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On Primal-Dual Interior-Point Methods for $P_*(\kappa)$ Linear Complementarity Problem Based on a New Proximity Function[1]

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1 Introduction

Primal and dual interior-point methods (IPMs) have been well known as the most effective methods for solving wide classes of optimization problems, for example, linear optimization (LO) problem, quadratic optimization problem (QOP), semidefinite optimization (SDO) problem, second-order cone optimization (SOCO) problem and convex optimization problem (CP).

The choice of parameter θ, so-called barrier update parameter, plays an important role in the both of theory and practice of IPMs. Usually, if θ is a constant which is independent of the dimension of the problem, then the algorithm is called a *large-update* method. If it depends on the dimension, then the algorithm is said to be a *small-update* method. Large-update methods are much more efficient than small-update methods in practice ([1]). The gap between theory and practice has been referred to as the irony of IPM methods ([17]). Recently, many authors have tried to reduce the gap of the worst-case iteration bound between large-update IPM and small-update IPM. Using self-regular proximity functions instead of classical logarithmic barrier functions, Peng et al. ([12, 13, 14, 15, 16]) improved the complexity of large-update IPMs for LO, SDO and SOCO.

Recently, Bai et al. ([2]) introduced a new class of kernel functions. The class was defined by some simple conditions on the kernel function and its derivatives, and presented a simple and unified computational scheme for the complexity analysis of kernel functions in the new class. The best iteration bound, which was given by Bai et al. ([2]), is $\mathcal{O}(\sqrt{n}\log n\log\frac{n}{\epsilon})$. Very recently, following the approach of Bai et al. ([2]) for LO, Cho et al. ([4]), Cho and Kim ([5]) and Cho ([6]) used new kernel functions to calculate the iteration bound for a $P_*(\kappa)$ linear complementarity problem (LCP).

The aim of this paper is to review main results of [8]. So, results of this paper are given from ones in [8] without proof. In this paper, we define a new class of proximity functions for the $P_*(\kappa)$ LCP by a new class of kernel functions which are modified by [7]. Using the new class of proximity functions for $P_*(\kappa)$ LCP, we formulate an algorithm for our large-update primal-dual IPM for the $P_*(\kappa)$ LCP and we give results for its complexity analysis, and then we present that the iteration bound in worst-case for our IPM is

$$\mathcal{O}((1+2\kappa)\sqrt{n}\log n\log\frac{n}{\epsilon}),$$

which is known as the best one.

We consider the following $P_*(\kappa)$ linear complementarity problem (LCP):

$$\begin{array}{lll} \text{(LCP)} & \text{Find} & (x,s)\in\mathbb{R}^n\times\mathbb{R}^n \\ & \text{such that} & x\geq 0,\ s=Mx+q\geq 0,\ xs=0, \end{array}$$

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where $M \in \mathbb{R}^{n\times n}$ is a $P_*(\kappa)$ matrix, $q \in \mathbb{R}^n$ and xs denote the componentwise product of the vectors x and s.

Now we recall the definition of $P_*(\kappa)$ matrix, which was given in [10].

Definition 1.1 *Let κ be a nonnegative number. A matrix $M \in \mathbb{R}^{n\times n}$ is called a $P_*(\kappa)$ matrix if*

$$(1+4\kappa)\sum_{i\in I_+(x)} x_i(Mx)_i + \sum_{i\in I_-(x)} x_i(Mx)_i \geq 0,$$

for all $x \in \mathbb{R}^n$, where $I_+(x) = \{i \in I \mid x_i(Mx)_i \geq 0\}$, $I_-(x) = \{i \in I \mid x_i(Mx)_i < 0\}$ and $I = \{1, \cdots, n\}$.

When we relax the complementarity condition to find an approximate solution for LCP, we get the following parameterized system:

$$\begin{cases} s = Mx + q, \\ xs = \mu e, \\ x > 0,\ s > 0, \end{cases} \tag{1}$$

where $\mu > 0$, $e = (1, \cdots, 1)^T \in \mathbb{R}^n$. In the sequel, we assume that LCP satisfies the interior-point condition (IPC), that is, there exists (x, s) such that $s = Mx + q$, $x > 0$ and $s > 0$. Since M is a $P_*(\kappa)$ matrix and LCP satisfies IPC, (1) has a unique solution for any $\mu > 0$ ([10]). The solution of (1) is denoted by $(x(\mu), s(\mu))$ for given $\mu > 0$ and it is called as μ-center for a given μ. Also, the solution set $\{(x(\mu), s(\mu)) \mid \mu > 0\}$ is called as the *central path* of LCP. As $\mu \to 0$, the sequence $(x(\mu), s(\mu))$ approach the solution (x, s) of LCP ([3, 10, 11, 19]).

