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A FREE BOUNDARY PROBLEM FOR A WEAK COMPETITION SYSTEM

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ABSTRACT. We study a free boundary problem for the Lotka-Volterra type weak competition model in a one-dimensional habitat. The main purpose of this study is to understand how these two competing species spread. We first establish a spreading-vanishing dichotomy. Then we provide some sufficient conditions for spreading success and spreading failure (vanishing), respectively. Finally, for the case of spreading success, we show that the asymptotic spreading speed, if it exists, is no larger than the minimal speed of traveling wavefront solutions for the competition model on the whole real line.

1. INTRODUCTION

In the study of the spreading (or invasion) phenomenon of species in a one-dimensional habitat, there have been a lot of works on the traveling fronts and two-front entire solutions. These are through the study of the Cauchy problem posed on the whole real line. However, in reality the support of the population of each individual species should be bounded initially. Therefore, it is quite natural to introduce a free boundary due to the changing of the support of each population with time.

For this purpose, Du and Lin [12] studied the following free boundary model with the logistic nonlinearity for one species:

$$(1.1)\qquad \begin{cases} u_t = du_{xx} + u(a - bu),\ 0 < x < h(t),\ t > 0, \\ u_x(0,t) = 0,\ u(h(t),t) = 0,\ t > 0, \\ h'(t) = -\mu u_x(h(t),t),\ t > 0, \\ h(0) = h_0,\ u(x,0) = u_0(x),\ 0 \le x \le h_0, \end{cases}$$

where u is the population density of the species, constants $d, a, b, \mu, h_0 > 0$, and $u_0 > 0$ in $[0, h_0)$ such that $u_0(h_0) = 0$. In [12], they established the spreading-vanishing dichotomy for (1.1), that is, either there holds the spreading success in the sense that $h(t) \to +\infty$ and $u(x,t) \to a/b$ as $t \to +\infty$, or the spreading failure (vanishing) occurs such that $h(t) \to h_\infty < +\infty$ for some $h_\infty > 0$ and $u(x,t) \to 0$ as $t \to +\infty$.

Motivated by the work of [12], a natural question is: *does there also exist a spreading-vanishing dichotomy for two species competition models?* In fact, there have been many

This paper is based on a joint work with Chang-Hong Wu [15].

studies for the following Lotka-Volterra type competition model:

$$u_t = u_{xx} + u(1 - u - kv), \quad x, t \in \mathbb{R}, \tag{1.2}$$

$$v_t = Dv_{xx} + rv(1 - v - hu), \quad x, t \in \mathbb{R}, \tag{1.3}$$

where $u(x,t), v(x,t)$ denote the population densities of two competing species and D, r, h, k are positive constants. Also, the global dynamics for the following related kinetic system (in the absence of diffusion) to (1.2)-(1.3) is well-known. Indeed, there are constant equilibria $\{(0,0), (1,0), (0,1)\}$ and in the case when both $h, k < 1$ or $h, k > 1$, we have the fourth equilibrium $(u^*, v^*) = ((1-k)/(1-hk), (1-h)/(1-hk))$. Moreover, the global dynamics for the kinetic system:

$$u_t = u(1 - u - kv), \quad t \in \mathbb{R},$$
$$v_t = rv(1 - v - hu), \quad t \in \mathbb{R},$$

by the phase plane analysis in $\{u, v > 0\}$, we have

(A). $(u,v)(t) \to (1,0)$ as $t \to \infty$, if $0 < k < 1 < h$;

(B). $(u,v)(t) \to (0,1)$ as $t \to \infty$, if $0 < h < 1 < k$;

(C). $(u,v)(t) \to$ one of $\{(1,0), (0,1), (u^*, v^*)\}$ as $t \to \infty$, if $h, k > 1$, depending on the initial value (this is the strong competition bistable case);

(D). $(u,v)(t) \to (u^*, v^*)$ as $t \to \infty$, if $0 < h, k < 1$ (this is the weak competition co-existence case).

