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Mathematical analysis of spin-coat model: maximal regularity theory and method of Newton polygon

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Abstract

An accurate model for the spin-coating process is presented and investigated from view point of the mathematical analysis. The method is based on the Hanzawa transform for the free boundary value problem to quasilinear evolution equations on a fixed layer domain. For solving, the maximal regularity approach is used. This is achieved by applying the Newton polygon method to the boundary symbol with Weis' Fourier multiplier theory.

1 Introduction

This note is a brief survey of the results related to [DGHSS11], mainly.

Problem We will analyze the spin-coating process, mathematically. Spin-coating has many applications, for example, in manufacturing micro-electronic devices or magnetic storage. Various models describing certain aspects have been developed in the engineering sciences as well as from physical or chemical point of views. The spin-coating process basically has the following three stages:

1. Deposition of the coating fluid onto the substrate.
2. Acceleration of the substrate up to its final.
3. Spinning of the substrate at a constant speed.

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In what follows, we zoom into the detail of stage 3, in particular, the fluid viscous forces dominate the thinning. We consider the Navier-Stokes equations (one-phase flow) in an infinite layer-like domain. Also, we shall impose the free boundary on the top, and Navier's partially-slip boundary on the bottom. The equations are as

$$
(SCP)\quad \begin{cases}
\rho(U_t + (U,\nabla)U) - \mu\Delta U + \nabla P & \\
\qquad = -\rho[2\Lambda\times U + \chi_R \Lambda\times(\Lambda\times x)] & \text{in } \bigcup_{t\in(0,T)}\{t\}\times\Omega(t), \\
\nabla\cdot U = 0 & \text{in } \bigcup_{t\in(0,T)}\{t\}\times\Omega(t), \\
-\mathbb{T}\nu = \sigma\kappa\nu, \qquad V = U\cdot\nu & \text{on } \bigcup_{t\in(0,T)}\{t\}\times\Gamma_+(t), \\
U' = c(\delta+h)^\alpha\partial_3 U', \quad U^3 = 0 & \text{on } (0,T)\times\Gamma_-, \\
U|_{t=0} = U_0 \quad (\nabla\cdot U_0 = 0) & \text{in } \Omega(0), \\
h|_{t=0} = h_0 \quad (|h_0| < \delta/2) & \text{in } \mathbb{R}^2.
\end{cases}
$$

Notations of differential are as $U_t := \partial_t U$, $\partial_t := \partial/\partial t$, $\partial_i := \partial/\partial x_i$ for $i = 1,2,3$, $\nabla := (\partial_1,\partial_2,\partial_3)$, $\nabla' := (\partial_1,\partial_2)$, $\Delta := \sum_{i=1}^3 \partial_i^2$. For vector fields a and b we denote $a\cdot b$ (or, (a,b) sometimes) by the scalar product in $\mathbb{R}^2$ or $\mathbb{R}^3$. The vector field $U = (U', U^3) = (U^1(t,x), U^2(t,x), U^3(t,x))$ denotes by the unknown velocity of the liquid at time $t \in (0,T)$ for some $T > 0$ and location $x \in \Omega(t)$; the scalar fields $P = P(t,x)$ and $h = h(t,x')$ stand for the unknown pressure and the unknown amplitude from the average δ of the region $\Omega(t)$, respectively. The region of the fluid at time t is represented by $\Omega(t) := \{x = (x', x_3); x' \in \mathbb{R}^2, x_3 \in (0, \delta + h(t,x'))\}$, which is an infinite layer-like domain in $\mathbb{R}^3$. The boundary of region is splited into two parts, the bottom is denoted by $\Gamma_-(t) := \Gamma_- := \{(x',0); x' \in \mathbb{R}^2\} := \mathbb{R}^2$, and the top surface stands for $\Gamma_+(t) := \{(x', h(t,x')); x' \in \mathbb{R}^2)\}$ at each time t. On the top surface we have used the outer normal ν and its tangential vector τ, those are defined by h explicitly (we will give them in below); the curvature on the top surface κ will also be done as well. The stress tensor is denoted by $\mathbb{T} := \mathbb{T}(U,P) := \mathbb{S}(U) + \mathbb{I}P$, where $\mathbb{S}(U) := \nabla U + (\nabla U)^T$ is the deformation tensor and $\mathbb{I}$ is the 3×3 identity matrix; M^T stands for the transposed matrix of M.

We put the following as positive constants: ρ is the density coefficient, μ is the viscosity, σ is the surface-tension, δ is the average of height; c and α are some positive constants due to Navier's partially-slip boundary condition; we usually take $\alpha \sim 5/3$ in experimentation, associated with the study of a nonintegrable singularity at the contact line in [DD74, HS71]. Also, V is the velocity vector field of the top surface, $\Lambda := (0,0,\Lambda_0)$, where $\Lambda_0 \in \mathbb{R}$ is the angular velocity of rotation, χ_R is a smooth cut-off function as $\chi_R \equiv 1$ in B_R and $\chi_R \equiv 0$ in B^c_{R+1} for some $R > 0$.

