_{id}^{obs})}{\Pr[C_i \in \mathcal{Q}_q | \boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0)]} \\
&= \frac{\int \nu_{id}^{obs} \Pr[C_i \in \mathcal{Q}_q | \boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0), \boldsymbol{\nu}_i] P^0(d\boldsymbol{\nu}_i)}{\Pr[C_i \in \mathcal{Q}_q | \boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0)]} \\
&= \int \nu_{id}^{obs} \frac{\sum_{j \in \mathcal{Q}_q} \sigma_{ij}(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0), \boldsymbol{\nu}_i; \boldsymbol{\theta}_d)}{\sum_{j \in \mathcal{Q}_q} \sigma_j(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0), \boldsymbol{\theta}_d, P^0)} P^0(d\boldsymbol{\nu}_i),
\end{aligned}
$$

we can form an identity, which is the basis for additional moment conditions

$$
\eta_{dq}^0 - \int \nu_{id}^{obs} \frac{\sum_{j \in \mathcal{Q}_q} \sigma_{ij}(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d^0, \boldsymbol{s}^0, P^0), \boldsymbol{\nu}_i; \boldsymbol{\theta}_d^0)}{\sum_{j \in \mathcal{Q}_q} \sigma_j(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d^0, \boldsymbol{s}^0, P^0), \boldsymbol{\theta}_d^0, P^0)} P^0(d\boldsymbol{\nu}_i) \equiv 0 \tag{13}
$$

for $q = 1, \ldots, N_p, d = 1, \ldots, D$.

Although P^0 is so far assumed known, we typically are not able to calculate the second term on the left hand side of (13) analytically and will have to approximate it by using the empirical distribution P^T of i.i.d. sample $\boldsymbol{\nu}_t, t = 1, \ldots, T$ from the underlying distribution P^0. The corresponding sample moments $\boldsymbol{G}_{J,T}^a(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0, \boldsymbol{\eta}^0)$, where a on the shoulder stands for additional, are

$$
\boldsymbol{G}_{J,T}^a(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0, \boldsymbol{\eta}^0) = \boldsymbol{\eta}^0 - \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{\nu}_t^{obs} \otimes \boldsymbol{\psi}_t(\boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0), \boldsymbol{\theta}_d, P^0) \tag{14}
$$

where

$$
\boldsymbol{\eta}^0 = (\eta_{11}^0, \ldots, \eta_{1N_p}^0, \ldots, \eta_{D1}^0, \ldots, \eta_{DN_p}^0)', \qquad \boldsymbol{\psi}_t(\boldsymbol{\xi}, \boldsymbol{\theta}_d, P) = \begin{pmatrix} \frac{\sum_{j \in \mathcal{Q}_1} \sigma_{tj}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\nu}_t; \boldsymbol{\theta}_d)}{\sum_{j \in \mathcal{Q}_1} \sigma_j(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P)} \\ \vdots \\ \frac{\sum_{j \in \mathcal{Q}_{N_p}} \sigma_{tj}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\nu}_t; \boldsymbol{\theta}_d)}{\sum_{j \in \mathcal{Q}_{N_p}} \sigma_j(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P)} \end{pmatrix}. \tag{15}
$$

The symbol $\otimes$ denotes the Kronecker product. The quantity $\boldsymbol{\psi}_t(\boldsymbol{\xi}, \boldsymbol{\theta}_d, P)$ is the consumer t's model-calculated purchasing probability of products with discriminating attribute q relative to the model-calculated market share of the same products. Note that these additional moments are again conditional on product characteristics $(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0))$, and thus depend on the indices J and T.

We use the set of the three moments, two from (11) and from (14) as

$$
\boldsymbol{G}_{J,T}(\boldsymbol{\theta}, \boldsymbol{s}^0, P^0, \boldsymbol{\eta}^0) = \begin{pmatrix} \boldsymbol{G}_J^d(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0) \\ \boldsymbol{G}_J^c(\boldsymbol{\theta}, \boldsymbol{s}^0, P^0) \\ \boldsymbol{G}_{J,T}^a(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0, \boldsymbol{\eta}^0) \end{pmatrix} \tag{16}
$$

to estimate $\boldsymbol{\theta}$ in theory. As pointed out in BLP (2004), we have two issues when evaluating $||\boldsymbol{G}_{J,T}(\boldsymbol{\theta}, \boldsymbol{s}^0, P^0, \boldsymbol{\eta}^0)||$. First, we assume P^0 is known so far, we typically are not able to calculate $\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P^0)$ analytically and have to approximate it by a simulator, say $\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P^R)$, where P^R is the empirical measure of i.i.d. sample $\boldsymbol{\nu}_r, r = 1, \ldots, R$ from the underlying distribution P^0. The sample $\boldsymbol{\nu}_r, r = 1, \ldots, R$ are assumed independent of the sample $\boldsymbol{\nu}_t, t = 1, \ldots, T$ in (14) for evaluating the additional moments. Simulated market shares are then given by

$$\sigma_j(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P^R) = \int \sigma_{ij}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\nu}_i; \boldsymbol{\theta}_d) dP^R(\boldsymbol{\nu}_i) \equiv \frac{1}{R} \sum_{r=1}^{R} \sigma_{rj}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\nu}_r; \boldsymbol{\theta}_d). \tag{17}$$