In general, IPMs consist of two strategies: the first one, which is called the inner iteration scheme, is to keep the iterative sequence in a certain neighborhood of the central path or to keep the iterative sequence in a certain neighborhood of the μ-center; the second one, which is called the outer iteration scheme, is to decrease the parameter μ to $\mu_+ := (1-\theta)\mu$, for some $\theta \in (0, 1)$.

2 Proximity Functions and Search Directions

Without loss of generality we assume that $(x(\mu), s(\mu))$ is known for some positive μ. Then we decrease μ to $\mu := (1-\theta)\mu$ for some fixed $\theta \in (0, 1)$. Let $F(x, s) = \begin{pmatrix} s - Mx - q \\ XSe - \mu e \end{pmatrix}$. Then $\nabla F(x, s) = \begin{pmatrix} -M & I \\ S & X \end{pmatrix}$ is a nonsingular matrix [10] for any positive diagonal matrices $X, S \in \mathbb{R}^{n\times n}$, where I is the n-dimensional identity matrix. Then we get the following Newton system having a unique solution $(\Delta x, \Delta s)$:

$$\begin{cases} -M\Delta x + \Delta s = 0, \\ s\Delta x + x\Delta s = \mu e - xs. \end{cases} \tag{2}$$

We define the following notation:

$$v := \sqrt{\frac{xs}{\mu}}.$$

Then the last equation in (2) is the following:

$$s\Delta x + x\Delta s = \mu e - \mu v^2 = \mu v(v^{-1} - v).$$

We consider a strictly convex function $\Psi(v)$ which has minimal at $v = e$ and $\Psi(e) = 0$, where e is the n-dimensional vector of ones. Then we replace $v^{-1} - v$ by $-\nabla\Psi(v)$. So, we have the following modified Newton system:

$$\begin{cases} -M\Delta x + \Delta s = 0, \\ s\Delta x + x\Delta s = -\mu v \nabla\Psi(v). \end{cases} \tag{3}$$

From the following notations:

$$dx := \frac{v\Delta x}{x}, \quad ds := \frac{v\Delta s}{s}$$

the second equation in (3) can be rewritten:

$$dx + ds = -\nabla\Psi(v). \tag{4}$$

For our IPM, we use the following a new class of kernel functions associated with $P_*(\kappa)$ LCP:

$$\psi(t) = \frac{t^2 - 1}{2} + \frac{e^{p(t^{-q}-1)} - 1}{pq}, \quad p \geqq 1 \text{ and } q \geqq 1 \quad \text{for } t > 0. \tag{5}$$

Then, we have

$$\begin{aligned}
\psi'(t) &= t - t^{-q-1} \cdot e^{p(t^{-q}-1)}, \\
\psi''(t) &= 1 + \left((q+1)t^{-q-2} + pqt^{-2q-2}\right) e^{p(t^{-q}-1)} > 1, \\
\psi'''(t) &= -\left((q+1)(q+2)t^{-q-3} + 3pq(p+2q+3)t^{-2q-3} + p^2q^2t^{-3q-3}\right) e^{p(t^{-q}-1)} < 0.
\end{aligned}$$

Furthermore, our new kernel function (5) satisfies

$$\lim_{t\to 0^+} \psi(t) = \lim_{t\to\infty} \psi(t) = \infty.$$

Note that $\psi(1) = \psi'(1) = 0$. Then $\psi(t)$ is determined

$$\psi(t) = \int_1^t \int_1^\xi \psi''(\zeta) d\zeta d\xi.$$

The proximity function(measure) for $P_*(\kappa)$ LCP is

$$\Phi(x, s; \mu) := \Psi(v) := \sum_{i=1}^{n} \psi(v_i). \tag{6}$$

For our large-update IPM for $P_*(\kappa)$ LCP, we discuss the above proximity functions instead of the right hand side of the equation (4). We define the norm-based proximity measure σ as follows:

$$\sigma := ||-\nabla\Psi(v)|| = ||dx + ds||.$$

Then the following proposition is easily checked from our new class of proximity functions:

