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On a population model with a free boundary and related elliptic problems

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1 Introduction

The spreading of invasive or new species is one of the most important topics in mathematical ecology. Since Skellam's work [12], a lot of researchers have studied the population dynamics of the species (see e.g. Shigesada-Kawasaki [11] and Cantrell-Cosner [1]). Recently Du and Lin [4] proposed a new mathematical model to understand the spreading of the species:

$$\begin{cases} u_t - du_{xx} = u(a-bu), & t>0,\ 0<x<h(t), \\ u_x(t,0)=0,\ u(t,h(t))=0, & t>0, \\ h'(t) = -\mu u_x(t,h(t)), & t>0, \\ h(0)=h_0,\ u(0,x)=u_0(x), & 0\le x\le h_0, \end{cases} \tag{1.1}$$

where $\mu,\ h_0,\ d,\ a$ and b are given positive numbers and u_0 is a nonnegative function. In (1.1), $u=u(t,x)$ represents a population density of the species in one dimensional habitat. A free boundary $x=h(t)$ is a spreading front of the species, while $x=0$ is the fixed boundary. The dynamics of the free boundary is determined by Stefan-like condition $h'(t)=-\mu u_x(t,h(t))$. This condition means that the spreading speed is proportional to the population pressure at the free boundary (the spreading front).

It is characteristic of this model that the asymptotic behaviors of solutions for (1.1) are divided into two cases:

(i) Spreading: $\lim_{t\to\infty} h(t)=\infty$ and $\lim_{t\to\infty} u(t,x)=a/b$ locally uniformly in $(0,\infty)$;

(ii) Vanishing: $\lim_{t\to\infty} h(t)\le(\pi/2)\sqrt{d/a}$ and $\lim_{t\to\infty}\|u(t,\cdot)\|_{C(0,h(t))}=0$.

Here the spreading means that the species succeed to spread to a whole region $(0,\infty)$, while the vanishing means that the species cannot survive in the region.

Such a model has been developed by many researchers. See e.g. Du-Guo [2], Du-Lou [5], Kaneko-Oeda-Yamada [8] and Kaneko-Yamada [9].

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In this article, we will set more realistic environments and consider a free boundary problem describing the population dynamics of biological species which desires a new environment in a limited area $(R_1, R_2) := \{x \in \mathbb{R}^N | \ R_1 < r < R_2\}$ $(R_1, R_2 > 0,\ r = |x|,\ N \in \mathbb{N})$. Here we allow $R_2 = \infty$. For simplicity, we assume that the distribution and the habitat of the species are radially symmetric. The problem **(FBP)** is given by (1.2) and (1.3):

$$\begin{cases} u_t - d\Delta u = uf(u), & t > 0,\ R_1 < r < h(t), \\ u(t, R_1) = 0 \ \ (\text{resp. } u_r(t, R_1) = 0), & t > 0, \\ u(t, h(t)) = 0, & t > 0, \\ h(t) \le R_2, & t > 0, \\ h(0) = h_0,\ u(0, r) = u_0(r), & R_1 \le r \le h_0 \end{cases} \tag{1.2}$$

and

$$h'(t) = \begin{cases} -\mu u_r(t, h(t)) & \text{for } \ t > 0 \text{ such that } h(t) < R_2, \\ 0 & \text{for } \ t > 0 \text{ such that } h(t) \ge R_2, \end{cases} \tag{1.3}$$

where μ, d and R_1 are positive constants, R_2 is a positive parameter and

$$\Delta u := u_{rr}(t, r) + \frac{(N-1)}{r} u_r(t, r).$$

Moreover initial data (u_0, h_0) satisfies $h_0 \in (R_1, R_2)$, $u_0 \in C^2(R_1, h_0)$, $u_0 > 0$ in (R_1, h_0) and

$$u_0(R_1) = u_0(h_0) = 0 \quad (\text{resp. } u_0'(R_1) = u_0(h_0) = 0).$$

We assume that the nonlinear function satisfies

$$\begin{aligned} & f \in C^1(\mathbb{R}),\ \ f(u) > 0 \ \text{ for } \ 0 \le u < 1,\ \ f(u) < 0 \ \text{ for } \ u > 1, \\ & \text{and } \ f'(u) < 0 \ \text{ for } \ u \ge 0. \end{aligned} \tag{1.4}$$

A typical example of this nonlinearity is a logistic term, $uf(u) = u(1-u)$.