To investigate the invasion and spreading phenomena, there are many interesting works on the traveling wave solutions and the asymptotic spreading speed for (1.2)-(1.3); see, for example, [30, 8, 14, 28, 18, 19, 20, 21] and [1, 2, 3, 23, 24, 31] with the references cited therein.

For this, we study a Lotka-Volterra type competition model with a free boundary. We are looking for the solution $(u, v, s) \in C^{2,1}(\Omega) \times C^{2,1}(\Omega) \times C^1([0,\infty))$, $\Omega := \{(x,t) \mid 0 \le x \le s(t),\ t > 0\}$, to the problem **(FBP)**:

$$\begin{cases} u_t = u_{xx} + u(1 - u - kv),\ 0 < x < s(t),\ t > 0, \\ v_t = Dv_{xx} + rv(1 - v - hu),\ 0 < x < s(t),\ t > 0, \\ u_x(0,t) = v_x(0,t) = 0,\ u(s(t),t) = v(s(t),t) = 0,\ t > 0, \\ s'(t) = -\mu[u_x(s(t),t) + \rho v_x(s(t),t)],\ t > 0, \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x),\ 0 \le x \le s_0,\ s(0) = s_0, \end{cases}$$

where $\mu, \rho > 0$ and (u_0, v_0, s_0) satisfies

$$\begin{cases} s_0 > 0,\ u_0, v_0 \in C^2([0,s_0]),\ u_0(x), v_0(x) > 0 \text{ for } x \in [0, s_0), \\ u_0(s_0) = v_0(s_0) = u_0'(0) = v_0'(0) = 0. \end{cases}$$

Here we assume that the expanding speed of the free boundary is proportional to the normalized population gradient at the free boundary, which is the well-known Stefan type condition. We call the free boundary $x = s(t)$ the *spreading front.* In the work [15], we only focus on the *weak competition* case: $0 < h, k < 1$. For the study of other free boundary problems for some biological models, we refer to, e.g., [4, 6, 7, 9, 10, 11, 13, 16, 17, 22, 25, 26, 27, 29] and references therein.

The outline of this paper is as follows. We first describe our main results obtained in [15] in §2. Then we give some ideas of the proofs of the main theorems in §3. Finally, some discussions are given in §4.

2. Main results

We now describe our main results obtained in [15] as follows.

Theorem 1. (FBP) *admits a unique global solution* $(u, v, s) \in C^{2,1}(\Omega)\times C^{2,1}(\Omega)\times C^1([0,\infty))$, *where* $\Omega := \{(x,t) : 0 \le x \le s(t),\ t > 0\}$, *such that* $0 < s'(t) \le \mu\Lambda$ *for all* $t \ge 0$ *with* $\Lambda > 0$ *depending only on* $D, r, \rho, u_0, v_0, s_0$, *and is independent of* μ. *More precisely, we have*

$$\Lambda := 2M_1 \max\{1, \|u_0\|_{L^\infty}\} + 2\rho M_2 \max\{1, \|v_0\|_{L^\infty}\},$$

$$M_1 := \max\left\{\frac{4}{3}, \frac{-4}{3}\left(\min_{x\in[0,s_0]} u_0'(x)\right)\right\},$$

$$M_2 := \max\left\{\sqrt{\frac{r}{2D}}, \frac{4}{3}, \frac{-4}{3}\left(\min_{x\in[0,s_0]} v_0'(x)\right)\right\}.$$

Note that the quantity $\mu\Lambda$ (in which Λ is independent of μ) is a priori bound for $s'(t)$ and this bound plays a crucial role to study the spreading-vanishing dichotomy.

