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Functions on singular surfaces

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1. Introduction. The purpose of this note is to announce my recent results in [8] with T. Fukui. Refer to [8] for details.

Let $f:(\mathbb{R}^2,\mathbf{0})\to(\mathbb{R}^3,\mathbf{0})$ be a smooth map-germ which defines a surface (possibly with singularities) in $\mathbb{R}^3$. We define families $H:(\mathbb{R}^2,\mathbf{0})\times S^2\to\mathbb{R}$ and $D:(\mathbb{R}^2,\mathbf{0})\times\mathbb{R}^3\to\mathbb{R}$ of functions on f by

$$H(u,v,\mathbf{w})=\langle f(u,v),\mathbf{w}\rangle,\ \text{and}\ D(u,v,p)=\|p-f(u,v)\|^2-t^2,$$

respectively, where S^2 is a unit sphere in $\mathbb{R}^3$, $\langle\ ,\ \rangle$ denotes the dot product in $\mathbb{R}^3$ and $t\in\mathbb{R}\setminus\{0\}$. We define $h_{\mathbf{w}}(u,v)=H(u,v,\mathbf{w})$ and $d_p(u,v)=D(u,v,p)$, which are the *height function* on f in the direction $\mathbf{w}$ and the *distance squared function* on f from the point p, respectively. The family H is a 2-parameter unfolding of $h_{\mathbf{w}}$, and the family D is 3-parameter unfolding of d_p. The analysis of the contact of a submanifold with degenerate objects (lines, planes, circles, spheres, etc.) is important to understand geometric properties of the submaifold. The contacts of a surface in $\mathbb{R}^3$ with planes and spheres are measured by singularities of the height functions $h_{\mathbf{w}}$ and the distance squared functions d_p on the surface, respectively. By using these kinds of techniques of the contacts, several researchers have studied the differential geometry of submanifolds in Euclidean space (see, for example [2, 3, 5, 15, 22]) and other ambient spaces.

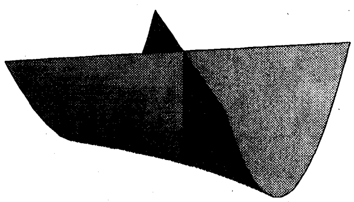

FIGURE. 1: The Whitney umbrella, $(u,v)\to(u,uv,v^2)$

H. Whitney [26] showed that smooth maps of $\mathbb{R}^2$ into $\mathbb{R}^3$ can have a singularity which are not avoidable by small perturbation. This singularity is called Whitney umbrella or cross-cap (Figure 1). Since the Whitney umbrella is a stable singularity, it is natural to seek its geometry. The extrinsic differential geometry of the Whitney umbrella is investigated in [4, 6, 7, 9, 10, 19, 20, 21, 24, 25], and in [11] its intrinsic properties are considered. It is shown for instance in [4, 25] that there are generically two types of Whitney umbrellas, labeled hyperbolic Whitney umbrella and elliptic Whitney umbrella (Figure 2), and these are characterized by the singularity type of their parabolic set in the source. The change from an elliptic to a hyperbolic Whitney umbrella occurs at a parabolic Whitney umbrella. In [6, 7], the singularities of the height and distance squared functions on Whitney umbrellas were studied in terms of its differential geometric properties via a blowing up. In [6], the singularities of sections of the Whitney umbrella by planes were studied and we showed the analogous theorem of J. A. Montaldi [17] for Whitney umbrellas. In [7], the criteria of singularities of wave-fronts and caustics of the Whitney umbrella were obtained, and the focal conic of the Whitney umbrella was introduced, which is the counterpart of focal points of smooth surfaces (Figure 2). The (generic) type of the focal conic determines the (generic) type of the Whitney umbrella, and vice-versa.

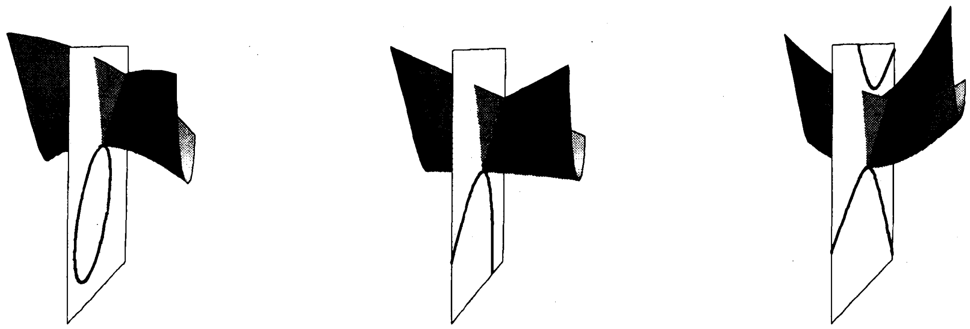

FIGURE. 2: The three types of the Whitney umbrella and their focal conics: hyperbolic Whitney umbrella (left), parabolic Whitney umbrella (center), elliptic Whitney umbrella (right).