We use $\times$ as the exterior product in $\mathbb{R}^3$. So, $2\Lambda \times U = 2\Lambda_0(-U^2, U^1, 0)$ stands for the Coriolis term, $\Lambda\times(\Lambda\times x) = -\Lambda_0^2(x', 0)$ is an unbounded function as the term of centrifugal force. In order investigate the fluid dynamics around the origin in terms of L_p-theory, we use the cut-off χ_R. The initial data $U_0 = U_0(x)$ and $h_0 = h_0(x')$ (and then the initial region $\Omega(0)$) are given as some functions enjoying a certain regularity. We assume that δ is relatively large (and initial amplitude of the top surface h_0 is relatively small) so that $|h_0| < \delta/2$, that is to say, the top surface does not touch to the bottom at least in some short time-interval. We have imposed the compatibility condition: $\nabla \cdot U_0 = 0$. Up to the situation, we will add further compatibility conditions on U_0.

Known results We now list-up several known results related to (SCP). Mathematical analysis of the Navier-Stokes equations in a layer-like domain with nonhomogeneous Dirichlet boundary condition was studied by Abe-Shibata [AS03], Abels-Wiegner [AW05] and Abels [Abels05a, Abels05b]. The proof is based on the semigroup theory due to the Helmholtz decompostion by Simader-Sohr [SS92] and the resolvent estimate by Farwig-Sohr [FS94]; see also Farwig [Farwig03]. In the case of more general situation, the reader may find the recent development in [Abe04, AY10, Abels06, Abels10, Kagei08, Kagei12, Shibata13] and the references therein.

For more information about mathematical theory on the Navier-Stokes equations in fixed domains in various setting, we refer to e.g., [Amann00, FKS05, Galdi94, GHHS12]. Also, in the rotational setting in the whole space the reader may find recent works by active researcher in e.g., [BMN01, CT07, GHH06, HS10, IT13].

A model problem for the free boundary in a bounded domain (one-phase flow, meaning, the out-side is adapted) with a surface tension has been observed by Solonnikov [Solonnikov87, Solonnikov99, Solonnikov04], and Shibata-Shimuzu [SS07]. Note that the surface tension plays a role for getting the smoothing effect. Without surface tension, even the time-local solvability is rather tough problem. Hataya obtained the precise analysis on it in [Hataya09]; see also Hosono-Kawashima [HK06].

The two-phase problem is a hot issue. The case of an ocean with infinite depth and bounded above by a free surface was treated by Beale [Beale84]. After Beale's work, its improvement in several direction have been done by, for instance, Beale-Nishida [BN85], Allain [Allain85, Allain87], Tani [Tani96] and Tani-Tanaka [TT95]. The two-phase problem of the Navier-

Stokes equations was investigated by Denisova [Denisova91, Denisova94], Sylvester [Sylvester90, Sylvester96], Tanaka [Tanaka93, Tanaka95] and Prüss-Simonett [PS09, PS10]. So far, many researcher continue to study related topics; see [AR09, LST12, Maekawa07, SS11] and the reference therein.

Main theorem We now state the main results in this note, that is, the time-local existence and uniqueness of strong solutions to (SCP) in the suitable setting of initial data and a rotation speed. The definition of function spaces will be given in Section 3.

Theorem 1.1 ([DGHSS11]). *Let $p > 5$, and let ρ, μ, σ, δ, R, c, α, Λ_0 be positive constants. Then there exist $\varepsilon > 0$ and $T > 0$ such that for all $U_0 \in W_p^{2-2/p}(\Omega(0))$ and all $h_0 \in W_p^{3-2/p}(\mathbb{R}^2)$ with $\nabla \cdot U_0 = 0$ on $\Omega(0)$ satisfying*

$$\|U_0\|_{W_p^{2-2/p}(\Omega(0))} + \|h_0\|_{W_p^{3-2/p}(\mathbb{R}^2)} < \varepsilon,$$

there exists a unique solution (U, P, h) to (SCP) *within the regularity classes*

$$\begin{aligned}
U &\in H_p^1(0,T; L_p(\Omega(t))^3) \cap L_p(0,T; H_p^2(\Omega(t))^3), \\
P &\in \{P \in L_p(\widehat{H}_p^1(\Omega(t))) : P \in W_p^{1/2-1/2p}(L_p(\Gamma_+(t))) \cap L_p(W_p^{1-1/p}(\Gamma_+(t)))\}, \\
h &\in W_p^{2-1/2p}(L_p(\mathbb{R}^2)) \cap H_p^1(W_p^{2-1/p}(\mathbb{R}^2)) \cap L_p(W_p^{3-1/p}(\mathbb{R}^2)).
\end{aligned}$$

Due to our assumption $p > 5$, by Sobolev embedding we have

$$h \in C(0,T; BUC^2(\mathbb{R}^2)) \quad \text{and} \quad \partial_t h \in C(0,T; BUC^1(\mathbb{R}^2)).$$

See e.g., [Amann95]. This implies that the regularity of $\Omega(t)$ is enough, that is to say, the normal velocity V of $\Gamma_+(t)$ and its mean curvature κ are well defined and continuous. In particular, the equations in the region and on the free boundaries can be understood in the classical sense, point-wisely. For U, we also note that

$$U(t) \in BUC^1(\Omega(t)) \quad \text{and} \quad \nabla U(t) \in BUC(\Omega(t)), \quad t \in (0,T).$$

The uniqueness of solutions obtained by Theorem 1.1 comes from the dual backward equations; this technique was developed by Lions-Masmoudi [LM01]. We deal with the nonlinear terms regarded as perturbation from the solutions to the linearized equations, as in Beale-Nishida [BN85].