Proposition 2.1 [8] *For any* $v_1,\ v_2 \in \mathbb{R}^n_{++}$,

$$\Psi\left(\sqrt{v_1 v_2}\right) \leqq \frac{1}{2}\left(\Psi(v_1) + \Psi(v_2)\right).$$

3 Algorithm and its Complexity Analysis

Now we explain our algorithm for the large-update primal-dual IPM for $P_*(\kappa)$ LCP. Assuming that a starting point in a certain neighborhood of the central path is available, we can set out from this point. Actually, by using the so-called self-dual embedding model, one can further get the point exactly on the central path corresponding to $\mu = 1$ as an initial point ([9, 14, ?]). Then, we will go to the outer "while loop". If μ satisfies $n\mu \geqq \epsilon$, then it is reduced by the factor $1 - \theta$, where $\theta \in (0, 1)$. Then, we make use of inner "while loop", and we repeat the procedure until we find iterates that are "close" to $(x(\mu), s(\mu))$, that is, the proximity $\Phi(x, s; \mu) < \tau$. Here, we apply Newton's method targeting at the new μ-centers to decide a search direction $(\Delta x, \Delta s)$. We return to the outer "while loop". The whole process is repeated until μ is small enough, say until $n\mu < \epsilon$.

The choice of the step size α is another crucial issue in the analysis of the algorithm. It has to be taken such that the closeness of the iterates to the current μ-center improves by a sufficient amount. In algorithm, the inner "while loop" is called the *inner iteration* and the outer "while loop" is called the *outer iteration.* Each outer iteration consists of an update of parameter μ and a sequence of (one or more) inner iteration. The total number of inner iterations is the worst-case iteration bound for our algorithm.

The algorithm for our large-update primal-dual IPM for $P_*(\kappa)$ LCP is given as follows:

Primal-Dual Algorithm for LCP
Inputs
A proximity parameter $\tau > 0$;
an accuracy parameter $\epsilon > 0$;
a variable damping factor α;
a fixed barrier update parameter $\theta \in (0,1)$;
(x^0, s^0) and $\mu^0 = 1$ such that $\Phi(x^0, s^0; \mu^0) \leqq \tau$.
begin
$x := x^0$; $s := s^0$; $\mu := \mu^0$;
while $n\mu \geqq \epsilon$ **do**
begin
$\mu := (1-\theta)\mu$;
while $\Phi(x, s; \mu) \geqq \tau$ **do**
begin
Solve the system (3) for Δx, Δs;
Determine a step size α;
$x := x + \alpha\Delta x$;
$s := s + \alpha\Delta s$;
end
end
end

3.1 Determining a default step size.

In this section, we compute the feasible step size α such that the proximity function is decreasing and the bound for the decrease during inner iterations and then given our default step size $\bar{\alpha}$; $\bar{\alpha} = (1+2\kappa)^{-1} \cdot \left(1 + 3\sigma(1+pq+q)(1+(1/p)\log 3\sigma)^{(q+1)/q}\right)^{-1}$. Since $P_*(\kappa)$ LCPs are generalization of LO problems, we lose the orthogonality of vectors dx and ds. So the analysis is different from LO case. After a damped step for fixed μ, we have new iterations

$$x_+ = x + \alpha\Delta x, \quad s_+ = s + \alpha\Delta s.$$

From the definitions of v, dx, and ds, we have

$$x_+ = \frac{x}{v}(v + \alpha dx), \quad s_+ = \frac{s}{v}(v + \alpha ds).$$

Then we get

$$v_+ = \sqrt{\frac{x_+ s_+}{\mu}} = \sqrt{(v+\alpha dx)(v+\alpha ds)}.$$

Throughout the paper, we assume that the step size α satisfies that the coordinates of the vectors $v + \alpha dx$ and $v + \alpha ds$ are positive. Hence by Proposition 2.1,

$$\Psi(v_+) = \Psi(\sqrt{(v+\alpha dx)(v+\alpha ds)}\,) \leq \frac{1}{2}(\Psi(v+\alpha dx) + (\Psi(v+\alpha ds)).$$