In the sequel it is often to use the following three quantities:

$$s_\infty := \lim_{t\to+\infty} s(t),$$

$$s_* := \min\left\{\frac{\pi}{2}, \frac{\pi}{2}\sqrt{\frac{D}{r}}\right\},$$

$$s^* := \begin{cases} \left(\frac{\pi}{2}\sqrt{\frac{D}{r}}\right)\frac{1}{\sqrt{1-h}} & \text{if } D < r; \\ \frac{\pi}{2}\frac{1}{\sqrt{1-k}} & \text{if } D > r; \\ \min\left\{\frac{\pi}{2}\frac{1}{\sqrt{1-k}}, \frac{\pi}{2}\frac{1}{\sqrt{1-h}}\right\} & \text{if } D = r. \end{cases}$$

Note that $s_* < s^*$.

We say that the two species *vanish eventually* if $s_\infty < +\infty$ and

$$\lim_{t\to+\infty} \|u(\cdot,t)\|_{C([0,s(t)])} = \lim_{t\to+\infty} \|v(\cdot,t)\|_{C([0,s(t)])} = 0;$$

we say that the two species *spread successfully* if $s_\infty = +\infty$ and the two species persist in the sense that

$$\liminf_{t\to+\infty} u(x,t) > 0 \text{ and } \liminf_{t\to+\infty} v(x,t) > 0$$

uniformly in any compact subset of $[0,+\infty)$.

In fact, we have the following simple criteria for the vanishing and spreading.

Theorem 2. *Let (u,v,s) be a solution of* **(FBP)**. *Then the followings hold.*

(i) *If $s_\infty \le s_*$, then the two species vanish eventually.*
(ii) *If $s_\infty > s^*$, then the two species spread successfully.*

Although Theorem 2 does not provide any information for spreading-vanishing when $s_* < s_\infty \le s^*$, but, if we add some restrictions on the parameters for **(FBP)**, e.g.,

$$A := \left\{0 < D < r,\ 0 < h \le 1 - \frac{D}{r},\ 0 < k < 1,\ \mu,\rho > 0\right\},$$
$$B := \left\{0 < r < D,\ 0 < k \le 1 - \frac{r}{D},\ 0 < h < 1,\ \mu,\rho > 0\right\}.$$

then we can obtain a spreading-vanishing dichotomy as follows.

Theorem 3. *Let (u,v,s) be a solution of* **(FBP)** *with $(D,h,k,r,\mu,\rho) \in A \cup B$. Then either $s_\infty \le s_*$ (and so the two species vanish eventually), or the two species spread successfully.*

Based on the previous results, we can provide some sufficient conditions for the spreading success and spreading failure via the initial data (u_0, v_0, s_0):

(i) *If $s_0 \ge s^*$, then the species u and v spread successfully.*
(ii) *Assume that $(D,h,k,r,\mu,\rho) \in A \cup B$. If $s_0 \ge s_*$, then the species u and v spread successfully.*
(iii) *If $s_0 < s_*$ and*

$$\max\{\ \|u_0\|_{L^\infty},\ \|v_0\|_{L^\infty}\ \} \le \cos\left(\frac{\pi}{2+\delta}\right)\frac{s_0^2\alpha\delta(2+\delta)}{2\pi\mu(1+\rho)},$$

then the species u and v vanish eventually, where

$$\delta := \frac{1}{2}\left[\frac{s_*}{s_0} - 1\right] > 0,$$
$$\alpha := \frac{1}{2}\min\left\{(\frac{\pi}{2})^2\frac{D}{(1+\delta)^2 s_0^2} - r,\ (\frac{\pi}{2})^2\frac{1}{(1+\delta)^2 s_0^2} - 1\right\} > 0.$$

In the case of spreading success, we have the following more precise asymptotic behavior.

Theorem 4. *Suppose that the two species spread successfully. Then*

$$(u,v)(x,t) \to \left(\frac{1-k}{1-hk}, \frac{1-h}{1-hk}\right) \quad \text{as } t \to +\infty,$$

uniformly in any compact subset of $[0, +\infty)$.

Our final result is to show that the asymptotic spreading speed (if it exists) for **(FBP)** with the weak competition is no larger than the minimal speed of traveling wavefront solutions to (1.2)-(1.3).