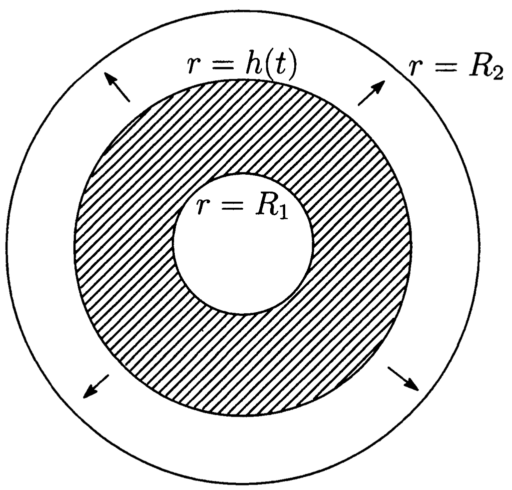

Figure 1. the habitat of species

In (FBP), we denote by $u(t,r)$ the population density of the species. The area is not initially occupied by the species, and the habitat of the species is described as $(R_1, h(t))$, where $r = h(t)$ is the free boundary representing the spreading front of the species. The condition (1.3) on $h(t)$ is same as that in (1.1) for $N = 1$ and $t > 0$ such that $h(t) < R_2$. However, when the free boundary reaches $r = R_2$ at some time $t = T^*$, it must stop at the point and we will consider a fixed boundary problem in $[R_1, R_2]$ for $t \geq T^*$. We also note that the region $[0, R_1] \cup [R_2, \infty]$ is a hostile environment, and the species cannot inhabit the region.

When $R_2 = \infty$, we can replace the Stefan-type condition of (1.3) by

$$h'(t) = -\mu u_r(t, h(t)), \quad t > 0. \tag{1.5}$$

Problem **(P)** given by (1.2) and (1.5) was studied by Kaneko [7].

It has been proved that the spreading behaviors (stationary states) of solutions for (FBP) and (P) are closely related to the following elliptic problems, respectively:

$$\text{(SP1)} \quad \begin{cases} d\Delta v + vf(v) = 0, \quad R_1 < r < R_2, \\ v(R_1) = v(R_2) = 0 \ (\text{resp. } v_r(R_1) = v(R_2) = 0) \end{cases}$$

and

$$\text{(SP2)} \quad \begin{cases} d\Delta v + vf(v) = 0, \quad R_1 < r < \infty, \\ v(R_1) = 0 \ (\text{resp. } v_r(R_1) = 0). \end{cases}$$

We will show such relations and present some results on (SP2) obtained in [7] in this paper.

The purposes of this article are as follows:

(i) Show the asymptotic behaviors of solutions for (FBP);

(ii) Make clear the differences on spreading and vanishing between (FBP) and (P);

(iii) Give some sufficient conditions for spreading and vanishing.

(iv) Show the existence and uniqueness of solutions for (SP2).

2 Main Results

2.1 Spreading and vanishing in a limited area

In this section, we discuss the existence and uniqueness of solutions for (FBP) and the asymptotic behaviors of solutions as $t \to \infty$. We first obtain the following results.

Theorem 2.1. *Let f satisfy* (1.4). *The free boundary problem* (FBP) *has a unique solution* (u, h) *satisfying*

$$\begin{aligned} 0 < u(t,r) \leq C_1 & \quad \textit{for} \;\; R_1 < r < h(t), \;\; t \geq 0, \\ 0 < h'(t) \leq \mu C_2 & \quad \textit{for} \;\; t \geq 0 \textit{ such that } h(t) < R_2, \\ h_0 < h(t) \leq R_2 & \quad \textit{for} \;\; t > 0, \end{aligned}$$

where C_1 and C_2 are positive constants depending only on $\|u_0\|_{C(R_1,h_0)}$ and $\|u_0\|_{C^1(R_1,h_0)}$, respectively. Moreover the limit of $h(t)$ as $t \to \infty$ exists and it belongs to $(h_0, R_2]$.

The proof of this theorem is almost similar to that for (P), and we omit the details here (see e.g. [2] and [7]).

