数理解析研究所講究録
第 1873 巻 2014 年 114-121

Superstring Theory and Triple Systems

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1 Introduction

It has been expected that there exists M-theory, which unifies string theories. In M-theory, some structures of 3-algebras were found recently. First, it was found that by using $u(N)\oplus u(N)$ Hermitian 3-algebra, we can describe a low energy effective action of N coincident supermembranes [1–5], which are fundamental objects in M-theory.

Second, recent studies have indicated that there also exist structures of 3-algebras in the Green-Schwartz supermembrane action, which defines full perturbative dynamics of a supermembrane. It had not been clear whether the total supermembrane action including fermions has structures of 3-algebras, whereas the bosonic part of the action can be described by using a tri-linear bracket, called Nambu bracket [6, 7], which is a generalization of Poisson bracket. If we fix to a light-cone gauge, the total action can be described by using Poisson bracket, that is, only structures of Lie algebra are left in this gauge [8]. However, it was shown under an approximation that the total action can be described by Nambu bracket if we fix to a semi-light-cone gauge [9]. In this gauge, the eleven dimensional space-time of M-theory is manifest in the supermembrane action, whereas only ten dimensional part is manifest in the light-cone gauge. Moreover, 3-algebra models of M-theory itself were proposed and have been studied in [9–13].

The hermitian (ϵ,δ)-Freudenthal-Kantor triple systems [14–36] are generalizations of the hermitian 3-algebras [1–9, 37–71]. The hermitian 3-algebras are special cases, where $K(a,b)=0$ or equivalently, $<abc>=-<cba>$, of the hermitian $(-1,-1)$-Freudenthal-Kantor triple systems of second order. And the hermitian 3-algebras are classified into the $u(N)\oplus u(M)$ and $sp(2N)\oplus u(1)$ hermitian 3-algebras [13, 43, 45, 46, 52]. Therefore, it is natural to extend these triple systems to more general hermitian $(-1,-1)$-Freudenthal-Kantor triple systems or hermitian generalized Jordan triple systems.

In the following section, we summarize some results concerning with the generalization of the hermitian 3-algebras in M-theory [72, 73].

2 Definitions

Let us start with a definition of a *-(ϵ,δ)-Freudenthal-Kantor triple system.

Definition. A triple system U is said to be a *-(ϵ,δ)-Freudenthal-Kantor triple system if relations (0)–(iv) satisfy;

0) U is a Banach space,

i) $[L(a,b),L(c,d)] = L(< abc >, d) + \varepsilon L(c, < bad >)$,
ii) $K(< abc >, d) + K(c, < abd >) + \delta K(a, K(c,d)b) = 0$,
where $L(a,b)c =< abc >$ and $K(a,b)c =< acb > -\delta < bca >$, $\varepsilon = \pm 1, \delta = \pm 1$,
iii) $< xyz >$ is **C**-linear operator on x, z and **C**-anti-linear operator on y,
iv) $< abc >$ continuous with respect to a norm $|| \, ||$ that is, there exists $K > 0$ such that

$$|| < xxx > || \leq K||x||^3 \text{ for all} x \in U.$$

Furthermore, a *-(ϵ, δ)-Freudenthal-Kantor triple system is said to be hermitian if it satisfies the following condition,
v) all operator $L(x,y)$ is a positive hermitian operator with a hermitian metric

$$(x,y) = tr \ L(x,y),$$

that is, $(L(x,y)z,w) = (z, L^*(x,y)w)$, and $(x,y) = \overline{(y,x)}$.

Let U be a *-(ϵ, δ)-Freudenthal-Kantor triple system. Then we may define the notation of tripotent as follows.

Definition. It is said to be a tripotent of U if

$$< ccc >= c, \ \ c \in U.$$

3 Tripotent basis

In this section, we give decomposition theorems based on the tripotent basis.

Theorem 1.1. Let U be a hermitian $(-1, \delta)$-Freudenthal-Kantor triple system. If $W \subset U$ is flat (that is, $L(x,y) = L(y,x)$ for all $x, y \in W$), then we have a decomposition,

$$W = \mathbf{R}e_1 \oplus \cdots \oplus \mathbf{R}e_n$$

where e_i are tripotents or bitripotens.

Proof. See [72, 73].