Theorem 5. *Let* (u, v, s) *be a solution of* **(FBP)** *with* $s_\infty = +\infty$. *Then*

$$\limsup_{t\to+\infty} \frac{s(t)}{t} \le c_{\min} = \max\{2, 2\sqrt{rD}\}.$$

Recall from [30] that for $c \ge c_{\min} := \max\{2, 2\sqrt{rD}\}$ there exist traveling wavefront solutions of (1.2)-(1.3) with $u = U(x-ct)$ and $v = V(x-ct)$, connecting $(0,0)$ with $\left(\frac{1-k}{1-hk}, \frac{1-h}{1-hk}\right)$, while no such positive wavefronts exist for $c < c_{\min}$. Thus $c_{\min}$ is called the minimal speed of traveling wavefronts.

3. Outline of Proofs

In this section, we shall provide some ideas of the proofs of the main results described in §2. First, to prove 1, we transform the free boundary problem into a fixed boundary value problem and apply the contraction mapping theorem. This method has been used in the works of Chen & Friedman [6] (see also Du & Lin [12]).

3.1. Some key lemmas for dichotomy.

Consider the problem (P_0):

$$u_t = Du_{xx} + ru(1-bu), \; x \in (0,l), \; t > 0,$$
$$u_x(0,t) = 0, \; u(l,t) = 0, \; \text{for } t > 0,$$

for given $b, r, D > 0$.

Lemma 3.1 ([5]). *Let* $l^* := \frac{\pi}{2}\sqrt{\frac{D}{r}}$. *Then we have:* (i) *all positive solutions of* (P_0) *tend to zero in* $C([0,l])$ *as* $t \to \infty$, *if* $l \le l^*$, (ii) *there exists a unique positive stationary solution* φ *of* (P_0) *such that all positive solutions of* (P_0) *approach* φ *in* $C([0,l])$ *as* $t \to \infty$, *if* $l > l^*$.

Lemma 3.2. *Let* (u, v, s) *be a solution of* **(FBP)**. *If* $s_\infty < +\infty$, *then*

$$s'(t) \to 0 \quad \text{as } t \to +\infty$$

Lemma 3.3. *Let* (u, v, s) *be a solution of* **(FBP)**. *If* $s_\infty > s^*$, *then* $s_\infty = +\infty$.

Lemma 3.4. *When* $D \ne r$, $s_\infty \notin \left(s_*, \max\left\{\frac{\pi}{2}, \frac{\pi}{2}\sqrt{\frac{D}{r}}\right\}\right]$.

From these lemmas, Theorems 2 and 3 can be proved.

3.2. **Long time behavior when $s_\infty = \infty$.** Firstly, the persistence for the two species can be established.

Lemma 3.5. *Let (u, v, s) be a solution of* **(FBP)** *with $s_\infty = +\infty$. Then*

(i) $\limsup_{t\to+\infty} u(x,t) \le 1$ *and* $\limsup_{t\to+\infty} v(x,t) \le 1$ *uniformly in* $x \in [0, +\infty)$,
(ii) $\liminf_{t\to+\infty} u(x,t) \ge 1-k$ *and* $\liminf_{t\to+\infty} v(x,t) \ge 1-h$ *uniformly in any compact subset of* $[0, +\infty)$.

Recall that $0 < h, k < 1$.

(i) Consider two sequences $\{\bar{u}_n\}_{n\in\mathbb{N}}$ and $\{\underline{v}_n\}_{n\in\mathbb{N}}$ defined as follows:

$$(\bar{u}_1, \underline{v}_1) := (1, 1-h),$$
$$(\bar{u}_{n+1}, \underline{v}_{n+1}) := (1 - k\underline{v}_n, 1 - h(1 - k\underline{v}_n)).$$

Then $\bar{u}_n > \bar{u}_{n+1} > 0$ and $\underline{v}_n < \underline{v}_{n+1} < 1$ for all $n \in \mathbb{N}$. Moreover,

$$(\bar{u}_n, \underline{v}_n) \to \left(\frac{1-k}{1-hk}, \frac{1-h}{1-hk}\right) \quad \text{as } n \to +\infty.$$