We next state the existence and uniqueness results of another type; the strong solution exists in an arbitrary time-interval if the initial data and the rotation speed are sufficiently small.

Theorem 1.2 ([DGHSS11]). *Let $p > 5$, $T > 0$, and let ρ, μ, σ, δ, R, c, α be positive constants. Then there exists $\varepsilon > 0$ such that for all $U_0 \in W_p^{2-2/p}(\Omega(0))$ and all $h_0 \in W_p^{3-2/p}(\mathbb{R}^2)$ with $\nabla \cdot U_0 = 0$ on $\Omega(0)$ satisfying.*

$$\|U_0\|_{W_p^{2-2/p}(\Omega(0))} + \|h_0\|_{W_p^{3-2/p}(\mathbb{R}^2)} + |\Lambda_0| < \varepsilon,$$

there exists a unique solution (U, P, h) to (SCP) in $(0, T)$ within the same regularity classes in Theorem 1.1.

2 Hanzawa transform

In this section we shall derive the linearized equations of (SCP). Hanzawa [Hanzawa81] introduced the transformation for fixing the region to solve the Stefan problem. So, the transform treated here is called 'Hanzawa transform', we use this terminology, throughout this paper.

We shall change the free boundary problem (SCP) in $\Omega(t)$ to a problem in the fixed domain $D := \mathbb{R}^2 \times (0, \delta)$. The top and bottom boundaries of D are given by $\Gamma_+ := \mathbb{R}^2 \times \{\delta\}$ and $\Gamma_- := \mathbb{R}^2 \times \{0\}$, respectively. To this end, we define

$$\Theta : (0,T) \times \mathbb{R}^2 \times (0,\delta) \to \bigcup_{t\in(0,T)} \{t\} \times \Omega(t), \qquad \Theta(t, x', y) := \left(t, x', \frac{yh(t,x')}{\delta}\right)$$

as well as $\theta(t, x', y) := (x', yh(t, x')/\delta)$. Hence, $\Theta(t, x', y) = (t, \theta(t, x', y))$ for all $t \in (0, T)$, $x' \in \mathbb{R}^2$ and $y \in (0, \delta)$. We define the transformed variables by

$$\begin{aligned} u'(t,x',y) &:= \begin{bmatrix} (\Theta^*U^1)(t,x',y) \\ (\Theta^*U^2)(t,x',y) \end{bmatrix} := U'(\Theta(t,x',y)), \\ u^3(t,x',y) &:= (\Theta^*U^3)(t,x',y) := U^3(\Theta(t,x',y)), \\ q(t,x',y) &:= (\Theta^*P)(t,x',y) := P(\Theta(t,x',y)). \end{aligned}$$

At $t = 0$, we also modify the initial velocity as

$$u_0(x', y) := (\theta^*U_0)(x', y) := U_0(\theta(0, x', y)).$$

Then, $D\Theta$ (the Jacobian of Θ) is of the form

$$D\Theta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ y\partial_t h/\delta & y\partial_1 h/\delta & y\partial_2 h/\delta & h/\delta \end{pmatrix}.$$

We can easily compute its inverse. By means of this coordinate transform, we assert for $j = 1, 2$

$$\begin{aligned}
\Theta^*\partial_t U &= \partial_t u - \frac{y}{h}(\partial_y u)\partial_t h,\\
\Theta^*\partial_j U &= \partial_j u - \frac{y}{h}(\partial_y u)\partial_j h,\\
\Theta^*\partial_j^2 U &= \partial_j^2 u - 2\frac{y}{h}(\partial_j\partial_y u)\partial_j h + \frac{y^2}{h^2}(\partial_y^2 u)(\partial_j h)^2 - y(\partial_y u)\frac{h\partial_j^2 h - 2(\partial_j h)^2}{h^2},\\
\Theta^*\partial_y U &= \frac{\delta}{h}\partial_y u,\\
\Theta^*\partial_y^2 U &= \frac{\delta^2}{h^2}\partial_y^2 u,\\
\Theta^*\Delta U &= [\Delta' + \frac{\delta^2}{h^2}\partial_y^2]u - 2\frac{y}{h}(\nabla' h, \nabla')\partial_y u + \frac{y^2}{h^2}|\nabla' h|^2\partial_y^2 u\\
&\quad - \frac{y}{h}(\partial_y u)\Delta' h + 2\frac{y}{h^2}(\partial_y u)|\nabla' h|^2,\\
\Theta^*(U\cdot\nabla)U &= ((\Theta^* u)\cdot(\nabla', \frac{\delta}{h})\partial_y)(\Theta^* u) - \frac{y}{h}(\partial_y(\Theta^* u))(u'\cdot\nabla')h,\\
\Theta^*\nabla P &= (\nabla', \frac{\delta}{h}\partial_y)q - \frac{y}{h}(\partial_y q)(\nabla', 0)h.
\end{aligned}$$