We define the odd power of x inductively as follows;

$$x^{(3)} :=< xxx >,$$
$$x^{(2n+1)} :=< xx^{(2n-1)}x > .$$

By using this theorem, we have

Theorem 1.2. Let U be a hermitian $(-1, \delta)$-Freudenthal-Kantor triple system. Then every $x \in U$ can be written uniquely

$$x = \lambda_1 e_1 + \lambda_2 e_2 + \cdots + \lambda_n e_n,$$

where the e_i are tripotents or bitripotents, which are linear combinations of power of x, and the λ_i satisfy

$$0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n.$$

Poof. See [72, 73].

4 Peirce decomposition

In this section, we give a theorem on a Peirce decomposition of a *-$(-1,-1)$-Freudenthal-Kantor triple system equipped with the tripotent $< ccc >= c$.

Theorem 2.1. Let U be a *-$(-1,-1)$-Freudenthal-Kantor triple system. Then, we have a decomposition with respect to a tripotent c (i.e.,$< ccc >= c$) as follows

$$U = U_1(c) \oplus U_{\frac{1}{2}}(c) \oplus U_0(c),$$

where

$$U_1(c) = \{x | (L(c,c) + R(c,c))x = 0,\ (R(c,c) - Id)x \neq 0\},$$
$$U_{\frac{1}{2}}(c) = \{x | (L(c,c) + R(c,c))x \neq 0,\ (R(c,c) - Id)x = 0\},$$
$$U_0(c) = \{x | (L(c,c) + R(c,c))x = 0,\ (R(c,c) - Id)x = 0\}.$$

Poof. See [72, 73].

5 Generalized hermitian 3-algebra

In this section, we extend the $u(N)\oplus u(M)$ 3-algebras to a hermitian $(-1,-1)$-Freudenthal-Kantor triple system.

Let $D^*_{N,M}$ be the set of all $N \times M$ matrices with operation

$$< xyz >= x\bar{y}^T z - z\bar{y}^T x + zx^T \bar{y},$$

where x^T and $\bar{x}$ mean transpose and conjugation of x, respectively.

Then $D^*_{N,M}$ is a hermitian $(-1,-1)$-Freudenthal-Kantor triple system. In fact, it satisfies the conditions (0), (i),(ii),(iii),(iv) and (v). This is an extension of the $u(N) \oplus u(M)$ hermitian 3-algebra, $< xyz >= x\bar{y}^T z - z\bar{y}^T x$, which is a basis for the effective action of the multiple membranes in M-theory.

One of the tripotents is given by

$$c = \begin{pmatrix} Id & 0 \\ 0 & 0 \end{pmatrix},$$

where Id is a $n \times n$ identity matrix $(n \leq N, M)$. Because any element is decomposed as

$$x = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(A - A^T) & B \\ 0 & D \end{pmatrix} + \begin{pmatrix} \frac{1}{2}(A + A^T) & 0 \\ C & 0 \end{pmatrix},$$

the Peirce decomposition is given by

$$\begin{aligned} U_1(c) &= \{\begin{pmatrix} \frac{1}{2}(A - A^T) & B \\ 0 & D \end{pmatrix}\}, \\ U_{\frac{1}{2}}(c) &= \{\begin{pmatrix} \frac{1}{2}(A + A^T) & 0 \\ C & 0 \end{pmatrix}\}, \\ U_0(c) &= 0. \end{aligned}$$

As in Theorem 1.1, we can expand any element as $x = \Sigma(\lambda_{ij} E_{ij} + \mu_{ij}\sqrt{-1}E_{ij})$, where E_{ij} means that (i,j) element is 1 and other element is zero, and E_{ij} and $\sqrt{-1}E_{ij}$ are tripotents, i.e., $< E_{ij}E_{ij}E_{ij} >= E_{ij}$, and $< \sqrt{-1}E_{ij}\sqrt{-1}E_{ij}\sqrt{-1}E_{ij} >= \sqrt{-1}E_{ij}$.

References

[1] J. Bagger, N. Lambert, "Modeling Multiple M2's," Phys. Rev. D75: 045020, 2007, hep-th/0611108.

[2] A. Gustavsson, "Algebraic structures on parallel M2-branes," Nucl. Phys. B811: 66, 2009, arXiv:0709.1260 [hep-th].