In [16], D. Mond showed that every $\mathscr{A}$-simple germ of a map from a 2-manifold to a 3-manifold is equivalent to one of the germs in Table 1.

TABLE 1.

Name	Normal form	$\mathscr{A}$-codim.
Immersion	$(u, v, 0)$	0
Whitney umbrella(S_0)	(u, v^2, uv)	2
$S_k^\pm$	$(u, v^2, v^3 \pm u^{k+1}v), k \geq 1$	$k+2$
$B_k^\pm$	$(u, v^2, u^2v \pm v^{2k+1}), k \geq 2$	$k+2$
$C_k^\pm$	$(u, v^2, u^2v \pm v^k v), k \geq 3$	$k+2$
F_4	$(u, v^2, u^3v + v^5)$	6
H_k	$(u, uv + u^{3k-1}, v^3), k \geq 2$	$k+2$

(When k is even, S_k^+ is equivalent to S_k^-, and C_k^+ to C_k^-.)

It is also of interest to investigate the singularities of the height and distance squared functions on singular surfaces defined by $f : (\mathbb{R}^2, \mathbf{0}) \to (\mathbb{R}^3, \mathbf{0})$ with $\mathscr{A}$-simple singularities more degenerate than the Whitney umbrella (see Table 1). Our main objective is to investigate the relations between the differential geometric properties of singular surfaces which have one of S_k, B_k, C_k, or F_4 singularity (shown in Table 1) and singularities of the height and distance squared functions on these singular surfaces. Work in the direction of the understanding the differential geometry of singular surfaces with a singularity of corank 1 was carried out, for instance, in [11, 13, 14, 21].

2. Preliminaries. In order to analyze the differential geometry of a surface, relevant parameterization of the surface are essential. The Whitney umbrella case was obtained in [25] (see also [7]).

Proposition 2.1. *Let $f : (\mathbb{R}^2, \mathbf{0}) \to (\mathbb{R}^3, \mathbf{0})$ be a map-germ of corank 1 at the origin. Then, after using rotations in the target and changes of coordinates in the source, we can reduce $j^k f(0,0)$ to*

$$\left(u,\ \frac{1}{2}v^2 + \sum_{i=2}^{k} \frac{b_i}{i!}u^i,\ \frac{1}{2}a_{20}u^2 + \sum_{i+j=3}^{k} \frac{a_{ij}}{i!j!}u^i v^j\right) \tag{2.1}$$

if $j^2 f(0,0)$ is $\mathscr{A}$-equivalent to $(u, v^2, 0)$, or

$$\left(u,\ uv + \sum_{i=3}^{k} \frac{b_i}{i!}v^i,\ \frac{1}{2}a_{20}u^2 + \sum_{i+j=3}^{k} \frac{a_{ij}}{i!j!}u^i v^j\right) \tag{2.2}$$

if $j^2 f(0,0)$ is $\mathscr{A}$-equivalent to $(u, uv, 0)$.

Proposition 2.2. *Let $f : (\mathbb{R}^2, \mathbf{0}) \to (\mathbb{R}^3, \mathbf{0})$ be a smooth map-germ of corank 1 and let $j^k f(0,0)$ be given in the form (2.1). Then the conditions for f to be $\mathscr{A}$-equivalent to S_k, B_k, C_k or F_4 are as follows:*

$$\begin{aligned}
S_1 &: \ a_{21} \neq 0,\ a_{03} \neq 0,\\
S_{k\geq 2} &: \ a_{21} = \cdots = a_{k,1} = 0,\ a_{k+1,1} \neq 0,\ a_{03} \neq 0,\\
B_2 &: \ a_{03} = 0,\ a_{21} \neq 0,\ 3a_{05}a_{21} - 5{a_{13}}^2 \neq 0,\\
B_{k\geq 3} &: \ a_{03} = 0,\ a_{21} \neq 0,\ 3a_{05}a_{21} - 5{a_{13}}^2 = 0,\\
&\quad\ \xi_3 = \cdots = \xi_{k-1} = 0,\ \xi_k \neq 0,\\
C_k &: \ a_{03} = 0,\ a_{21} = \cdots = a_{k-1,1} = 0,\ a_{k,1} \neq 0,\ a_{13} \neq 0,\\
F_4 &: \ a_{03} = 0,\ a_{21} = 0,\ a_{31} \neq 0,\ a_{13} = 0,\ a_{05} \neq 0,
\end{aligned}$$

where ξ_m depends on the $(2m+1)$-jet of the third component of (2.1).