Here $\nabla = (\nabla', \partial_y)$ as well as $\Delta = \Delta' + \partial_y^2$. The fourth equation of (SCP) is transformed via the outer normal ν of $\Gamma_+(t)$ given by

$$\nu := \frac{1}{\sqrt{1+|\nabla' h|^2}}(-\partial_1 h, -\partial_2 h, 1)$$

into

$$\partial_t h = \Theta^* V_\nu \sqrt{1+|\nabla' h|^2} = \Theta^*\nu\cdot u^3\sqrt{1+|\nabla' h|^2} = -(u'\cdot\nabla')h + u^3.$$

In order to compute the transformed stress tensor on Γ_+, we note first that the outer normal ν on the free surface $\Gamma_+(t)$ and the outer normal $\nu_D = (0,0,1)$ on Γ_+ are related through

$$\nu = \frac{1}{\sqrt{1+|\nabla' h|^2}}(\mathbb{I}+K)\nu_D \quad \text{with} \quad K := \begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ -\partial_1 h & -\partial_2 h & 0 \end{pmatrix}.$$

Employing this representation, we compute the transformed stress tensor on the top boundary to be equal to

$$\Theta^*\mathbb{T}(U,P) = \nabla(\Theta^* u)D\theta^{-1} + D\theta^{-T}\nabla(\Theta^* u) - \mathbb{I}q.$$

Writing $D\theta^{-1}$ as $D\theta^{-1} = I_{\frac{\delta}{h}}(\mathbb{I}+K)$ with $I_{\frac{\delta}{h}} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \delta/h \end{pmatrix}$, we obtain

$$
\begin{aligned}
&\Theta^*\mathbb{T}(U,P)\nu \\
&= \frac{1}{\sqrt{1+|\nabla' h|^2}}\Big[\nabla(\Theta^* u)(I_{\frac{\delta}{h}}(\mathbb{I}+K)) + (I_{\frac{\delta}{h}}(\mathbb{I}+K))^T\nabla(\Theta^* u) - \mathbb{I}q\Big](\mathbb{I}+K)^T\nu_D \\
&= \frac{1}{\sqrt{1+|\nabla' h|^2}}\Big[(\nabla', \frac{\delta}{h}\partial_y)(\Theta^* u) + ((\nabla', \frac{\delta}{h}\partial_y)(\Theta^* u)) - \mathbb{I}q\Big]\nu_D \\
&\quad + \frac{1}{\sqrt{1+|\nabla' h|^2}}\big[-\nabla'(\Theta^* u)\nabla' h + \frac{\delta}{h}|\nabla' h|^2\partial_y(\Theta^* u) - ((\nabla', \frac{\delta}{h}\partial_y)u')(\nabla' h, 0) \\
&\quad - \frac{\delta}{h}\partial_y u^3(\nabla' h, 0) + \frac{\delta}{h}(\nabla' h, \partial_y u')(\nabla' h, 0) + q(\nabla' h, 0)\big].
\end{aligned}
$$

Analogously, the mean curvature κ of $\Gamma_+(t)$ is given by

$$
\kappa = -\nabla' \cdot \left(\frac{\nabla' h}{\sqrt{1+|\nabla' h|^2}}\right) = -\frac{\Delta' h}{\sqrt{1+|\nabla' h|^2}} + \sum_{j,k=1}^{2}\frac{\partial_j h \partial_k h}{(1+|\nabla' h|^2)^{\frac{3}{2}}}\partial_j\partial_k h.
$$

The transformed Navier's partially slip condition on the bottom reads as

$$
u' = \Theta^* U' = c(\delta+h)^\alpha \Theta^* \partial_y U' = c(\delta+h)^\alpha \frac{\delta}{h}\partial_y(\Theta^* u) = c\delta(\delta+h)^{\alpha-1}\partial_y(\Theta^* u).
$$

Summarizing, the transformed equations (TE) are

$$
\left\{
\begin{array}{rcll}
u_t - \Delta u + \nabla q &=& \chi_R \Lambda \times (\Lambda \times (x', y)) + F_1(u,q,h) & \text{in } (0,T)\times D, \\
\nabla \cdot u &=& F_d(u,h) & \text{in } (0,T)\times D, \\
\mathbb{T}(u,q)\nu_D - \sigma\Delta' h \nu_D &=& G_+(u,q,h) & \text{on } (0,T)\times\Gamma_+, \\
\partial_t h - u^3 &=& H(u,h) & \text{on } (0,T)\times\Gamma_+, \\
u' - c\delta^\alpha \partial_y u' &=& G_-(u,h) & \text{on } (0,T)\times\Gamma_-, \\
u^3 &=& 0 & \text{on } (0,T)\times\Gamma_-, \\
u|_{t=0} &=& u_0 & \text{in } D, \\
h|_{t=0} &=& h_0 & \text{in } \mathbb{R}^2.
\end{array}
\right.
$$