We next prepare some positive number R^* to show the asymptotic behaviors of solutions. Let $\lambda_1(d, R_1, l)$ be the least eigenvalue of

$$\begin{cases} -d\Delta\phi = \lambda\phi, & R_1 < r < l, \\ \phi(R_1) = \phi(l) = 0. \end{cases}$$

Here l is a given positive number. It is well known that $\lambda_1(d, R_1, l)$ is continuous and decreasing with respect to l, and moreover it satisfies

$$\lim_{l \to R_1 + 0} \lambda_1(d, R_1, l) = +\infty \quad \text{and} \quad \lim_{l \to +\infty} \lambda_1(d, R_1, l) = 0.$$

Thus, for given d, R_1 and f, there exists a positive number $R^* = R^*(d, R_1, f(0))$ such that

$$f(0) = \lambda_1(d, R_1, R^*) \quad \text{and} \quad f(0) > \lambda_1(d, R_1, l) \;\; \text{for} \;\; l > R^*. \tag{2.1}$$

Theorem 2.2. *Let f satisfy* (1.4) *and let (u, h) be any solution of* (FBP). *Then there exists $R^* = R^*(d, R_1, f(0)) > 0$ determined by* (2.1) *with the following properties.*

(I) *Suppose $R_2 \leq R^*$. Then*

$$\textit{Vanishing}: \;\; \lim_{t\to\infty} h(t) \leq R_2 \;\; \textit{and} \;\; \lim_{t\to\infty} \|u(t,\cdot)\|_{C(R_1,h(t))} = 0$$

occurs for any initial data.

(II) *Suppose $R_2 > R^*$. Then either* (A) *or* (B) *holds true:*

(A) *Spreading* : $h(t) = R_2$ *for all* $t \geq T$ *with some* $T \in (0, \infty)$ *and* $\lim_{t\to\infty} u(t,r) = v(r)$ *uniformly in* $[R_1, R_2]$, *where* v *is a unique positive solution of* (SP1);

(B) *Vanishing* : $\lim_{t\to\infty} h(t) \leq R^*$ *and* $\lim_{t\to\infty} \|u(t,\cdot)\|_{C(R_1,h(t))} = 0$.

We need the following lemma.

Lemma 2.1. *Let* (u, h) *be any solutions of* (FBP). *If* $\lim_{t\to\infty} h(t) < R_2$, *then*

$$\lim_{t\to\infty} \|u(t,\cdot)\|_{C(R_1,h(t))} = 0.$$

We can easily prove this lemma by using [7, Theorem 2].

Proof of Theorem 2.2. Suppose that $R_2 \le R^*$. Let $(\overline{u}, \overline{h})$ be a solution of

$$\begin{cases} \overline{u}_t - d\Delta\overline{u} = \overline{u}f(\overline{u}), & t > 0,\ R_1 < r < R_2, \\ \overline{u}(t, R_1) = 0,\ \overline{u}(t, R_2) = 0, & t > 0, \\ \overline{u}(0, r) = u_0(r), & R_1 \le r \le R_2. \end{cases} \tag{2.2}$$

Then the standard comparison principle shows

$$u(t, r) \le \overline{u}(t, r) \quad \text{for} \quad t > 0,\ R_1 \le r \le h(t).$$

Since $\|\overline{u}(t,\cdot)\|_{C(R_1,R_2)}$ converges to 0 as $t \to \infty$ (cf. Henry [6]), we have

$$\lim_{t\to\infty} \|u(t,\cdot)\|_{C(R_1,h(t))} = 0.$$

We next assume $R_2 > R^*$. Since $h(t)$ is strictly increasing for $t > 0$ as long as $h(t) < R_2$, we find that $h_\infty := \lim_{t\to\infty} h(t) < R_2$ or $h(T) = R_2$ for some $T \in (0, \infty]$.