For given $\mu > 0$, by letting $f(\alpha)$ be the difference between the proximity measures before and after one step by a function of the step size, i.e.,

$$f(\alpha) = \Psi(v_+) - \Psi(v),$$

we have

$$f(\alpha) \leq f_1(\alpha),$$

where $f_1(\alpha) = \frac{1}{2}(\Psi(v + \alpha dx) + \Psi(v + \alpha ds)) - \Psi(v)$.
Here,

$$f(0) = f_1(0) = 0.$$

For notational convenience, we denote by dx_i and ds_i are i-th components of vectors dx and ds, respectively. By taking the derivative of $f_1(\alpha)$ with respect to α, we have

$$f_1'(\alpha) = \frac{1}{2}\sum_{i=1}^{n}(\psi'(v_i + \alpha dx_i)dx_i + \psi'(v_i + \alpha ds_i)ds_i).$$

From (4) and the definition of σ,

$$f_1'(0) = -\frac{\sigma^2}{2}.$$

By differentiating $f_1'(\alpha)$ with respect to α, we obtain

$$f_1''(\alpha) = \frac{1}{2}\sum_{i=1}^{n}(\psi''(v_i + \alpha dx_i)dx_i^2 + \psi''(v_i + \alpha ds_i)ds_i^2).$$

Here, since M is a $P_*(x)$ matrix, $M\Delta x = \Delta s$ from (2), for $\Delta x \in R^n$, $dxds = \frac{1}{\mu}\Delta x \Delta s$ and $\mu > 0$,

$$(1 + 4\kappa)\sum_{i \in I_+} dx_i ds_i + \sum_{i \in I_-} dx_i ds_i \geqq 0, \tag{7}$$

where $I_+ = \{i \in I : dx_i ds_i \geqq 0\}$, $I_- = \{i \in I : dx_i ds_i < 0\}$ and $I_- = I - I_+$.
By the definition of σ,

$$\sigma^2 = \sum_{i \in I}(dx_i + ds_i)^2 \geqq \sum_{i \in I_+}(dx_i + ds_i)^2 \geqq 4\sum_{i \in I_+} dx_i ds_i.$$

By combining the above inequality and (7), we obtain

$$(1 + 4\kappa)\sigma^2 \geqq 4(1 + 4\kappa)\sum_{i \in I_+} dx_i ds_i \geqq -4\sum_{i \in I_-} dx_i ds_i. \tag{8}$$

The following result follows from the above inequalities (7) and (8), follows from the method [5].

Lemma 3.1 [5, 10] *Suppose that* (7) *holds. Then*

(i) $\sum_{i \in I}(dx_i^2 + ds_i^2) \leqq (1 + 2\kappa)\sigma^2$,

(ii) $\|dx\| \leqq \sigma\sqrt{1 + 2\kappa}$ *and* $\|ds\| \leqq \sigma\sqrt{1 + 2\kappa}$.

For $v = (v_1, \cdots, v_n)^T \in \mathbb{R}^n$, we define $v_{min} = min\{v_1, \cdots, v_n\}$.

Lemma 3.2 [8] *Suppose that the kernel function is defined by* (5). *Then*

$$f_1''(\alpha) \leqq \frac{(1 + \kappa)\sigma^2}{2}\psi''(v_{\min} - \alpha\sigma\sqrt{1 + 2\kappa}).$$

Since $f_1(0) = 0$ and $f_1'(0) = -\frac{\sigma^2}{2}$, by Lemma 3.2,

$$f(\alpha) \leqq f_1(\alpha) := f_1(0) + f_1'(0)\alpha + \int_0^\alpha \int_0^\xi f_1''(\zeta)d\zeta d\xi$$
$$\leqq f_2(\alpha) := f_1(0) + f_1'(0)\alpha + \frac{(1+\kappa)\sigma^2}{2}\int_0^\alpha \int_0^\xi \psi''(v_{\min} - \zeta\sigma\sqrt{1+2\kappa})d\zeta d\xi.$$