Second, we are not necessarily able to observe the true market shares $\boldsymbol{s}^0$. Instead, the vector of given observed market shares, $\boldsymbol{s}^n$, are typically constructed from n i.i.d. draws from the population of consumers, and hence is not equal to the population value $\boldsymbol{s}^0$ in general. The observed market share of product j is

$$s_j^n = \frac{1}{n} \sum_{i=1}^{n} 1(C_i = j), \tag{18}$$

where the indicator variable $1(C_i = j)$ takes one if $C_i = j$ and zero otherwise. Since C_i denotes the choice of randomly sampled consumer i, they are i.i.d. across i.

We substitute $\boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R)$ given as a solution of $\boldsymbol{s}^n - \boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P^R) = \boldsymbol{0}$ for (11) to obtain

$$\boldsymbol{G}_J^d(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R) = J^{-1} \sum_{j=1}^{J} \boldsymbol{z}_j^d \xi_j(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R). \tag{19}$$

Furthermore, substituting $\boldsymbol{\omega}(\boldsymbol{\theta}, \boldsymbol{s}^n, P^R) = (\omega_1(\boldsymbol{\theta}, \boldsymbol{s}^n, P^R), \ldots, \omega_J(\boldsymbol{\theta}, \boldsymbol{s}^n, P^R))'$ obtained from evaluating (7) at $\boldsymbol{\xi} = \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R)$ and $P = P^R$ for (11) gives

$$\boldsymbol{G}_J^c(\boldsymbol{\theta}, \boldsymbol{s}^n, P^R) = J^{-1} \sum_{j=1}^{J} \boldsymbol{z}_j^c \omega_j(\boldsymbol{\theta}, \boldsymbol{s}^n, P^R). \tag{20}$$

In addition, we have another issue when evaluating the additional moments in (14). In general, we do not know the conditional expectation of demographics η_{dq}^0, instead, we have its estimate η_{dq}^N from independent sources, which is typically estimated from the sample of N consumers. The sample counterparts we can calculate for the additional moments are thus

$$\boldsymbol{G}_{J,T}^a(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R, \boldsymbol{\eta}^N) = \boldsymbol{\eta}^N - \frac{1}{T} \sum_{t=1}^{T} \boldsymbol{\nu}_t^{obs} \otimes \boldsymbol{\psi}_t(\boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R), \boldsymbol{\theta}_d, P^R) \tag{21}$$

for $\boldsymbol{\theta}_d \in \Theta_d$. As a result, the actual sample-based objective function we minimize in the GMM estimation is the sum of norm of $\boldsymbol{G}_J^d(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R)$, $\boldsymbol{G}_J^c(\boldsymbol{\theta}, \boldsymbol{s}^n, P^R)$, and $\boldsymbol{G}_{J,T}^a(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R, \boldsymbol{\eta}^N)$, that is, the norm of

$$G_{J,T}(\boldsymbol{\theta}, \boldsymbol{s}^n, P^R, \boldsymbol{\eta}^N) = \begin{pmatrix} G_J^d(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R) \\ G_J^c(\boldsymbol{\theta}, \boldsymbol{s}^n, P^R) \\ G_{J,T}^a(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R, \boldsymbol{\eta}^N) \end{pmatrix}. \tag{22}$$

Notice that the first two moments G_J^d and G_J^c in (22) are sample moments averaged over products $j = 1, \ldots, J$, while the third moment $G_{J,T}^a$ is averaged over consumers $t = 1, \ldots, T$. Note also that in the expression $G_{J,T}(\boldsymbol{\theta}, \boldsymbol{s}^n, P^R, \boldsymbol{\eta}^N)$, there exist five distinct randomness: one from the draws of the product characteristics $(\boldsymbol{x}_{1j}, \xi_j, \mathbf{w}_{1j}, \omega_j)$, two from the sampling processes not controlled by econometrician of consumers for $\boldsymbol{s}^n$ and $\boldsymbol{\eta}^N$, two from the empirical distributions P^R and P^T employed by econometrician. The impact of these randomnesses on the estimate of $\boldsymbol{\theta}$ are decided by the relative size of the sample—J, n, N, R and T. Now we are going to operationalize the sampling and the simulation errors in the following.