For the sake of simplicity, we take $\rho = \mu = 1$. Here, the new functions on the right hand side above are given by

$$
\begin{aligned}
F_1 :=& -(u,\nabla)u + \frac{y\partial_t h}{\delta+h}\partial_y u + \left(\frac{\delta^2}{(\delta+h)^2} - 1\right)\partial_y^2 u - \frac{2y}{\delta+h}(\nabla' h, \nabla')\partial_y u \\
&+ \frac{y^2}{(\delta+h)^2}|\nabla' h|^2\partial_y^2 u - \frac{y\Delta' h}{\delta+h}\partial_y u + \frac{2y}{(\delta+h)^2}|\nabla' h|^2\partial_y u + \frac{y\partial_y q}{\delta+h}(\nabla' h, 0)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{hu^3}{\delta+h}\partial_y u+\frac{y(u',\nabla')h}{\delta+h}\partial_y u+\frac{h}{\delta+h}(0,0,\partial_y q),\\
F_d &:= \frac{h}{\delta+h}\partial_y u^3+\frac{y}{\delta+h}(\partial_y u',\nabla')h,\\
G_+ &:= (1-\frac{\delta}{h})\partial_y(u',2u^3)+\sigma(\frac{1}{\sqrt{1+|\nabla' h|^2}}-1)\Delta' h\cdot\nu_D\\
&-\sum_{j,k=1}^{2}\frac{\sigma\partial_j h\partial_k h\partial_j\partial_k h}{(1+|\nabla' h|^2)^{\frac{3}{2}}}\nu_D-\frac{\sigma}{\sqrt{1+|\nabla' h|^2}}(\Delta' h-\sum_{j,k=1}^{2}\frac{\partial_j h\partial_k h\partial_j\partial_k h}{1+|\nabla' h|^2})(\nabla' h,0)\\
&-\big[-(\nabla' h,\nabla')u+\frac{\delta}{h}|\nabla' h|^2\partial_y u-((\frac{\delta}{h}\partial_y)u',\nabla')\nabla' h-\frac{\delta}{h}(\partial_y u^3)(\nabla' h,0)\\
&+\frac{\delta}{h}(\partial_y u',\nabla')h(\nabla' h,0)+q(\nabla' h,0)\big],\\
H &:= -(u',\nabla')h,\\
G_- &:= c\delta(h^{\alpha-1}-\delta^{\alpha-1})\partial_y u.
\end{aligned}
$$

Obviously, the toughest part in our analysis is to deal with the fourth equation (where H appears in the right hand side), because that is a quasilinear equation of hyperbolic type. In what follows, we rather discuss the transformed equations (TE) than (SCP).

3 Maximal regularity estimates

We first give the definition of function spaces. Let $s \in \mathbb{R}$, $1 < p < \infty$, $J \subset \mathbb{R}$ be an open interval, Ω a domain in $\mathbb{R}^n$ for $n \in \mathbb{N}$ with regular boundary $\Gamma := \partial\Omega$, and let X be a Banach space. As usual, $L_p(\Omega)$ is the Lebesgue space with integral exponent p in Ω. We denote by $H_p^s(\Omega)$ the Bessel potential space in Ω of differential order s and integral exponent p as well as $H_p^s(J;X)$ is the space of X-valued Bessel potential functions in J. The Slobodeckij space $W_p^s(J;X)$ is defined as $W_p^s(J;X) := B_{p,p}^s(J;X)$, where $B_{p,p}^s$ denotes the corresponding Besov space. Moreover, for $T \in (0,\infty)$, we sometimes omit the notation of the time-interval $J = (0,T)$ as $L_p(H_p^s(\Omega)) := L_p(0,T;H_p^s(\Omega))$, if no confusion occurs likely.

Let ${}_0W_p^s(0,T;X)$ be the zero-trace subspace of $W_p^s(X)$ at $t = 0$ defined for $s \geq 0$ with $s - 1/p \notin \mathbb{N}_0 := \mathbb{N} \cup \{0\}$ as

$$
{}_0W_p^s(0,T;X) := \begin{cases} \{u \in W_p^s(0,T;X) : u(0) = \ldots = u^{(k)}(0) = 0\}, \\ \qquad\qquad \text{if} \quad s \in (k+\frac{1}{p}, k+1+\frac{1}{p}) \quad \text{for} \quad k \in \mathbb{N}_0, \\ W_p^s(0,T;X), \quad \text{if} \quad 0 \leq s < \frac{1}{p}. \end{cases}
$$