Remark 2.3. Each of these criteria of Proposition 2.2 includes the condition that

$$a_{21} \neq 0 \text{ or } a_{m,1} = 0\ (2 \leq m \leq n),\ a_{n+1,1} \neq 0 \text{ for some } n. \tag{2.3}$$

Let $f : (\mathbb{R}^2, \mathbf{0}) \to (\mathbb{R}^3, \mathbf{0})$ be a smooth map-germ of corank 1 at the origin $\mathbf{0}$. At the singular point $\mathbf{0}$, the tangent plane degenerates to a line, that is, the image of $(df)_{\mathbf{0}}$ is a line, which is the *tangent line.* The plane passing through the singular point $\mathbf{0}$ perpendicular to the tangent line is called the *normal plane.* There exists non-zero vector $\eta \in T_0\mathbb{R}^2$ such that $(df)_{\mathbf{0}}(\eta) = 0$. We call η the *null vector* (cf. [12, 23]). A regular plane curve in the parameter space passing through $(0,0)$ whose tangent vector is transverse to η is called a *tangential curve.* Let $\gamma(t)$ be the parameterization of a tangential curve with $\gamma(0) = (0,0)$. Clearly, $f \circ \gamma$ is tangent to the tangent line of f at the singular point $\mathbf{0}$. If $j^2 f(0,0)$ is $\mathscr{A}$-equivalent to $(u, v^2, 0)$, then there exists a plane passing through the singular point $\mathbf{0}$ spanned by the tangent line and $\eta\eta f(0,0)$, where $\eta\eta f$ is the twice times directional derivative of f with respect to η. We call the plane the *principal plane.* We remark that the definitions of the tangent line, normal plane and principal plane are independent of the choice of coordinates on the source. Let $j^2f(0,0)$ be $\mathscr{A}$-equivalent to $(u, v^2, 0)$. The singular point $\mathbf{0}$ is a *inflection point* if $f\circ\gamma$ have at least 3-point contact (inflectional tangent) with the principal plane at $\mathbf{0}$. We denote Γ by a family of tangential curves γ. A member Γ_0 of the family is a *characteristic tangential curve* if the curvature, of the projection of $f\circ\Gamma_0$ to the principal plane, at $\mathbf{0}$ has the extremum value κ_0. Note that tangential curves tangent to the characteristic tangential curve are characteristic tangential curves. If the singular point $\mathbf{0}$ is a inflection point, the singular point $\mathbf{0}$ is a *degenerate inflection point* if $\kappa_0 = 0$. We remark that the definitions of the inflection and degenerate inflection points are independent of the choice of the coordinates on the source. We also remark that the ways of definitions of the inflection and degenerate inflection points differ here from those in [21].

FIGURE. 3: The tangent line, the normal plane and the principal plane of S_1^- (left) and S_1^+ (right).

Proposition 2.4. *Let* $f : (\mathbb{R}^2, \mathbf{0}) \to (\mathbb{R}^3, \mathbf{0})$ *be a smooth map-germ of corank* 1 *at the origin, and let* $j^kf(0,0)$ *be given in the form* (2.1)*. Then the origin is an inflection* (*resp. degenerate inflection*) *point if and only if* $a_{20} = 0$ (*resp.* $a_{20} = b_2 = 0$)*. Moreover, tangential curves tangent to* ∂_u *at* $(0,0)$ *are the characteristic tangential curves.*

Let $f : (\mathbb{R}^2, \mathbf{0}) \to (\mathbb{R}^3, \mathbf{0})$ be a smooth map-germ of corank 1 at the origin, and let $j^kf(0,0)$ be given in the form (2.1) with the assumption (2.3). We set the map $\Pi_{n+1} : \mathbb{R}\times S^1 \to \mathbb{R}^2$ with

$$\Pi_{n+1}(r,\theta) = (r\cos\theta, r^{n+1}\cos^n\theta\sin\theta) \quad (n = 1 \text{ if } a_{21} \neq 0).$$