Note that $f_2(0) = 0$. Furthermore, since $f_2'(\alpha) = -\frac{\sigma^2}{2} + \frac{\sigma\sqrt{1+2\kappa}}{2}\left(\psi'(v_{\min}) - \psi'(v_{\min} - \alpha\sigma\sqrt{1+2\kappa})\right)$, we have $f_2'(0) = -\frac{\sigma^2}{2}$ which is the same value of the $f_1'(0)$, and $f_2''(\alpha) = \frac{(1+\kappa)\sigma^2}{2}\sigma^2\psi''(v_{\min} - \alpha\sigma\sqrt{1+2\kappa})$, which is increasing on $\alpha \in [0, \frac{v_{min}}{2\sigma\sqrt{1+2\kappa}})$. Using $f_1'(0) = f_2'(0)$ and $f_1''(\alpha) \leqq f_2''(\alpha)$, we can easily check that

$$f_1'(\alpha) = f_1'(0) + \int_0^\alpha f_1''(\xi)d\xi \leqq f_2'(\alpha).$$

This relation gives that

$$f_1'(\alpha) \leqq 0, \quad \text{if } f_2'(\alpha) \leqq 0.$$

To compute the feasible step size α such that the proximity measure is decreasing when we take a new iterate for fixed μ, we want to calculate the step size α which satisfies that $f_2'(\alpha) \leqq 0$ holds with α as large as possible. Since $f_2''(\alpha) > 0$, that is, $f_2'(\alpha)$ is monotonically increasing at α, the largest possible value at α satisfying $f_2'(\alpha) \leqq 0$ occurs when $f_2'(\alpha) = 0$, that is,

$$-\psi'(v_{min} - \alpha\sigma\sqrt{1+2x}) + \psi'(v_{min}) = \frac{\sigma}{\sqrt{1+2\kappa}}. \tag{9}$$

Since $\psi''(t)$ is monotonically decreasing, the derivative of the left hand side in (9) with respect to $v_{\min}$ is

$$-\psi''(v_{\min} - \alpha\sigma\sqrt{1+2\kappa}\,) + \psi''(v_{\min}) < 0.$$

So, the left hand side in (9) is decreasing at $v_{\min}$. This implies that if $v_{\min}$ gets smaller, then α gets smaller with fixed σ. Note that

$$\sigma = \|\nabla\Psi(v)\| = \sqrt{\sum_{i\in I}\psi'(v_i)^2} \geqq |\psi'(v_{\min})| \geqq -\psi'(v_{\min})$$

and the equality is true if and only if $v_{\min}$ is the only coordinate in v which is different from 1 and $v_{\min} < 1$, that is, $\psi'(v_{\min}) < 0$.
Hence, the worse situation for the largest step size occurs when $v_{\min}$ satisfies

$$-\psi'(v_{\min}) = \sigma. \tag{10}$$

In that case, the largest α satisfying (9) is minimal. For our purpose, we need to deal with the worse case and so we assume that (10) is hold.

From now on, we denote that $\rho : [0,\infty) \to (0,1]$ is the inverse function of the restriction of $-\psi'(t)$ in the interval $(0,1]$. Then, (10) implies

$$v_{\min} = \rho(\sigma). \tag{11}$$

By using (9) and (10), we immediately obtain

$$-\psi'(v_{\min} - \alpha\sigma\sqrt{1+2\kappa}) = \left(1 + \frac{1}{\sqrt{1+2\kappa}}\right)\sigma.$$

By the definition of ρ,

$$v_{\min} - \alpha\sigma\sqrt{1+2\kappa} = \rho\left(\left(1 + \frac{1}{\sqrt{1+2\kappa}}\right)\sigma\right).$$

By applying (11), the largest step size α of the worse case is given as follows:

$$\alpha^* = \frac{\rho(\sigma) - \rho\left((1 + \frac{1}{\sqrt{1+2\kappa}})\sigma\right)}{\sigma\sqrt{1+2\kappa}}. \tag{12}$$

For the purpose of finding an upper bound of $f(\alpha)$, we need a default step size $\bar{\alpha}$ that is the lower bound of the α^* and consists of σ.