[3] J. Bagger, N. Lambert, "Gauge Symmetry and Supersymmetry of Multiple M2-Branes," Phys. Rev. D77: 065008, 2008, arXiv:0711.0955 [hep-th].

[4] O. Aharony, O. Bergman, D. L. Jafferis, J. Maldacena, "N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals," JHEP 0810: 091, 2008, arXiv:0806.1218 [hep-th]

[5] J. Bagger, N. Lambert, "Three-Algebras and N=6 Chern-Simons Gauge Theories," Phys. Rev. D79: 025002, 2009, arXiv:0807.0163 [hep-th]

[6] Y. Nambu, Generalized Hamiltonian dynamics, Phys. Rev. D7 (1973) 2405.

[7] H. Awata, M. Li, D. Minic, T. Yoneya, On the Quantization of Nambu Brackets, JHEP 0102 (2001) 013.

[8] B. de Wit, J. Hoppe, H. Nicolai, On the Quantum Mechanics of Supermembranes, Nucl. Phys. B305 (1988) 545.

[9] M. Sato, "Model of M-theory with Eleven Matrices," JHEP 1007 (2010) 026, arXiv:1003.4694 [hep-th]

[10] M. Sato, " Covariant Formulation of M-Theory," Int. J. Mod. Phys. A24 (2009), 5019, arXiv:0902.1333 [hep-th]

[11] M. Sato, "Supersymmetry and the Discrete Light-Cone Quantization Limit of the Lie 3-algebra Model of M-theory," Phys. Rev. D85 (2012) 046003, arXiv:1110.2969 [hep-th]

[12] M. Sato, "Zariski Quantization as Second Quantization," Phys. Rev. D85 (2012) 126012, arXiv:1202.1466 [hep-th]

[13] M. Sato, "3-Algebras in String Theory," Linear Algebra - Theorems and Applications, Edited by Hassan Abid Yasser, InTech, Croatia, 2012

[14] N.Kamiya,;A structure theory of Freudenthal-Kantor triple systems,J.Alg. 110 (1987) 108-123.

[15] N.Kamiya,;A structure theory of Freudenthal-Kantor triple systems II,Comm. Math.Univ.Sancti Pauli.,38(1989)41-60.

[16] N.Kamiya,;A structure theory of Freudenthal-Kantor triple systems III, Mem.Fac. Sci.Shimane Univ. 23(1989)33-51.

[17] N.Kamiya,;The construction of all Simple Lie algebras over C from balanced Freudenthal-Kantor triple systems, Contributions to General Algebra 7,Verlag Hoder-Pichler-Tempsky, Wien,Veralg.G.Teubner,Stutugart(1991) 205-213.

[18] N.Kamiya,;On Freudenthal-Kantor triple systems and generalized structurable algebras, Proceeding in International conference of nonassociative algebras and its applications, Mathematics and Its Applications 303(1994) 198-203,Kluwer Academic Publisher,

[19] N.Kamiya,;On the Peirce decompostions for Freudenthal-Kantor triple systerns, Comm. in Alg. 25,(6),(1997),1833-1844.

[20] N.Kamiya, Examples of Peirce decomposition of generalized Jordan triple system of second order,-balanced cases-, Contemmporary Math., vol.391,AMS.(2005), Noncommutative Geometry and Representation theory an Mathematical Physics,(ed)Fuchs, p157-165.

[21] N.Kamiya and S.Okubo, On δ-Lie supertriple systems associated with (ε, δ)-Freudenthal-Kantor triple systems, Proc. Edinb. Math.Soc. 43(2000),243-260.

[22] I.L.Kantor and N.Kamiya, A Peirce decomposition for generalized Jordan triple systems of second order, Comm in Alg.,31(2003),5875-5913.

[23] I.L.Kantor,;Models of exceptional Lie algebras, Soviert Math.Dokl.14(1973) 254-258.

[24] I.L.Kantor,;Some generalization of Jordan algebras,Trudy Sem.Vektor, Tensor Anal.16(1972)407-499(Russian).

[25] W.Kaup,;Algebraic characterization of symmetric complex Banach manifoleds, Math. Ann.228. (1977)39-64.