The spaces ${}_0H^s_p(0,T;X)$ are defined, analogously. The homogeneous version of the above spaces will be denoted by $\widehat{H}^s_p(0,T;X)$ and $\widehat{W}^s_p(0,T;X)$. We furthermore set ${}^0\widehat{H}^1_p(0,T;\Gamma) := \{\varphi \in \widehat{H}^1_p(\Omega) : \gamma\varphi = 0 \text{ on } \Gamma\}$ and

$$ {}_0\widehat{H}^{-1}_p(0,T;\Gamma) := \left({}^0\widehat{H}^1_{p'}(0,T;\Gamma)\right)'. $$

Here, γ denotes the trace operator $u \mapsto u|_\Gamma$, $p' = p/(p-1)$ is the Hölder conjugate exponent of p, and X' means the topological dual space of X.

It is the aim of this section to prove maximal regularity estimates for the linearized problem of (TE).

To this end, we introduce the function space $\mathbb{F}$ associated with the right hand side of (TE) as

$$ \mathbb{F} := \mathbb{F}_1 \times \mathbb{F}_d \times \mathbb{G}_+ \times \mathbb{H} \times \mathbb{G}_- \times \mathbb{I}_u \times \mathbb{I}_h $$

with

$$ \begin{aligned} \mathbb{F}_1 &:= \mathbb{F}_1(0,T;D) := L_p(0,T;L_p(D)^3), \\ \mathbb{F}_d &:= \mathbb{F}_d(0,T;D) := H^1_p({}_0\widehat{H}^{-1}_p(D)) \cap H^{1/2}_p(L_p(D)) \cap L_p(H^1_p(D)), \\ \mathbb{G}_+ &:= \mathbb{G}_+(0,T;\Gamma^+) := W^{1/2-1/2p}_p(L_p(\Gamma^+)^3) \cap L_p(W^{1-1/p}_p(\Gamma^+)^3), \\ \mathbb{H} &:= \mathbb{H}(0,T;\mathbb{R}^2) := W^{1-1/2p}_p(L_p(\mathbb{R}^2)) \cap L_p(W^{2-1/p}_p(\mathbb{R}^2)), \\ \mathbb{G}_- &:= \mathbb{G}_-(0,T;\Gamma_-) := W^{1/2-1/2p}_p(L_p(\Gamma_-)^2) \cap L_p(W^{1-1/p}_p(\Gamma_-)^2), \\ \mathbb{I}_u &:= \mathbb{I}_u(D) := W^{2-2/p}_p(D)^3, \\ \mathbb{I}_h &:= \mathbb{I}_h(\mathbb{R}^2) := W^{3-2/p}_p(\mathbb{R}^2). \end{aligned} $$

We also put the similar solution classes:

$$ \mathbb{E} := \mathbb{E}_u \times \mathbb{E}_q \times \mathbb{E}_h $$

with

$$ \begin{aligned} \mathbb{E}_u &:= H^1_p(0,T;L_p(D)^3) \cap L_p(0,T;H^2_p(D)^3), \\ \mathbb{E}_q &:= \{q \in L_p(0,T;\widehat{H}^1_p(D)) : q \in W^{1/2-1/2p}_p(L_p(\Gamma^+)) \cap L_p(W^{1-1/p}_p(\Gamma^+))\}, \\ \mathbb{E}_h &:= W^{2-1/2p}_p(L_p(\mathbb{R}^2)) \cap H^1_p(W^{2-1/p}_p(\mathbb{R}^2)) \cap L_p(W^{3-1/p}_p(\mathbb{R}^2)). \end{aligned} $$

We are now position to state the maximal regularity estimates for the solutions to the linearized (TE) around the trivial solution when $\Lambda_0 = 0$.

Proposition 3.1. *Let $\Lambda_0 = 0$, $T > 0$, $p \in (1,\infty)$ with $p \neq 3/2, 3$. Then there exists a unique solution $(u,q,h) \in \mathbb{E}$ to* (TE) *if and only if the terms in* *the right hand side and initial data* $(F_1, F_d, G_+, H, G_-, u_0, h_0) \in \mathbb{F}$ *and satisfy the following compatibility conditions*

$$G_+(0) = \partial_y u_0' + \nabla' u_0^3 \text{ on } \Gamma_+, \quad G_-(0) = u_0' - c\delta^\alpha \partial_y u_0' \text{ on } \Gamma_- \quad \text{if } p > 3,$$
$$u_0^3 = 0 \text{ on } \Gamma_-, \quad F_d(0) = \nabla \cdot u_0 \quad \text{if } p > 3/2.$$

We can derive the maximal regularity estimates in above classes as follows:

Lemma 3.2. *Let* $p \in (1, \infty)$ *with* $p \neq 3/2, 3$, $\Lambda = F_1 = F_d = G_+ = G_- = u_0 = h_0 = 0$, *and let* $H \in \mathbb{H}$. *Then there exist* $T > 0$, $C > 0$ *and a solution* $(u, q, h) \in \mathbb{E}$ *of* (TE) *in* $(0, T)$ *satisfying*

$$\|(u, q, h)\|_{\mathbb{E}} \leq C\|H\|_{\mathbb{H}}.$$

Note that one can apply the lemma of superposition for the solutions to the linearized equations of (TE). For making this note short, we skip its detail of other cases.