Lemma 3.3 [8] *Let $\sigma \geqq 1$. Then, for $0 < t < \rho(2\sigma)$,*

$$\psi''(t) \leqq 1 + 3\sigma(1 + pq + q)(1 + \frac{1}{p}\log 3\sigma)^{\frac{q+1}{q}}.$$

Theorem 3.1 [8] *Let α^* be as defined in* (12). *Then*

$$\alpha^* \geqq \frac{1}{1+2\kappa} \cdot \frac{1}{1 + 3\sigma\,(1 + pq + q)\left(1 + \frac{1}{p}\log 3\sigma\right)^{\frac{q+1}{q}}}.$$

For using $\bar{\alpha}$ as the default step size in the Algorithm, define the $\bar{\alpha}$ as follows

$$\bar{\alpha} = \frac{1}{1+2\kappa} \cdot \frac{1}{1 + 3\sigma\,(1 + pq + q)\left(1 + \frac{1}{p}\log 3\sigma\right)^{\frac{q+1}{q}}}. \tag{13}$$

3.2 Iteration bound

We need to count how many inner iterations are required to return to the situation where $\Psi(v) \leqq \tau$ after a μ-update. We denote the value of $\Psi(v)$ after μ-update as Ψ_0; the subsequent values in the same outer iteration are denote as Ψ_k, $k = 1, \cdots$. If K denotes the total number of inner iterations in the outer iteration, we then have

$$\Psi_0 \leqq L = \mathcal{O}(n), \quad \Psi_{K-1} > \tau, \quad 0 \leqq \Psi_K \leqq \tau$$

and according to Theorem **??**,

$$\Psi_{k+1} \leqq \Psi_k - \frac{1}{1+2\kappa} \cdot \frac{1}{2 + 6\sqrt{2}\,(1 + pq + q)\left(1 + \frac{1}{p}\log 3\sqrt{2\Psi_0}\right)^{\frac{q+1}{q}}} \Psi_k^{\frac{1}{2}}.$$

At this stage we invoke the following lemma from Lemma 14 in [13] without proof.

Lemma 3.4 [13] *Let t_0, $t_1, \cdots, t_K$ be a sequence of positive numbers such that*

$$t_{k+1} \leqq t_k - \beta t_k^{1-\gamma}, \quad k = 0, 1, \cdots, K-1,$$

where $\beta > 0$ and $0 < \gamma \leqq 1$. Then

$$K \leqq \lfloor \frac{t_0^\gamma}{\beta\gamma} \rfloor.$$

Letting $t_k = \Psi_k$, $\beta = \frac{1}{1+2\kappa} \cdot \frac{1}{2+6\sqrt{2}(1+pq+q)\left(1+\frac{1}{p}\log 3\sqrt{2\Psi_0}\right)^{\frac{q+1}{q}}}$ and $\gamma = \frac{1}{2}$, we can get the following lemma from Lemma 3.4.

Lemma 3.5 [8] *Let K be the total number of inner iterations in the outer iteration. Then we have*

$$K \leqq 2(1+2\kappa)\left(2 + 6\sqrt{2}\,(1 + pq + q)\left(1 + \frac{1}{p}\log 3\sqrt{2}\sqrt{\Psi_0}\right)^{\frac{q+1}{q}}\right)\Psi_0^{1/2},$$

where Ψ_0 is the result of the after the $\mu-$update in outer iteration.

Now we estimate the main result which is the total complexity for our algorithm.

Theorem 3.2 [8] *If $\tau \geq 1$, the total number of iterations is not more than*

$$\left\lceil 2(1+2\kappa)\left(2+6\sqrt{2}\,(1+pq+q)\left(1+\frac{1}{p}\log 3\sqrt{2}\sqrt{\Psi_0}\right)^{\frac{q+1}{q}}\right)\Psi_0^{1/2}\right\rceil\left\lceil\frac{1}{\theta}\log\frac{n}{\epsilon}\right\rceil.$$

Since $\Psi_0^{1/2} = \mathcal{O}(\sqrt{n})$, if we take $p = \mathcal{O}(\log n)$ and $q = 1$, then we can get the best known upper bound for the total number of inner iterations in the outer iteration is

$$\mathcal{O}((1+2\kappa)\sqrt{n}\log n).$$

Also, we take for θ a constant (not depending on n), namely $\frac{1}{\theta} = \Theta(1)$. With $\tau = \mathcal{O}(n)$, the best complexity of the primal-dual interior-point method for $P_*(\kappa)$ linear complementarity problem based on our new proximity function with $p = \log n$ and $q = 1$, is given by

$$\mathcal{O}\Big((1+2\kappa)\sqrt{n}\log n\log\frac{n}{\epsilon}\Big).$$

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