[26] I.Loos,;Bounded symmetric domains and Jordan pairs,Mathematical Lectures.lnvine: University of California at Irwine,1977.

[27] K.Meyberg,;Lecture on algebras and triple systems,Lecture Notes,the Univ. ofVirginia,1972.

[28] E.Neher,;On the radical of J*-triples, Math.Z.211. (1992)323-332.

[29] B.N.Allison,; A class of nonassociative algebras with involution containing class of Jordan algebras, Math.Ann., 1978,237(2),133-156.

[30] N.Jacobson,;Structure and representations of Jordan algebras. Amer. Math. Soc.,Providence.R.1.,1968.

[31] S.Okubo,;lntroduction to Octonion and Other Non-associative Algebras in Physics, Cambridge Univ.Press.1995.

[32] 1.Satake,;Algebraic structure of symmetric domains,Princeton University press.1980.

[33] R.D.Shafer,;An introduction to Non-associative Algebras,Academicpress, 1966.

[34] N.Kamiya and D.Mondoc; A new class of nonassociative algebras with involution, Proc.Japan Acad.,84 Ser.A(2008),no5,68-72,

[35] N.Kamiya and S.Okubo,; A construction of simple Lie superalgebras of certain type from triple systems, Bull.Australia Math.Soc. vol.69,(2004) 113-123.

[36] N.Kamiya and S.Okubo,; Construction of Lie superalgebrasD(2,1,α), $G(3)$ and $F(4)$ from some triple systems. Proc. Edinburgh Math. Soc. (2003) 46, 87-98.

[37] V. T. Filippov, n-Lie algebras, Sib. Mat. Zh. 26, No. 6, (1985) 126140.

[38] E. Bergshoeff, E. Sezgin, P.K. Townsend, Supermembranes and Eleven-Dimensional Supergravity, Phys. Lett. B189 (1987) 75.

[39] D. Minic, "M-theory and Deformation Quantization," arXiv:hep-th/9909022.

[40] J. Figueroa-O'Farrill, G. Papadopoulos, "Pluecker-type relations for orthogonal planes," J. Geom. Phys. 49 (2004) 294, math/0211170]

[41] G. Papadopoulos, "M2-branes, 3-Lie Algebras and Plucker relations," JHEP 0805 (2008) 054, arXiv:0804.2662 [hep-th]

[42] J. P. Gauntlett, J. B. Gutowski, "Constraining Maximally Supersymmetric Membrane Actions," JHEP 0806 (2008) 053, arXiv:0804.3078 [hep-th]

[43] D. Gaiotto, E. Witten, "Janus Configurations, Chern-Simons Couplings, And The Theta-Angle in N=4 Super Yang-Mills Theory," arXiv:0804.2907[hep-th]

[44] Y. Honma, S. Iso, Y. Sumitomo, S. Zhang, "Janus field theories from multiple M2 branes," Phys.Rev.**D78**:025027,2008, arXiv:0805.1895 [hep-th].

[45] K. Hosomichi, K-M. Lee, S. Lee, S. Lee, J. Park, "N=5,6 Superconformal Chern-Simons Theories and M2-branes on Orbifolds," JHEP 0809 (2008) 002, arXiv:0806.4977[hep-th]

[46] M. Schnabl, Y. Tachikawa, "Classification of N=6 superconformal theories of ABJM type," arXiv:0807.1102[hep-th]

[47] S. Mukhi, C. Papageorgakis, M2 to D2, JHEP 0805 (2008) 085.

[48] J. Gomis, G. Milanesi, J. G. Russo, "Bagger-Lambert Theory for General Lie Algebras," JHEP 0806: 075, 2008, arXiv:0805.1012 [hep-th].

[49] S. Benvenuti, D. Rodriguez-Gomez, E. Tonni, H. Verlinde, "N=8 superconformal gauge theories and M2 branes," JHEP 0901: 078, 2009, arXiv:0805.1087 [hep-th].

[50] P.-M. Ho, Y. Imamura, Y. Matsuo, "M2 to D2 revisited," JHEP 0807: 003, 2008, arXiv:0805.1202 [hep-th].