The unit normal vector $\tilde{\mathbf{n}} = \mathbf{n}\circ\Pi_{n+1}$ of f in (r,θ) is extendible near the exceptional set $\{(r,\theta) \in \mathbb{R}\times S^1 \mid r\cos\theta = 0\}$ and

$$\tilde{\mathbf{n}}(0,\theta) = \frac{(0, -a_{n+1,1}\cos\theta, (n+1)!\sin\theta)}{\mathscr{A}(\theta)},$$

where $\mathscr{A}(\theta) = \sqrt{(a_{n+1,1}\cos\theta)^2 + ((n+1)!\sin\theta)^2}$. The principal curvatures $\tilde{\kappa}_i = \kappa_i\circ\Pi_{n+1}$ of f in (r,θ) are expressed as follows:

$$\tilde{\kappa}_1(r,\theta) = \frac{-a_{n+1,1}b_2\cos\theta + (n+1)!a_{20}\sin\theta}{\mathscr{A}(\theta)} + O(r),$$

$$\tilde{\kappa}_2(r,\theta) = \frac{1}{r^{2n+2}}\left(\frac{-((k+1)!)^2 a_{n+1,1}}{\cos^{2n-1}\theta\mathscr{A}(\theta)^3} + O(r)\right).$$

The vector $\mathbf{v}_i = (N-\kappa_i G)\partial_u - (M-\kappa_i F)\partial_v$ $(i = 1,2)$ is a principal vector relative to κ_i. The principal vectors $\tilde{\mathbf{v}}_i$ in (r,θ) are given by

$$\begin{aligned}\tilde{\mathbf{v}}_1 =& \left(-\frac{a_{n+1,1}}{\mathscr{A}(\theta)} + O(r)\right)\partial_r \\ &+\left(-\frac{a_{n+2,1}\cos^2\theta\sin\theta + (n+2)(n+1)!\,a_{12}\cos\theta\sin^2\theta}{(n+2)\mathscr{A}(\theta)} + O(r)\right)\partial_\theta, \\ \tilde{\mathbf{v}}_2 =& -\frac{(n+1)!\,a_{n+1,1}(a_{20}a_{n+1,1}\cos\theta + (n+1)!\,b_2\sin\theta)\cos^{2-n}\theta}{\mathscr{A}(\theta)^3 r^{2n+1}} \\ &\left((\sin\theta\, r + O(r^2))\partial_r + (\cos\theta + O(r))\partial_\theta\right).\end{aligned}$$

We can describe the asymptotic behavior of the ridge and sub-parabolic curves. Here, the *ridge* (resp. *sub-parabolic*) *curve* is the locus of points where one principal curvature has an extremum along lines of the same (resp. other) principal curvature. It is known that the ridge and sub-parabolic curves of a regular surface in $\mathbb{R}^3$ correspond, respectively, to the cuspidal edges and parabolic set of caustics of the surface (see, for instance [3, 18]).

3. Singularities of height functions. Even at a singular point of a singular surface f, we can define, in the parameter space, the parabolic set of f as the zero set of

$$(\langle f_{uu}, f_u \times f_v\rangle\langle f_{vv}, f_u \times f_v\rangle - \langle f_{uv}, f_u \times f_v\rangle^2)(u,v).$$

The parabolic set of f has a singularity. The singularities of the parabolic set of singular surfaces with one of $\mathscr{A}$-simple singularities of $\mathscr{A}_e$-codim.≤ 3 are investigated in [21].

Theorem 3.1. *Let a smooth map-germ $f : (\mathbb{R}^2, \mathbf{0}) \to (\mathbb{R}^3, \mathbf{0})$ be $\mathscr{A}$-equivalent to one of S_k, B_k, C_k or F_4 singularity, and let the origin be not inflection point.*

(1) *There is a branch P, of the parabolic set of f, which is a characteristic tangential curve.*

(2) *Let $\mathscr{L}(t)$ be a parameterization of P on f with $\mathscr{L}(0) = (0,0,0)$, and let $\mathbf{b}(t)$ and $\tau(t)$ denote by the unit binormal vector and the torsion of $\mathscr{L}(t)$, respectively. Then the height function $h_{\mathbf{w}}$ on f in $\mathbf{w} = \pm\mathbf{b}(0)$ has a singularity at $(0,0)$ of type*

$$\begin{aligned}A_2 &\Longleftrightarrow \tau(0) \neq 0 \\ A_3 &\Longleftrightarrow \tau(0) = 0,\ \tau'(0) \neq 0, \\ A_{\geq 4} &\Longleftrightarrow \tau(0) = \tau'(0) = 0.\end{aligned}$$

(3) *The branch P has at least m-point contact with its tangent line at $(0,0)$, where m is as shown in the following table:*

$\mathscr{A}$-type	S_k	B_k	C_k	F_4
m	$k+1$	2	k	3

For Whitney umbrellas, an analogs theorem was obtained in [21]. If we replace the assumption that the origin is not inflection point with the assumption that the origin is degenerate inflection point and $\hat{\gamma}$ has 3-point contact with the principal plane at the origin, Theorem 3 holds except that $h_{\mathbf{w}}$ on f in $\mathbf{w} = \pm\mathbf{b}(0)$ does not have an A_2 singularity at $(0,0)$.