When $h_\infty < R_2$, it holds from Lemma 2.1 that

$$\lim_{t\to\infty} \|u(t,\cdot)\|_{C(R_1,h(t))} = 0. \tag{2.3}$$

To complete the proof of part (B), we must show $h_\infty \le R^*$. Otherwise there exists some $T_1 > 0$ such that $l := h(T_1) \in (R^*, R_2)$. Consider a solution $\underline{u}(t, r)$ of

$$\begin{cases} \underline{u}_t - d\Delta\underline{u} = \underline{u}f(\underline{u}), & t > 0,\ R_1 < r < l, \\ \underline{u}(t, R_1) = 0,\ \underline{u}(t, l) = 0, & t > 0, \\ \underline{u}(T_1, r) = u(T_1, r), & R_1 \le r \le l. \end{cases}$$

Then the comparison principle shows

$$u(t, r) \ge \underline{u}(t, r) \quad \text{for} \quad t \ge T_1,\ R_1 \le r \le l.$$

Since $\underline{u}(t, r)$ converges to the unique positive solution $q(r)$ of

$$\begin{cases} d\Delta q + qf(q) = 0, & R_1 < r < l, \\ q(R_1) = q(l) = 0 \end{cases}$$

as $t \to \infty$, we have

$$\liminf_{t\to\infty} u(t, r) \ge q(r) > 0 \quad \text{for} \quad R_1 < r < l.$$

This is a contradiction to (2.3), and hence $h_\infty \le R^*$ if $h_\infty < R_2$.

We next consider the case that $h(T) = R_2$ for some $T \in (0, \infty]$. To prove part (A), we will show $T < \infty$. Indeed, by the assumption, there exists some $T_2 > 0$ such that $h(T_2) > R^*$. Let $(v(t,r), s(t))$ be a solution of (P) with initial data $(u(T_2, r), h(T_2))$. Then we can easily show by a comparison principle (see [7, Lemma 3]) that

$$s(t) \le h(t+T_2) \text{ for } t \ge 0 \text{ and } v(t,r) \le u(t+T_2, r) \text{ for } t \ge 0, R_1 \le r \le s(t).$$

By [7, Theorem 5], we find that $s(T_3) = R_2$ for some $T_3 < \infty$. Hence it holds for $T := T_2 + T_3$ that

$$h(t) = R_2 \text{ for } t \ge T \text{ and } u(T,r) > 0 \text{ for } R_1 < r < R_2.$$

Thus we consider a fixed boundary problem with initial data $u(T,r)$, and obtain the uniform convergence of u to the positive solution of (SP1) as $t \to \infty$. □

By Theorem 2.2, when $R_2 \le R^*$, vanishing occurs for any initial data. We can also give sufficient conditions for spreading and vanishing when $R_2 > R^*$.

Proposition 2.1. *Suppose $R_2 > R^*$. Let (u,h) be any solution of* (FBP). *Then the following results hold true:*

(i) *Suppose $h_0 \ge R^*$. Then spreading occurs.*

(ii) *Suppose $h_0 < R^*$. There exists a positive function w in $[R_1, h_0]$ such that, if $u_0(r) \le w(r)$ in $[R_1, h_0]$, then vanishing occurs and $\|u(t,\cdot)\|_{C(R_1, h(t))} = O(e^{-\beta t})$ for some $\beta > 0$ as $t \to \infty$.*

Proof. We first prove part (i). Since $h(t)$ is strictly increasing and $h_0 \ge R^*$ by the assumption, we see $h(t) > R^*$ for all $t > 0$. By Theorem 2.2, we have

$$h(t) = R_2 \text{ for } t \ge T \text{ and } T < \infty, \ \lim_{t \to \infty} u(t,r) = v(r) \text{ uniformly in } [R_1, R_2].$$

It remains to prove part (ii). Define $(v(t,r), s(t))$ by

$$s(t) = s_0(1 + \delta(1 - e^{-\alpha t})) \text{ and } v(t,r) = \varepsilon_0 e^{-\beta t} \varphi\Big(\frac{s_0}{s(t)} r; \gamma\Big),$$

where $s_0 \in [h_0, R^*)$ and $\varphi(y;\gamma)$ is an eigenfunction corresponding to the least eigenvalue for the problem:

$$\begin{cases} -d\Delta_y \varphi = \lambda_1 \varphi, \quad \varphi > 0, \quad \gamma < y < s_0, \\ \varphi(\gamma) = \varphi(s_0) = 0 \ \ (\text{resp. } \varphi_y(\gamma) = \varphi(s_0) = 0) \end{cases}$$

with γ sufficiently close to R and $0 < \delta < \min\{R_1/\gamma - 1, R_2/s_0 - 1\}$. Choosing suitable constants α, β, δ, ε_0 and small initial data such that $u_0(r) \le w(r) :=$