[51] M. A. Bandres, A. E. Lipstein, J. H. Schwarz, "Ghost-Free Superconformal Action for Multiple M2-Branes," JHEP 0807: 117, 2008, arXiv:0806.0054 [hep-th]

[52] P. de Medeiros, J. Figueroa-O'Farrill, E. Me'ndez-Escobar, P. Ritter, "On the Lie-algebraic origin of metric 3-algebras," Commun. Math. Phys. 290 (2009) 871, arXiv:0809.1086 [hep-th]

[53] S. A. Cherkis, V. Dotsenko, C. Saeman, "On Superspace Actions for Multiple M2-Branes, Metric 3-Algebras and their Classification," Phys. Rev. D79 (2009) 086002, arXiv:0812.3127 [hep-th]

[54] P.-M. Ho, Y. Matsuo, S. Shiba, "Lorentzian Lie (3-)algebra and toroidal compactification of M/string theory," arXiv:0901.2003 [hep-th]

[55] P. de Medeiros, J. Figueroa-O'Farrill, E. Mendez-Escobar, P. Ritter, "Metric 3-Lie algebras for unitary Bagger-Lambert theories," JHEP 0904: 037, 2009, arXiv:0902.4674 [hep-th]

[56] H. Nishino, S. Rajpoot, Triality and Bagger-Lambert Theory, Phys. Lett. B671 (2009) 415, arXiv:0901.1173.

[57] A. Gustavsson, S-J. Rey, Enhanced N=8 Supersymmetry of ABJM Theory on R(8) and R(8)/Z(2), arXiv:0906.3568 [hep-th].

[58] O. Aharony, O. Bergman, D. L. Jafferis, Fractional M2-branes, JHEP 0811 (2008) 043, arXiv:0807.4924.

[59] M. Hanada, L. Mannelli, Y. Matsuo, Large-N reduced models of supersymmetric quiver, Chern-Simons gauge theories and ABJM, arXiv:0907.4937 [hep-th].

[60] G. Ishiki, S. Shimasaki, A. Tsuchiya, Large N reduction for Chern-Simons theory on S^3, Phys. Rev. D80 (2009) 086004, arXiv:0908.1711.

[61] H. Kawai, S. Shimasaki, A. Tsuchiya, Large N reduction on group manifolds, arXiv:0912.1456 [hep-th].

[62] G. Ishiki, S. Shimasaki, A. Tsuchiya, A Novel Large-N Reduction on S^3: Demonstration in Chern-Simons Theory, arXiv:1001.4917 [hep-th].

[63] Y. Pang, T. Wang, From N M2's to N D2's, Phys. Rev. D78 (2008) 125007, arXiv:0807.1444.

[64] J. DeBellis, C. Saemann, R. J. Szabo, Quantized Nambu-Poisson Manifolds in a 3-Lie Algebra Reduced Model, JHEP 1104 (2011) 075, arXiv:1012.2236.

[65] M. M. Sheikh-Jabbari, Tiny Graviton Matrix Theory: DLCQ of IIB Plane-Wave String Theory, A Conjecture , JHEP 0409 (2004) 017, arXiv:hep-th/0406214.

[66] J. Gomis, A. J. Salim, F. Passerini, Matrix Theory of Type IIB Plane Wave from Membranes, JHEP 0808 (2008) 002, arXiv:0804.2186.

[67] K. Hosomichi, K. Lee, S. Lee, Mass-Deformed Bagger-Lambert Theory and its BPS Objects, Phys.Rev. D78 (2008) 066015, arXiv:0804.2519.

[68] L. Smolin, "M theory as a matrix extension of Chern-Simons theory," Nucl.Phys.**B591**(2000) 227, hep-th/0002009.

[69] L. Smolin, "The cubic matrix model and a duality between strings and loops," hep-th/0006137.

[70] T. Azuma, S. Iso, H. Kawai, Y. Ohwashi, "Supermatrix Models," Nucl.Phys.**B610**(2001) 251, hep-th/0102168.

[71] J. Palmkvist, "Unifying N = 5 and N = 6," JHEP 1105 (2011) 088, arXiv:1103.4860.

[72] N. Kamiya, M. Sato, "Hermitian generalised Jordan triple systems and certain applications to field theory," preprint.

[73] N.Kamiya and M.Sato, "Hermitian (ϵ, δ)-Freudenthal-Kantor triple systems and certain applications of *-generalized Jordan triple systems to field theory," preprint