4. Singularities of distance squared functions. Let $f : (\mathbb{R}^2, \mathbf{0}) \to (\mathbb{R}^3, \mathbf{0})$ be a smooth map-germ of corank 1 at the origin $\mathbf{0}$, and let $j^2 f(0,0)$ be $\mathscr{A}$-equivalent to $(u, v^2, 0)$. The locus of points p where the distance squared function d_p has a degenerate singularity at $(0,0)$ is called the *focal locus*. The focal locus can be considered as an analogy of the focal conic of Whitney umbrellas (cf. [7, Lemma 3.3]).

Theorem 4.1. *Let $f : (\mathbb{R}^2, \mathbf{0}) \to (\mathbb{R}^3, \mathbf{0})$ be a smooth map-germ of corank* 1 *at the origin* $\mathbf{0}$, *and let $j^2 f(0,0)$ be $\mathscr{A}$-equivalent to $(u, v^2, 0)$.*

(1) *The origin is not an inflection point if and only if the focal locus is a pair of two intersecting lines (a unique line and a line).*

(2) *The origin is a non-degenerate inflection point if and only if the focal locus is a two parallel lines (a unique line and a line).*

(3) *The origin is a degenerate inflection point if and only if then the focal locus is a unique line.*

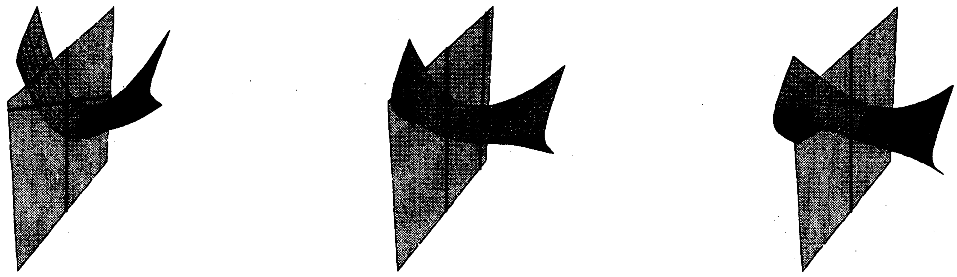

FIGURE. 4: Focal loci of S_1^+.

Wave-fronts are described by Huygens's principle as follows: The *wave-front* of a surface at a instant is the envelope of spherical wavelets from points on the surface at the instant. Even if a surface is smooth its wave-fronts in general have singularities. The *caustic* of a surface is the envelope of the normal rays to the surface. The caustic can be thought of the set of the singularities of all wave-fronts. The wave-front and caustic of a surface are the discriminant and bifurcation sets of the family D of the distance squared function d_p on the surface, respectively.

Theorem 4.2. *Assume that $f : (\mathbb{R}^2, \mathbf{0}) \to (\mathbb{R}^3, \mathbf{0})$ is a smooth map-germ of corank* 1 *at the origin whose k-jet $j^k f(0,0)$ is given in the form* (2.1). *Moreover, let f be $\mathscr{A}$-equivalent to one of S_k, B_k, C_k or F_4 singularity, and let the origin be not degenerate inflection point. The singularities of fronts and caustics of f at $p = \tilde{\mathbf{n}}(0,\theta)/\tilde{\kappa}_1(0,\theta)$ $(\cos\theta \neq 0)$ are shown in Table 2 and 3, respectively.*

TABLE 2. *Singularities of fronts.*

condition for $(0,\theta)$	*singularity*
not ridge relative to $\tilde{\mathbf{v}}_1$	*cuspidal edge*
1st ridge relative to $\tilde{\mathbf{v}}_1$, *not sub-parabolic relative to* $\tilde{\mathbf{v}}_2$	*swallowtail*

TABLE 3. *Singularities of caustics.*

condition for $(0,\theta)$	*singularity*
not ridge relative to $\tilde{\mathbf{v}}_1$	*non-singular*
1st ridge relative to $\tilde{\mathbf{v}}_1$	*cuspidal edge*

Here, $(0,\theta)$ *is a 1st ridge relative to* $\tilde{\mathbf{v}}_1$ *if the ridge curve relative to* $\tilde{\mathbf{v}}_1$ *is transverse to* $\tilde{\mathbf{v}}_1$ *at* $(0,\theta)$.

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