In order to derive the maximal regularity estimates we consider the corresponding resolvent equations. By Laplace transform (roughly speaking, $\mathcal{L}^{-1} : \partial_t \mapsto \lambda \in \mathbb{C}$) and Fourier transform (roughly speaking, $\mathcal{F}^{-1} : x' \mapsto \xi' \in \mathbb{R}^2$), we deduce the Stokes resolvent problem

$$\text{(RP)} \qquad \begin{cases} \lambda \hat{u} + |\xi'|^2 \hat{u} - \partial_y^2 \hat{u} + (i\xi', \partial_y)\hat{q} = \hat{F}_1 & \text{in } D, \\ i\xi' \cdot \hat{u}' + \partial_y \hat{u}^3 = \hat{F}_d & \text{in } D, \\ \lambda \hat{h} + \hat{u}^3 = \hat{H} & \text{on } \Gamma_+, \\ \partial_y \hat{u}' + i\xi' \hat{u}^3 = \hat{G}_+' & \text{on } \Gamma_+, \\ 2\partial_y \hat{u}^3 - \hat{q} + \sigma|\xi'|^2 \hat{h} = \hat{G}_+^3 & \text{on } \Gamma_+, \\ \hat{u}' - c\delta^\alpha \partial_y \hat{u}' = \hat{G}_- & \text{on } \Gamma_-, \\ \hat{u}^3 = 0 & \text{on } \Gamma_-. \end{cases}$$

Here, we take $\Lambda = 0$ and $i := \sqrt{-1}$.

To solve (RP), we put 'Ansatz' for $(\hat{u}, \hat{q}, \hat{h})$ for $\lambda \in \mathbb{C}$ as follows:

$$\begin{aligned} \hat{h} &= \hat{h}(\xi'), \\ \hat{q}(\xi', y) &= \hat{\psi}_-(\xi') e^{-|\xi'|y} + \hat{\psi}_+(\xi') e^{|\xi'|y}, \\ \hat{u}'(\xi', y) &= \hat{\phi}_-'(\xi') e^{-\omega y} - \int_0^\delta k_-(\xi', y, s) i\xi' \hat{q}(\xi', s) ds \\ &\quad + \hat{\phi}_+'(\xi') e^{\omega y} - \int_0^\delta \ell_-(\xi', y, s) i\xi' \hat{q}(\xi', s) ds, \\ \hat{u}^3(\xi', y) &= \hat{\phi}_-^3(\xi') e^{-\omega y} - \int_0^\delta k_+(\xi', y, s) \partial_y \hat{q}(\xi', s) ds \end{aligned}$$

$$+ \hat{\phi}_+^3(\xi')e^{\omega y} - \int_0^\delta \ell_+(\xi', y, s)\partial_y \hat{q}(\xi', s)ds.$$

Here, for $\lambda \in \mathbb{C}$ and $\xi' \in \mathbb{R}^2$ we denoted $\omega := \sqrt{\lambda + |\xi'|^2}$, and for $y, s \in (0, \delta)$

$$k_\pm(\xi', y, s) := \frac{1}{2\omega}\big(e^{-\omega|y-s|} \pm e^{-\omega(y+s)}\big), \qquad \ell_\pm(\xi', y, s) := \frac{1}{2\omega}\big(e^{\omega|y-s|} \pm e^{\omega(y+s)}\big).$$

Our task is to find 9 functions $(\hat{h}, \hat{\psi}_-, \hat{\psi}_+, \hat{\phi}_-^1, \hat{\phi}_+^1, \hat{\phi}_-^2, \hat{\phi}_+^2, \hat{\phi}_-^3, \hat{\phi}_+^3)$ by equations and boundary conditions.

In what follows, we take $\delta = c = \alpha = 1$ for simplicity. Computing the inverse of 9×9 matrix, we have e.g.,

$$\hat{h} = \frac{\omega^3 + \lambda|\xi'| + 3\omega|\xi'|^2}{\lambda(\omega^3 + \lambda|\xi'| + 3\omega|\xi'|^2) + \sigma|\xi'|^3(\omega + |\xi'|)}\hat{H} =: \frac{m_1}{m_2}\hat{H}.$$

Note that $m_2(\lambda, |\xi'|) = \lambda(\omega^3 + \lambda|\xi'| + 3\omega|\xi'|^2) + \sigma|\xi'|^3(\omega + |\xi'|)$ contains the parabolic scale ($\lambda \sim |\xi'|^2$) and the hyperbolic scale ($\lambda \sim |\xi'|$) at once. So, it is difficult to apply the standard methods, at least directly, for guarantee non-zero of the denominators and for deriving the resolvent estimate.