2.4 The Sampling and Simulation Errors

The sampling error, $\boldsymbol{\epsilon}^n$, is defined as the difference between the observed market shares $\boldsymbol{s}^n$ and the true market share $\boldsymbol{s}^0$. Specifically, its component ϵ_j^n for the product j is

$$\epsilon_j^n \equiv s_j^n - s_j^0 = \frac{1}{n}\sum_{i=1}^{n}\{1(C_i = j) - s_j^0\} = \frac{1}{n}\sum_{i=1}^{n}\epsilon_{ji} \tag{23}$$

for $j = 1, \ldots, J$, where $\epsilon_{ji} \equiv 1(C_i = j) - s_j^0, i = 1, \ldots, n$ are the differences of the sampled consumer's choice from the population market share (s_j^0) of the same choice and are assumed independent across i.

Note that from (2), for any $\boldsymbol{\theta}_d \in \Theta_d$, the unique solutions $\boldsymbol{\xi}$ for $\boldsymbol{s}^n - \boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P^R) = \mathbf{0}$ and $\boldsymbol{s}^0 - \boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P^0) = \mathbf{0}$ are written as $\boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R)$ and $\boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0)$ respectively. So, substituting these $\boldsymbol{\xi}$s back into $\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P^R)$ and $\boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P^0)$ retrieves $\boldsymbol{s}^n$ and $\boldsymbol{s}^0$ respectively, or $\boldsymbol{s}^n = \boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R), \boldsymbol{\theta}_d, P^R)$ and $\boldsymbol{s}^0 = \boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0), \boldsymbol{\theta}_d, P^0)$ for any $\boldsymbol{\theta}_d \in \Theta_d$. Similarly, if we evaluate (2) with the observed (true) market share $\boldsymbol{s}^n$ $(\boldsymbol{s}^0)$ and the underlying (empirical) population P^0 (P^R) of consumers, the resulting $\boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^0)$ $(\boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^R))$ satisfies $\boldsymbol{s}^n = \boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^0), \boldsymbol{\theta}_d, P^0)$ $(\boldsymbol{s}^0 = \boldsymbol{\sigma}(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^R), \boldsymbol{\theta}_d, P^R))$ for all $\boldsymbol{\theta}_d \in \Theta_d$. These facts are used to define the simulation errors below.

The simulation process generates the simulation error $\boldsymbol{\epsilon}^R(\boldsymbol{\theta}_d)$, which is for any $\boldsymbol{\theta}_d$ the difference between the simulated market shares in (17) from the P^R and those from the P^0. The simulation error ϵ_j^R for product j with sample of R consumers is defined as follows.

$$\begin{aligned}\epsilon_j^R(\boldsymbol{\theta}_d) &\equiv \sigma_j(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0), \boldsymbol{\theta}_d, P^R) - \sigma_j(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0), \boldsymbol{\theta}_d, P^0) \qquad (24)\\ &= \frac{1}{R}\sum_{r=1}^{R} \epsilon_{jr}^*(\boldsymbol{X}, \boldsymbol{\xi}(\boldsymbol{\theta}_d, \boldsymbol{s}^0, P^0), \boldsymbol{\theta}_d)\end{aligned}$$

for $j = 1, \dots, J$, where $\epsilon_{jr}^*(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d) = \sigma_{rj}(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\nu}_r; \boldsymbol{\theta}_d) - \sigma_j(\boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\theta}_d, P^0), r = 1, \dots, R$ are independent across r conditional on $(\boldsymbol{X}, \boldsymbol{\xi})$ by the simulating process.

We also assume N independent consumer draws with their purchasing histories are used to construct the additional information $\boldsymbol{\eta}^N = (\eta_{11}^N, \dots, \eta_{1N_p}^N, \dots, \eta_{D1}^N, \dots, \eta_{DN_p}^N)'$ and define the sampling error $\boldsymbol{\epsilon}^N$ in the additional information $\boldsymbol{\eta}^N$ itself as follows.

$$\boldsymbol{\epsilon}^N \equiv \boldsymbol{\eta}^N - \boldsymbol{\eta}^0 = \frac{1}{N}\sum_{i'=1}^{N} \boldsymbol{\epsilon}_{i'}^{\#}. \qquad (25)$$

In short, we assume here that $\boldsymbol{\eta}^N$ is the average of N conditionally independent random variables given the set of product characteristics $(\boldsymbol{X}, \boldsymbol{\xi})$ of all products.

Since we use the sample of T draws of consumer to evaluate the additional moments, this also induces the sampling error in $\boldsymbol{G}_{J,T}^a(\boldsymbol{\theta}_d, \boldsymbol{s}^n, P^R, \boldsymbol{\eta}^N)$ in (21). Note that quantities n and N are are normally *beyond the control of* econometrician. On the other hand quantities R and T are are both *chosen by* econometrician.

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