$\varepsilon_0\varphi(r)$, we can regard (v,s) as an upper solution of (FBP) (see the proof of [7, Theorem 5]), and we have

$$h(t) \le s(t) \ \text{ and } \ u(t,r) \le v(t,r) \ \text{ for } \ t>0, \ R_1 \le r \le h(t).$$

Hence we conclude $\lim_{t\to\infty} h(t) \le s_0(1+\delta) < R_2$ and $\|u(t,\cdot)\|_{C(R_1,h(t))} = O(e^{-\beta t})$ as $t \to \infty$. □

We can show a threshold on initial data which separates spreading and vanishing. Let $\phi \in C^2(R_1,h_0)$ satisfy $\phi > 0$ in (R_1,h_0) and $\phi(R_1) = \phi(h_0) = 0$ (resp. $\phi_r(R_1) = \phi(h_0) = 0$). Then we have the following result.

Corollary 2.1. *Suppose $h_0 < R^* < R_2$. Consider the solution of* (FBP) *with initial data (u_0,h_0). Then, there exists a number $\sigma^* = \sigma^*(u_0,h_0) \in (0,\infty]$ such that spreading occurs if $u_0 > \sigma^*\phi$, while vanishing occurs if $u_0 \le \sigma^*\phi$.*

The proof is almost similar to that in [7, Corollary 1].

2.2 An elliptic problem in an exterior domain

In this section, we will discuss elliptic problem (SP2). We remark that (SP2) is concerned with the stationary state of solutions for (P) (which is (FBP) with $R_2 = \infty$). In other words, when spreading occurs, the solutions converge to some solution of (SP2) as $t \to \infty$. Moreover the stationary state is uniquely determined because of the unique existence of solutions for (SP2). The proofs of results in this section are shown in [7].

Our purpose of this section is to prove the following theorem.

Theorem 2.3. *Let f satisfy* (1.4). *Then there exists a unique positive solution v of* (SP2). *The solution satisfies $v_r(r) > 0$ for all $r \ge R_1$ and $\lim_{r\to\infty} v(r) = 1$ with $v_r(r) = o(1/r^{N-1})$ as $r \to \infty$.*

We need the following proposition.

Proposition 2.2. *Suppose that f satisfies* (1.4). *Let $v \in C^2(R_1,\infty)$ be any positive solution of* (SP2). *Then $v_r(r) > 0$ for all $r \ge R$ and $\lim_{r\to\infty} v(r) = 1$ with $v_r(r) = o(1/r^{N-1})$ as $r \to \infty$.*

Proof of Theorem 2.3. We first prove the existence of solutions for the problem by the standard monotone method. Let

$$w(r) = \begin{cases} \phi(r), & r \in [R_1, l], \\ 0, & r \in (l,\infty), \end{cases}$$

where l is a positive number satisfying $l > R^*$ and ϕ is a positive solution of

$$\begin{cases} -d\Delta\phi = \lambda_1\phi, & R_1 < r < l, \\ \phi(R_1) = \phi(l) = 0. \end{cases}$$

Then we find that, for any small $\delta > 0$, δw is a lower solution of (SP2) in the distribution sense. On the other hand, $v \equiv 1$ is an upper solution of (SP2). Hence, by the standard monotone method (see Sattinger [10] and Smoller [13]), there exists a solution v such that $\delta w(r) \le v(r) \le 1$ for $r \in [R, \infty)$. Moreover v satisfies (SP2) in the classical sense.

We next prove the uniqueness of solutions for (SP2). Since δ is any sufficiently small positive number, the uniqueness of solutions v for (SP2) satisfying $\delta w(r) \le v(r) \le 1$ for $r \in [R, \infty)$ enables us to get the conclusion. Suppose that w_* (resp. w^*) is a minimal (resp. maximal) positive solution of (SP2), which is generated by $\delta w(r)$ (resp. 1). Then

$$d(r^{N-1}w_{*,r}(r))_r + r^{N-1}w_*(r)f(w_*(r)) = 0, \quad R_1 < r < \infty, \quad w_*(R_1) = 0$$
$$(resp.\ d(r^{N-1}w^*_r(r))_r + r^{N-1}w^*(r)f(w^*(r)) = 0, \quad R_1 < r < \infty, \quad w^*(R_1) = 0)$$