(ii) Consider two sequences $\{\underline{u}_n\}_{n\in\mathbb{N}}$ and $\{\bar{v}_n\}_{n\in\mathbb{N}}$ defined as follows:

$$(\underline{u}_1, \bar{v}_1) := (1-k, 1),$$
$$(\underline{u}_{n+1}, \bar{v}_{n+1}) := (1 - k(1 - h\underline{u}_n), 1 - h\underline{u}_n).$$

Then $\underline{u}_n < \underline{u}_{n+1} < 1$ and $\bar{v}_n > \bar{v}_{n+1} > 0$ for all $n \in \mathbb{N}$. Moreover,

$$(\underline{u}_n, \bar{v}_n) \to \left(\frac{1-k}{1-hk}, \frac{1-h}{1-hk}\right) \quad \text{as } n \to +\infty.$$

Lemma 3.6. *Let (u, v, s) be a solution of* **(FBP)** *with $s_\infty = +\infty$. Then for each $n \in \mathbb{N}$,*

$$\underline{u}_n \le \liminf_{t\to+\infty} u(x,t) \le \limsup_{t\to+\infty} u(x,t) \le \bar{u}_n,$$
$$\underline{v}_n \le \liminf_{t\to+\infty} v(x,t) \le \limsup_{t\to+\infty} v(x,t) \le \bar{v}_n,$$

uniformly in any compact subset of $[0, +\infty)$.

Then Theorem 4 can be proved by using the above lemma.

3.3. **Upper bound for the asymptotic spreading speed.** The proof of Theorem 5 is based on the following comparison principle for **(FBP)**.

Lemma 3.7. *Let (u, v, s) be a solution of* **(FBP)**. *Also assume that $(w_1, w_2, \sigma) \in C^{2,1}(\mathcal{D}) \times C^{2,1}(\mathcal{D}) \times C^1([0, \infty))$, where $\mathcal{D} := \{(x,t):\ 0 \le x \le \sigma(t),\ t > 0\}$, satisfying the following:*

$$\begin{cases} w_{1,t} \ge w_{1,xx} + w_1(1 - w_1) \text{ in } \mathcal{D}, \\ w_{2,t} \ge D w_{2,xx} + r w_2(1 - w_2) \text{ in } \mathcal{D}, \\ w_{i,x}(0,t) \le 0,\ w_i(\sigma(t), t) = 0,\ t > 0,\ i = 1, 2, \\ \sigma'(t) \ge -\mu(1+\rho) w_{i,x}(\sigma(t), t),\ t > 0,\ i = 1, 2. \end{cases}$$

If $w_1(x,0) \ge u_0(x)$, $w_2(x,0) \ge v_0(x)$ for all $x \in [0, s_0]$ and $\sigma(0) \ge s_0$, then $\sigma(t) \ge s(t)$ for all $t \ge 0$, $w_1(x,t) \ge u(x,t)$ and $w_2(x,t) \ge v(x,t)$ for all $x \in [0, s(t)]$, $t \ge 0$.

4. Discussion

In this paper, we study a Lotka-Volterra type model with weak competition and with a free boundary. The model describes that two species u and v competing with each other in a one-dimensional habitat. We assume that the species initially occupy a bounded region and have a tendency to expand their territory together. We first obtain a sufficient condition for spreading success and spreading failure via $s_\infty := \lim_{t \to +\infty} s(t)$. We then establish a spreading-vanishing dichotomy for given suitable initial data under certain parameter regime. If the size of initial habitat is too small, and initial populations are small enough, it causes no population can survive eventually, while they can coexist if the size is large enough. This phenomenon suggests that the size of the initial habitat is important to the survival for the two species (cf. [5]). Finally, we provide an upper bound for the asymptotic spreading speed. It would be very interesting if one can realize how the asymptotic spreading speed depends on the parameters in the free boundary problem. Moreover, the other choices of free boundary conditions are under investigations.

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