4 Method of Newton polygon

The method of Newton polygon was developed by Euler, and is the way to seek the direction for the biggest gradient of a complex-value function. More precisely, taking $\lambda \sim |\xi'|^k$ for $k > 0$, we observe which is the dominant terms. We refer to the description of Newton polygon approach in [GV92] and [DMV98].

In here, we give a brief idea of Newton polygon. We settle

$$z := |\xi'| \in \mathbb{R}_+, \quad \omega := \sqrt{\lambda + z^2}, \quad \lambda \sim z^k \text{ for } k > 0.$$

So, 5 terms of $m_2(\lambda, z) = \lambda(\omega^3 + \lambda z + 3\omega z^2) + \sigma z^3(\omega + z)$ are of ordering as

- $0 < k < 1 \Rightarrow m_2 \sim z^{k+3} + z^{2k+1} + z^{k+3} + z^4 + z^4,$
- $k = 1 \Rightarrow m_2 \sim z^4 + z^3 + z^4 + z^4 + z^4,$
- $1 < k < 2 \Rightarrow m_2 \sim z^{k+3} + z^{2k+1} + z^{k+3} + z^4 + z^4,$
- $k = 2 \Rightarrow m_2 \sim z^5 + z^5 + z^5 + z^4 + z^4,$
- $k > 2 \Rightarrow m_2 \sim z^{5k/2} + z^{2k+1} + z^{3k/2+2} + z^{k/2+3} + z^4.$

Note that the highest order terms dominate. In fact, putting (r, s) the exponents of the terms $\lambda^r z^s$, every point (r, s) in above is in the convex hull

$N(P)$ of 4 points $P := \{(0,0),(5/2,0),(1,3),(0,4)\}$ in clockwise direction; this convex hull $N(P)$ is so-called Newton polygon. In addition, all coefficient of five terms is a positive real number. So, we can see that there exist $\lambda_0 > 0$ and $\varphi \in (0,\pi)$ such that $m_2(\lambda, |\xi'|) \neq 0$ for all $\lambda \in \lambda_0 + \Sigma_\varphi$ and $\xi' \in \mathbb{R}^2$. Here, $\Sigma_\varphi := \{\lambda \in \mathbb{C}; |\arg \lambda| < \varphi\}$ is the sector region in the complex plane.

Moreover, the method of Newton polygon leads us to derive the maximal regularity estimates due to the Fourier multiplier theory by Kalton-Weis [KW01]. Weis' Fourier multiplier theory is a strong tool for mathematical analysis on fluid dynamics. Idneed, there are a lot of papers as application of it, for example, [DHP03, DHP07, DSS08, PS12, SS11].

Let $M_2 := \mathcal{L}^{-1}\mathcal{F}^{-1}m_2\mathcal{F}$ be Fourier multiplier of symbol m_2.

Lemma 4.1. *If $1 < p < \infty$, $r, s \geq 0$, then there exists $\rho_0 \geq 0$ such that*

$$M_2 \in \mathit{Isom}(\mathcal{K}^{r+5/2}_{p,\rho}(\bar{\mathcal{K}}^s_p) \cap \mathcal{K}^{r+1}_{p,\rho}(\bar{\mathcal{K}}^{s+3}_p) \cap \mathcal{K}^r_{p,\rho}(\bar{\mathcal{K}}^{s+4}_p), \mathcal{K}^r_{p,\rho}(\bar{\mathcal{K}}^s_p))$$

for all $\rho \geq \rho_0$, where $\mathcal{K}^r_{p,\rho}(\bar{\mathcal{K}}^s_p) := {}_0\mathcal{K}^r_{p,\rho}(\mathbb{R}_+; \bar{\mathcal{K}}^s_p(\mathbb{R}^2))$; $\mathcal{K}, \bar{\mathcal{K}} \in \{H, B_{p,}\}$; ${}_0H^r_{p,\rho}(\mathbb{R}_+; \bar{\mathcal{K}}) := \Big\{u; e^{-\rho t}\partial_t^k u \in L_p(\mathbb{R}_+; \bar{\mathcal{K}}), {}^\forall k \in \mathbb{N}_0,\ 0 \leq k \leq r,\ u(0) = 0\Big\}$.

The proof of this lemma is shown by [DGHSS11], we omit the detail. We would emphasize that (r, s) the differential orders in the image of isomorphism is added to the the corner of $N(P)$ in the domain. By this lemma, we therefore prove that for $H \in B^{1-1/2p}_{p,p}(L_p(\mathbb{R}^2)) \cap L_p(B^{2-1/p}_{p,p}(\mathbb{R}^2))$,

$$\mathcal{L}^{-1}\mathcal{F}^{-1}\frac{m_1}{m_2}\hat{H} = h \in B^{2-1/2p}_{p,p}(L_p(\mathbb{R}^2)) \cap H^1_p(B^{2-1/p}_{p,p}(\mathbb{R}^2)) \cap L_p(B^{3-1/p}_{p,p}(\mathbb{R}^2))$$

in $(0,T)$ for $T < \infty$. This calculation completes the proof of Lemma 3.2.

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