with

$$w_*(r) \le w^*(r) \ \text{ for } \ R_1 < r < \infty.$$

Multiplying the equation by w^* (resp. w_*) and subtracting the both sides of the equations, we obtain

$$\begin{aligned} &r^{N-1}w^*(r)w_*(r)\{f(w^*(r)) - f(w_*(r))\} \\ &\qquad = d\{(r^{N-1}w_{*,r}(r))_r w^*(r) - (r^{N-1}w^*_r(r))_r w_*(r)\}. \end{aligned}$$

Moreover integrating the equation in $[R_1, \rho]$ for $\rho > R_1$ leads to

$$\begin{aligned} &\frac{1}{d}\int_{R_1}^{\rho} r^{N-1}w^*(r)w_*(r)\{f(w^*(r)) - f(w_*(r))\}\, dr \\ &\qquad = \int_{R_1}^{\rho} (r^{N-1}w_{*,r}(r))_r w^*(r) - (r^{N-1}w^*_r(r))_r w_*(r)\, dr. \end{aligned}$$

Integrating by parts the right-hand side of the above identity implies

$$\begin{aligned} &\frac{1}{d}\int_{R_1}^{\rho} r^{N-1}w^*(r)w_*(r)\{f(w^*(r)) - f(w_*(r))\}\, dr \\ &\qquad = \rho^{N-1}w_{*,r}(\rho)w^*(\rho) - \rho^{N-1}w^*_r(\rho)w_*(\rho). \end{aligned}$$

By Proposition 2.2, it holds that

$$\begin{aligned} &\lim_{\rho\to\infty} \rho^{N-1}w_{*,r}(\rho) = \lim_{\rho\to\infty} \rho^{N-1}w^*_r(\rho) = 0, \\ &\lim_{\rho\to\infty} w^*(\rho) = \lim_{\rho\to\infty} w_*(\rho) = 1. \end{aligned}$$

Taking $\rho \to \infty$, we have

$$\int_{R_1}^{\infty} r^{N-1}w^*(r)w_*(r)\{f(w^*(r)) - f(w_*(r))\}\, dr = 0.$$

It follows from $f'(u) < 0$ for $u \geq 0$ and $w^* \geq w_* > 0$ in $[R_1, \infty)$ that $w^* \equiv w_*$ in $[R_1, \infty)$, and we complete the proof. □

We will show the existence and uniqueness of positive solutions for (SP2) under the Neumann boundary condition at $r = R_1$.

Theorem 2.4. *Suppose that f satisfies* (1.4). *Then there exists a unique positive solution $v \equiv 1$ for* (SP2) *under the Neumann boundary condition at $r = R_1$.*

We can prove this theorem by the following proposition.

Proposition 2.3. *Suppose that f satisfies* (1.4). *Let $v \in C^2(R_1, \infty)$ be any positive solution of* (SP2) *under the Neumann boundary condition at $r = R_1$. Then $v \equiv 1$.*

3 Concluding Remarks

In this section, we will give some remarks.

(i) We can extend the results on spreading and vanishing to the case of general nonlinearity. When we consider a bistable term like $uf(u) = u(u-c)(1-u)$ $(0 < c < 1/2)$, we also get spreading and vanishing behaviors, but it is different from the logistic case.

(ii) If the area and the distribution of the species are not radially symmetric, then the problem becomes more complicated. The case $R_2 = \infty$ was discussed by Du-Guo [3].

(iii) For general nonlinearities, we may define spreading and vanishing of solutions for (FBP) as follows.

Definition 3.1. Let (u, h) be any solution of (FBP).

(I) *Spreading of species* is the case when

$$h(t) = R_2 \ \text{for}\, t \geq T \,\text{with some}\, T \in (0, \infty], \ \lim_{t\to\infty} u(t,x) > 0 \,\text{for}\, R_1 < r < R_2;$$

(II) *Vanishing of species* is the case when

$$\lim_{t\to\infty} h(t) \leq R_2 \quad \text{and} \quad \lim_{t\to\infty} \|u(t,\cdot)\|_{C(R_1,h(t))} = 0.$$

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